Educational Signaling under Different Education Systems*

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Abstract

We consider a two-period signaling model in which an informed worker has to decide whether she invests in education or participates in the labor market in the first period. When the rate at which the cost of education decreases with the worker´s productivity is sufficiently high (low), the worker´s incentives to invest in education become stronger (weaker) when the worker is more patient, when future prospects in the labor market are better, or when the cost of education decreases. Those results are robust to the worker´s risk preferences and to the specification of the prior distribution function of worker´s productivities.

Keywords: Education, risk preferences, selective educational system, separating equilibrium, signaling.

JEL Classification: C72, C79, D82, D83, I21, J01.

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1 Introduction

In this article, we introduce a theoretical model in order to analyze the effects of changes in the labor market conditions on workers´ incentives to invest in education. Specifically, we consider a two-period signaling game in which a worker has private information on her productivity and decides whether she invests in education or not in the first period. If the worker does not invest in education, she will participate in the labor market in both periods, but if she invests in education, she will only work in the second period because she abandons the labor market during the investment period. When people decide whether they invest in education or not, they do not know the wages they will receive after obtaining their diploma because the labor market conditions in the future are uncertain. In order to introduce this real-world feature into the model, we assume that the monetary value of the worker´s productivity will depend on the price at which the product can be sold in the market, and the worker will have to choose her level of investment in the first period without knowing the price of the product in the second period. Additionally, the positive correlation between the prices of the product in both periods will allow workers to use the current labor market conditions in order to predict those conditions in the future.

In this setting, we focus on a separating equilibrium in which only a worker with an ability greater than a certain threshold invests in education and this equilibrium may arise under two types of assumptions. First, we analyze a selective educational system in which the cost of education decreases sufficiently with the worker´s ability and in this context, the separating equilibrium is obtained because the worker uses the educational investment as a signal of her productivity. Second, we consider a non-selective educational system in which the rate at which the cost of education decreases with the worker´s ability is sufficiently low, and in this setting, the separating equilibrium is also obtained, but now the main reason why the worker with a low ability does not invest in education is the high opportunity cost of that investment.

In our selective educational system, we obtain that the worker´s incentives to invest in education will become stronger when the worker is more patient, when the expected future price is greater or when the cost of education goes down. Interestingly, in our non-selective educational system, an increase in the worker´s level of patience, a rise in the expected future price and a lower cost of education will lead to a new separating equilibrium in which fewer worker´s types invest in education. In this new equilibrium, those worker´s types with the lowest ability among educated types decide to work instead of investing in education and consequently, the pool of non-educated types has a greater expected ability and employers pay a greater wage to those uneducated types in equilibrium. For this reason, the opportunity cost of education, which is the wage paid to uneducated types in the first period, goes up and compensates the greater profit from education in the new equilibrium. Finally, we found that these results are robust to the specification of the prior distribution of worker´s abilities and to the worker´s risk preferences.

In the selective educational system considered, an exogenous rise in the price of the product in period zero will lead to an increase in wages among non-educated workers who participate in the labor market in that period. When the positive correlation between the prices in periods zero and one is sufficiently low, those greater wages in period zero will mainly cause a greater opportunity cost of schooling and the worker´s incentives to invest in education will be weaker. These results might explain previous empirical literature. For example, Hillman and Orians (2013) show that college enrollment is countercyclical, which suggests that the lower the wages, the greater the number of students who enroll in college. Similarly, Petrongolo and San Segundo (2002), Dellas and Sakellaris (2003), Giannelli and Monfardini (2003) and Tumino and Taylor (2015) show evidence suggesting that greater unemployment rates among youngsters cause a lower opportunity cost of education and a greater probability that they stay at school or university. However, when the positive correlation between the prices in periods zero and one is sufficiently high in our model, an exogenous increase in the price of the product in period zero will give rise to greater wages in that period, which will lead to much greater prices and wages in period one. As a result, the expected return to education will go up and consequently, the worker´s incentives to invest in education will be stronger. These results explain some evidence about the relationship between unemployment rates and educational investment. For instance, Petrongolo and San Segundo (2002), showed that higher unemployment rates among adults predict lower returns to education, which causes a lower probability of being enrolled into the schooling system. Similarly, Gilpin et al. (2015) found that enrollment and degree completion in for-profit colleges is positively related to employment growth and wages in related occupations.

Although a meta-regression analysis conducted by Havranek *et al.* (2018) concluded that researchers have shown positive effects of a rise in tuition fees on the demand for education less often than they should, empirical literature of the effect of an exogenous reduction in the cost of college on the enrollment and degree completion is mixed. For example, McPherson and Schapiro (1991) obtained a positive relationship between the net cost of education and enrollment among middle and upper-income students and a negative relationship among low-income individuals. Furthermore, when McPherson and Schapiro (1991) estimated their regressions for private and public universities separately, their results showed that an increase in the cost of education at private institutions reduced the enrollment rate among low and medium-income students, but does not affect the enrollment rate significantly among high-income students. At public institutions, which are much less costly than private colleges, an increase in the net cost of education did not affect the enrollment rate significantly among low-income students and increased the enrollment rate among medium and high-income students. Likewise, Bruckmeier and Wigger (2014) found a negative effect of the introduction of academic fees on the enrollment rate in some German states, but they found a positive effect in those states in which students were eligible to non-interest-bearing loans, in those in which a high percentage of students do not pay tuition fees due to fee exemptions, and in some rich states where the real price of tuition fees was low. Similarly, Castleman and Long (2016) obtained a positive causal impact of grants that reduce the cost of education on college access and graduation, while Cohodes and Goodman (2014) showed that a tuition grant program that reduces the cost of college reduces the spending on higher education and the probability of degree completion. Interestingly, Cohodes and Goodman (2014) found that direct relationship between the cost of education and educational achievement in colleges in which the total cost of education, including fees, room, board and books, is much lower than in other colleges. Additionally, those students who received the grant had a high income and got better grades at school than those who did not receive the grant. Using all that empirical literature, we conclude that there is a negative impact of an increase in the cost of education on enrollment and graduation rates in costly educational institutions and among low-income students, whereas the opposite occurs in those educational settings where students bear a lower cost of education and/or they belong to high-income classes. Our theoretical results suggest some explanation for that evidence. In particular, when there is an exogenous reduction in the cost of education, we obtain that the worker´s incentives to invest in education become stronger in the selective educational setting, but the opposite occurs in the non-selective setting.

We consider a signaling game with only two possible messages (levels of education). Although there are more than two levels of education in the real world, we can interpret our game as a model of the decision about completing an additional level of education or starting to work. In addition to this, our model is more general than it may seem at first sight. In models of entry deterrence, an incumbent has private information on its cost and makes a decision in order to signal its cost to a potential entrant (Milgrom and Roberts, 1982). If that decision is an investment in order to increase the incumbent´s production capacity that requires interrupting the economic activity for one period, there will also be an opportunity cost of that decision which is the profit lost during the investment period. Similarly, in models of conspicuous consumption, a consumer has private information on his income or wealth and uses the consumption of a "conspicuous" good in order to signal that private information to an audience (Ireland, 1994). If that consumer has to save some money for one period in order to buy an expensive "conspicuous" good, he may need to stop attending social events and participating in other signaling activities during the saving period, in which case there will also be an opportunity cost of the signal.

This article is organized as follows. In the next section, we briefly describe the contribution of our model to previous literature. In the third section, we present the model. In the fourth, we obtain our main results with a risk-neutral worker and under the assumption of a uniform prior distribution of worker´s abilities, whereas we analyze the robustness of those results to different assumptions about the worker´s risk preferences and to a different specification of the prior distribution of abilities in the fifth and sixth sections, respectively. Finally, we summarize the main conclusions in the seventh section and the proofs of all lemmas and propositions are included in the appendix.

2 Literature review

Since the seminal paper written by Spence (1973), theoretical models on the use of education as a signal of workers´ productivity in the labor market have proliferated. For example, Spence (1974) compares the results of the model when employers compete for hiring workers, when the employer is a monopsonist, in the efficient case, or with the maximization of different social welfare functions. Likewise, Spence (1976) analyzes what happens when employers offer two types of jobs and compete with respect to the signaling prerequisites for jobs as well as with salaries. Furthermore, Forges (1990) studies the effects of adding costless communication to an educational signaling model, Bedard (2001) adds a set of workers with constrained access to university, Frazis (2002) considers a model in which workers are unsure of their ability and Feltovich *et al.* (2002) introduce a model in which employers receive a noisy signal of workers´ productivity. Similarly, Spence (2002) also extends his seminal work by assuming that education increases the worker´s productivity, by introducing some companies which may learn the worker´s productivity at a cost, or by allowing the cost of education to increase with the worker´s productivity. Moreover, Gallice (2009) introduces a signaling model with two independent cohorts of workers who play the game only once, and some of those workers´ utility depend on the difference between their educational choice and the average education chosen by all the individuals belonging to the same cohort. Finally, Daley and Green (2014) develop a signaling model in which firms observe the worker´s level of education and their grade, which is a random variable, Perri (2019) considers a model in which workers may be employed in two sectors: a secondary sector where a worker´s productivity is the same regardless of her ability and a primary sector where that productivity is positively correlated with ability, and Jungbauer and Waldman (2023) study what happens when the employer cannot observe the educational level chosen by the worker, but that worker may send a true or false message about that education level to the employer, and the cost of that message depends on the worker´s level of honesty, which is private information.

In those models, workers choose a level of education and receive the returns to their educational investment in the same period and consequently, they do not take into account the opportunity cost generated by the wages lost during the educational period. For this reason, previous literature on educational signaling cannot explain the interaction between the labor market outcomes and workers´ incentives to use their level of education as a signaling device.

Although a few researchers have studied some interactions between the labor market conditions and workers´ incentives to signal through education, they have focused on the relationship between the use of education as a signal and current employers´ decisions about retaining or promoting their employee to a better paid job when retention or promotion may signal workers´ productivity in the labor market (Waldman 1984, 1990, 2016). These models also assume that workers receive their wages in the same period regardless of the level of education they choose and consequently, they ignore the opportunity cost of education during the investment period.

In this article, we introduce several changes into previous models. First, we consider a two-period signaling game instead of a one-shot game. Second, the cost of education will not only include the monetary cost in our model, but also the opportunity cost of education which is the wage lost during the investment period. Finally, we will introduce some uncertainty into the model because the worker does not know the price at which she will sell her physical productivity after obtaining the diploma and consequently, education is a risky decision in our model.

Our model shares some similarities with that introduced by Kurlat and Scheuer (2021), in which they assume that firms can directly observe imperfect information on workers´ types and the quality of that information is heterogeneous across firms. Due to the introduction of those assumptions, they found that signaling decreases if the cost is higher, if the demand for workers increases, or if firms´ expertise improves. In our model, we also analyze the effects of an exogenous change in the cost of signaling (cost of education). Furthermore, an exogenous change in the price of the product in period zero in our model is equivalent to a change in the demand for workers in their model. Their results about the effects of an exogenous change in the cost of the signal on signaling are similar to those obtained in our model when the cost of education decreases sufficiently with the worker´s ability. However, the introduction of the opportunity cost of education allows us to obtain just the opposite relationship between signaling and that parameter when the relation between the cost of education and the worker´s ability is weaker. Besides, due to the introduction of correlation between the prices in both periods, our results suggest that a change in the demand for labor today may affect the educational investment differently depending on that correlation. Additionally, in their model, education is a risky decision because equally productive and educated workers may receive different wages. The reason for this risk is that firms imperfectly evaluate workers´ productivities. In our model, the workers´ educational decision is risky because the conditions in the labor market after obtaining the diploma are uncertain. Lastly, unlike Kurlat and Scheuer´s model, we also analyze what happens when the worker is not risk neutral and when the prior distribution of worker´s abilities is not uniform.

3 Model

A worker has private information on her productivity, $t \in T = [t_0, t_n]$, where $0 < t_0 < t_n$. We assume that the prior distribution of this productivity is represented by a uniform distribution function and this is common knowledge. After observing her type, the worker chooses one of two possible levels of education: $e \in M = \{e_0, e_1\}$.

If the worker chooses e_0, it means that she does not study at the university and is employed in a company in periods 0 and 1. For simplicity, the cost of this level of education is assumed to be equal to zero. In order to make the model as simple as possible, we will assume that companies are short-lived[1] and therefore those companies that hire workers in period zero are different from those hiring workers in period 1. As companies compete a la Bertrand for hiring the worker, they will pay a wage equal to $P_0 E(t|e_0)$ and $P_1 E(t|e_0)$ to an uneducated worker in periods 0 and 1 respectively, where $E(t|e_0)$ is the expected physical productivity among those workers who chose e_0 and P_t is the price at which each unit of product can be sold in the market in period $t \in \{0,1\}$. In period zero, when the worker decides whether she invests in education or not, she does not know the price of the product in period one and we assume that it follows a simple process: $P_1 = \rho P_0 + \varepsilon$, where $\rho > 0$ represents the

[1] In the online appendix, we extend the model to a setting in which employers may learn the worker´s productivity after one period of labor experience.

positive correlation between the prices in both periods and ε is a random variable whose density function is $f(\cdot)$ and it represents fluctuations of the price around its long-run tendency. This density function is continuous and differentiable and is common knowledge. Moreover, the prior distribution of worker´s types and the distribution of ε are statistically independent. We assume that the support of ε is a closed interval: $\varepsilon \in [\bar{\varepsilon} - \Delta, \bar{\varepsilon} + \Delta]$. In order to avoid negative prices in period 1, we assume that $\Delta < \rho P_0 + \bar{\varepsilon}$.

Finally, when the worker chooses e_1, she will incur a cost of that level of education in period 1: $c(t, e_1)$. This cost increases with the level of education that the worker has to achieve in order to get the diploma, that is, $\frac{\partial c(t,e_1)}{\partial e_1} > 0$. In other words, , e_1 is a parameter that measures the value of the monetary cost of education that does not depend on the worker´s ability such as the tuition fees, the cost of books, etc. Therefore, the greater e_1, the greater the monetary cost of education in our model. As usual in models of educational signaling, we assume that the cost of education decreases with the worker´s type, that is, $\frac{\partial c(t,e_1)}{\partial t} < 0$. This relationship arises because there are components of the monetary cost of education that depend on the worker´s ability, such as the monetary value of the psychological cost of education, and part of the tuition fees and the cost of accommodation because low-ability students will need more time to pass the subjects and will pay for those tuition fees and for accommodation for more years. If the worker chooses e_1, she will not work until completing the high level of education in period 1 and the competitive company will offer her a wage equal to the monetary value of her productivity, which is $P_1 E(t|e_1)$. As the price in period 1 is not known in period 0, the educational investment is risky.

Following standard notation, the discount factor will be represented by δ.

In this setting, we will obtain the perfect Bayesian equilibrium of the game, which satisfies the following conditions:

i. Each worker´s type will choose the level of education, e, that maximizes the discounted sum of utility levels: $u(t,e) = U[P_0 E(t|e_0) 1(e = e_0)] + \delta U[P_1 E(t|e) - c(t,e)]$, where $U(\cdot)$ is the worker´s utility function and $1(e = e_0)$ represents an indicator function, which is equal to one when the worker chooses the low level of education and zero otherwise.

ii. Given the level of education chosen by the worker, in each period the company will pay her a wage which is equal to the expected monetary value of productivity among those worker´s types with that level of education in equilibrium: $P_t E(t|e)$.

iii. The company´s beliefs must be consistent with the Bayesian rule in the equilibrium path. For example, when those worker´s types lower than t^* choose e_0 and those types greater than or equal to t^* choose e_1 in a separating equilibrium, the probability assigned by the company to type t after observing an educational level of e is given by $\mu(t|e) = \begin{cases} \frac{1}{t^* - t_0} & if\ e = e_0 \\ \frac{1}{t_n - t^*} & if\ e = e_1 \end{cases}$.

As shown by Mailath (1988), when the set of possible productivities of a worker forms a continuum, workers´ behavior in a separating equilibrium is completely determined because there is a unique separating equilibrium. In this article, we analyze the effects of a change in the labor market conditions on the separating equilibrium obtained.

4 Risk-neutral worker

In this section, we analyze the workers´ incentives to invest in education when they are risk-neutral.

The worker´s utility function, $U: \mathbb{R} \to \mathbb{R}$, is differentiable and strictly increasing, that is $U'(x) > 0\ \forall x \in \mathbb{R}$. In order to simplify the model, the worker´s utility from no money is normalized to be equal to zero, that is, $U(0) = 0$. Additionally, due to risk-neutrality[2], $U''(x) = 0 \; \forall x \in \mathbb{R}$.

In this model, the level of education may reveal some information on the worker´s productivity in the labor market for two reasons. First, the educational investment has an opportunity cost, which is equal to the wage lost during the investment period, and that wage lost is the wage received by non-educated types in equilibrium, which increases with the expected productivity of those non-educated types. As a result, when the expected type among non-educated workers is sufficiently high, only the highest worker´s types invest in education because their cost is lower than the additional wage received by educated workers. Second, as the cost of education decreases with the worker´s type, high-ability workers may invest in education in order to signal their higher productivity in the labor market.

In this article, we analyze what happens in two types of educational systems. First, we consider a scenario in which the rate at which the cost of education decreases with the worker´s productivity is sufficiently low. This setting arises in those contexts with undemanding universal educational systems in which even low ability students may complete a high level of education by putting a bit more effort than high ability students. This setting may also appear in educational institutions dominated by high-income students because rich low-ability students might find it easy to spend more money on support tuition and additional help in order to complete high levels of education. Second, we consider a scenario in which the rate at which the cost of education decreases with the worker´s productivity is sufficiently high. This set-up is more likely to appear in those contexts with selective educational systems in which only high-ability students may complete the highest level of education.

<u>Non-selective educational system</u>. In this setting, we assume that the lowest type of worker will prefer the high to the low level of education when the employer identifies her type after

[2] In the next section, we analyse the robustness of our results to the worker´s risk-preferences.

observing the low level and pays the ex-ante expected wage after observing the high. This assumption can be written as:

Assumption 1. $U(t_0P_0) + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U[t_0(\rho P_0 + \varepsilon)]f(\varepsilon)d\varepsilon < \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left[\frac{t_0+t_n}{2}(\rho P_0 + \varepsilon) - c(t_0, e_1)\right] f(\varepsilon)d\varepsilon.$

This assumption is satisfied when the cost of education among the lowest types of worker is not too high. Similarly, we assume that the highest type of worker prefers the low to the high level of education when the wage among non-educated workers is the monetary value of the ex-ante productivity and the wage among educated workers is equal to the value of the productivity of the highest type, that is:

Assumption 2. $U\left(\frac{t_0+t_n}{2}P_0\right) + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left[\frac{t_0+t_n}{2}(\rho P_0 + \varepsilon)\right] f(\varepsilon)d\varepsilon > \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U[t_n(\rho P_0 + \varepsilon) - c(t_n, e_1)]f(\varepsilon)d\varepsilon.$

This assumption holds when the cost of education among the highest-ability workers is not too low.

Both assumptions 1 and 2 imply that the cost of the high level of education does not decrease with the worker´s type too much. This idea is reinforced by introducing the next assumption:

Assumption 3. $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < \frac{P_0}{2\delta}\ \forall t \in [t_0, t_n].$

In this scenario, there may be a pooling equilibrium in which all worker´s types choose the low level of education and another pooling equilibrium in which all types choose the high level and those equilibria may survive standard refinements[3]. However, as we are interested in the use of education as a signaling device, we focus on a separating equilibrium in which only those worker´s types greater than a certain threshold invest in education.

[3] The online appendix includes an analysis of pooling equilibria in our setting.

Selective educational system. In this scenario, we assume that the lowest type of worker will prefer the low to the high level of education when the employer identifies her type after observing the former and pays the ex-ante expected wage after observing the latter. This assumption can be written as:

Assumption 1´. $U(t_0P_0) + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U[t_0(\rho P_0 + \varepsilon)]f(\varepsilon)d\varepsilon > \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left[\frac{t_0+t_n}{2}(\rho P_0 + \varepsilon) - c(t_0, e_1)\right] f(\varepsilon)d\varepsilon.$

This assumption is satisfied when $c(t_0, e_1)$ is sufficiently high. Likewise, we assume that the highest type will prefer the high to the low level of education when the employer identifies her type after observing the high level of education and pays the monetary value of the ex-ante expected productivity after observing the low level, that is:

Assumption 2´. $U\left(\frac{t_0+t_n}{2}P_0\right) + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left(\frac{t_0+t_n}{2}(\rho P_0 + \varepsilon)\right) f(\varepsilon)d\varepsilon < \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U[t_n(\rho P_0 + \varepsilon) - c(t_n, e_1)]f(\varepsilon)d\varepsilon.$

This assumption holds providing that $c(t_n, e_1)$ is sufficiently low.

Now assumptions 1´ and 2´ imply that the cost of education decreases sufficiently with the worker´s type, and this idea is reinforced by the following assumption:

Assumption 3´. $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > \frac{P_0}{2\delta}\ \forall t \in [t_0, t_n].$

In this scenario, although there may be pooling equilibria as well, they may be discarded by standard refinements, such as D1.

A Simple Example of the Non-selective Educational System. As the non-selective scenario is not standard and will lead to striking results, I will use an example to illustrate this context. Imagine that $T = [0,10]$ and the prior distribution of worker´s types is uniform with that support. Additionally, $\delta = 1$, $P_0 = 1$ and the distribution of ε is uniform with support

$[-0.5, 0.5]$, that is, $\bar{\varepsilon} = 0$ and $\Delta = 0.5$. Finally, $\rho = 1.1$, $c(t, e_1) = 2 - \frac{t}{10}$ and $U(x) = x$. This example satisfies assumptions 1-3 and there is a separating equilibrium in which those types lower than $t^* = 8.75$ choose e_0 and those greater than t^* choose e_1.

In this example, the employers should offer uneducated workers a wage equal to the monetary value of the expected physical productivity of types lower than t^* and offer educated workers a wage equal to the monetary value of the expected physical productivity of types greater than t^*. In particular, the wage paid to an uneducated worker would be 4.375 ($\frac{0+8.75}{2}$) in period zero and 4.8125 (4.375×1.1) would be the expected wage paid to an uneducated worker in period one. Similarly, the expected wage paid to an educated worker in period one must be equal to 10.3125 ($1.1 \times \frac{8.75+10}{2}$). With those wage offers, only those worker´s types greater than 8.75 will have incentives to invest in education and companies will be able to separate low from high ability workers.

In both scenarios, a separating equilibrium will arise. In that equilibrium, there will be a worker´s type, t^*, who will be indifferent between both levels of education, those types lower than t^* will choose e_0 and those types greater than t^* will choose e_1. Under the assumptions of both scenarios, this is the unique separating equilibrium with a threshold. In that equilibrium, the indifferent worker´s type, t^*, will satisfy the following equation:

$$U\left(\frac{t_0+t^*}{2}P_0\right) + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right) f(\varepsilon) d\varepsilon = \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left[\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right] f(\varepsilon) d\varepsilon \quad (1)$$

The left-hand side of this equation shows the expected utility obtained by those workers´ types who choose to work in period 0, which is equal to the sum of the discounted levels of utility obtained by those types of worker, whereas the right-hand side shows the expected utility obtained by those worker´s types who complete the high level of education in period

1, which is equal to the discounted expected utility obtained in that period from the wage received minus the cost of education.

Now, we perform some comparative statics analysis in our general model. Under the assumptions of the non-selective and selective educational scenarios, it is straightforward to see that there is a unique separating equilibrium. In that equilibrium, the effect of a change in the parameters of the model on the worker´s incentives to invest in education will depend on the scenario considered.

PROPOSITION 1. In the non-selective and selective educational systems, there exists a unique $t^* \in [t_0, t_n]$ *such that there is a separating equilibrium in which those worker´s types lower than* t^* *choose* e_0 *and those greater than* t^* *choose* e_1*. In the non-selective educational system,* t^* *increases with* δ *and with* ρ *and decreases with* e_1*. In the selective setting,* t^* *decreases with* δ *and with* ρ *and increases with* e_1*.*

Under the assumptions of our settings, there is a unique separating equilibrium and it is easy to understand the intuition of the comparative statics results. In the non-selective educational system, the signaling role of education is weak and the opportunity cost plays the main role in the model. In our separating equilibrium, t^* is the worker´s type for which the additional wage received from education is equal to the total cost of education, which includes the monetary cost and the opportunity cost. In this setting, when the expected profit from education goes up because of a greater discount rate (greater δ), a greater expected price of the product in period 1 (greater ρ), or a lower cost of education (lower e_1), the indifferent type´s opportunity cost of education should go up in order to compensate the rise in the expected future profit. For this reason, the indifferent worker´s type must increase so that the average productivity of those worker´s types who do not invest in education is higher. In other words, when the expected profit from investing in education increases, another equilibrium arises in which the lowest-ability types among educated workers in the initial equilibrium decide to work instead of studying and consequently, the expected productivity of the pool of uneducated workers and the wage received by them go up. As a result, the opportunity cost of education is higher in the new equilibrium compensating the additional profit from the educational investment.

However, in the selective scenario, the role of education as a signaling device outweighs the effect of its opportunity cost. As a result, when δ or ρ goes up, or when e_1 goes down, the discounted profit from educational signaling increases. Therefore, more worker´ types invest in education (t^* goes down).

In our model, the worker makes her educational decision in period 0 and a change in wages in that period affects her incentives to invest in education. As the wage paid in period 0 depends on the price of the product in that period, we analyze the effect of an exogenous change in that price on the worker´s incentives to invest in education in our next proposition.

PROPOSITION 2. There exists a certain threshold, $K \in \mathbb{R}^+$*, such that* $\frac{\partial t^*}{\partial P_0} \lesseqgtr 0$ *when* $\rho \lesseqgtr K$ *in the non-selective educational scenario and* $\frac{\partial t^*}{\partial P_0} \gtreqless 0$ *when* $\rho \lesseqgtr K$ *in the selective educational scenario.*

In this model, the effect of an improvement in the labor market today on the worker´s incentives to study will depend on the effect of that improvement on the expected prospects of the future labor market. In particular, when a rise in the current wage barely affects the wage expected by the worker in the future ($\rho < K$), a greater wage today will mainly imply a greater opportunity cost of studying. On the contrary, when a greater wage today increases significantly the expected wage in the future ($\rho > K$), a greater wage today will mainly imply a greater expected profit from the educational investment. Then, in the selective scenario, a rise in P_0 will lead to an increase in the opportunity cost of education and a lower proportion of educated types (greater t^*) when $\rho < K$ and to a greater expected profit from education and a greater proportion of educated types (lower t^*) when $\rho > K$. In the non-selective scenario, when $\rho < K$, the increase in the opportunity cost of education caused by a rise in

P_0 has to be compensated with a reduction in that opportunity cost for the indifferent type, which gives rise to a lower proportion of educated types (lower t^*). Similarly, when $\rho > K$, the increase in the expected profit from education caused by a rise in P_0 has to be compensated with a greater opportunity cost of education for the indifferent type and for this reason, t^* increases.

5 Results with different risk preferences

In this section, we analyze what happens when the worker is not risk-neutral. Now, the worker´s utility function is $U(x)$, which is continuous and twice differentiable, and we assume that $U'(x) > 0 \; \forall x \geq 0$. Once again, as a normalization, we assume that $U(0) = 0$ and therefore $U(x) > 0 \; \forall x > 0$.

In this section, the worker may be risk-averse, in which case $U''(x) < 0 \; \forall x > 0$, or risk-lover: $U''(x) > 0 \; \forall x > 0$.

As in the previous section, we consider the non-selective and selective scenarios:

Non-selective Educational System. Assumptions 1 and 2 are the same as those described in section four and we substitute assumption 3 with the following condition:

Assumption 4. $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < \frac{P_0}{2\delta \overline{U}(t)} U'\left(\frac{t_0+t}{2} P_0\right) - \frac{1}{\overline{U}(t)} \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \frac{\rho P_0+\varepsilon}{2} \left\{U'\left[\frac{t+t_n}{2}(\rho P_0 + \varepsilon) - c(t,e_1)\right] - U'\left(\frac{t_0+t}{2}(\rho P_0 + \varepsilon)\right)\right\} f(\varepsilon)\, d\varepsilon \; \forall t \in [t_0, t_n]$.

Where $\overline{U}(t) = \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U'\left[\frac{t+t_n}{2}(\rho P_0 + \varepsilon) - c(t,e_1)\right] f(\varepsilon) d\varepsilon > 0$.

Selective Educational System. In this set-up, we impose assumptions 1´ and 2´ and substitute assumption 3´ with the following condition:

Assumption 4´. $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > \frac{P_0}{2\delta \overline{U}(t)} U'\left(\frac{t_0+t}{2}P_0\right) - \frac{1}{\overline{U}(t)}\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} \frac{\rho P_0+\varepsilon}{2}\left\{U'\left[\frac{t+t_n}{2}(\rho P_0+\varepsilon) - c(t,e_1)\right] - U'\left(\frac{t_0+t}{2}(\rho P_0+\varepsilon)\right)\right\} f(\varepsilon)\, d\varepsilon \ \forall t \in [t_0, t_n]$.

When the worker is risk-neutral, $\overline{U}(t) = \kappa = U'(x)\ \forall x$ and assumptions 4 and 4´ are equivalent to assumptions 3 and 3´ respectively.

Assuming that the prior distribution function of the worker´s productivity is uniform, $t \sim U[t_0, t_n]$, and that it is independent of the distribution of ε, once again, the worker´s type, t^*, who is indifferent between studying and not studying in a separating equilibrium will be given by equation (1).

Before obtaining our results, we define the following equilibrium ratio:

$$\theta^* = \frac{\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U'\left(\frac{t_0+t^*}{2}(\rho P_0+\varepsilon)\right) f(\varepsilon) d\varepsilon}{\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U'\left(\frac{t^*+t_n}{2}(\rho P_0+\varepsilon) - c(t^*,e_1)\right) f(\varepsilon) d\varepsilon} > 0 \qquad (2)$$

The numerator of this ratio is the expected marginal utility of the future income received by uneducated types in our equilibrium, while the denominator is the expected marginal utility of the future income received by educated types in that equilibrium. Thus, θ^* is the ratio of the expected marginal utilities of uneducated to educated types in our separating equilibrium. This ratio depends on the worker´s level of risk aversion or risk loving. For example, when the worker is risk neutral, $\theta^* = 1$. It also depends on the cost of the high level of education. For example, if the level of risk-aversion (risk-loving) is high and the cost of education is sufficiently low, the denominator is much lower (higher) than the numerator and $\theta^* > 1$ ($\theta^* < 1$). Finally, θ^* depends on the type of equilibrium obtained and the rate at which the cost of education decreases with the worker´s type. For instance, in an equilibrium in which only the most talented risk-averse workers study (t^* close to t_n) and the cost of education decreases significantly with the worker´s type, the denominator will be lower than the numerator and $\theta^* > 1$. Depending on this equilibrium ratio, we distinguish two types of separating equilibria:

Definition 1: *Equilibrium with overpaid educated workers*. We say that an equilibrium exhibits overpaid educated workers as long as $\theta^* > 1$.

Definition 2. Equilibrium with underpaid educated workers. We say that an equilibrium exhibits underpaid educated workers providing that $\theta^* < 1$.

When $\theta^* > 1$, the expected marginal utility from income in period 1 among uneducated types in equilibrium is greater than that expected marginal utility among educated types. Therefore, an increase in future income increases worker´s utility by less among educated than among uneducated workers. The opposite occurs when $\theta^* < 1$.

Another interesting equilibrium ratio is the ratio of expected marginal utilities of present and future income among uneducated workers:

$$\lambda^* = \frac{U'\left(\frac{t_0+t^*}{2}P_0\right)}{\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U'\left(\frac{t_0+t^*}{2}(\rho P_0+\varepsilon)\right) f(\varepsilon)d\varepsilon} > 0 \tag{3}$$

This is the marginal utility from current income among non-educated types divided by the expected marginal utility from future income among those types. Then, λ^* measures the relative value of the present in comparison to the value of the future among uneducated types in equilibrium. When $\lambda^* > 1$, the uneducated worker values the present more than the future in equilibrium and the opposite occurs when $\lambda^* < 1$.

Our next proposition shows the effects of changes in δ, e_1, ρ and P_0 on t^* when the worker is not risk neutral.

PROPOSITION 3. Regardless of whether the worker is risk averse or risk lover, we obtain the following results:

i. *In the non-selective educational scenario (under assumptions 1, 2 and 4),* t^* *increases with* δ *and decreases with* e_1*. In the selective educational scenario (under assumptions 1´, 2´ and 4´),* t^* *decreases with* δ *and increases with* e_1.

ii. *In an equilibrium with underpaid educated workers,* $\frac{\partial t^*}{\partial \rho} > 0$ *in the non-selective scenario and* $\frac{\partial t^*}{\partial \rho} < 0$ *in the selective setting.*

iii. *In an equilibrium with overpaid educated workers* $\frac{\partial t^*}{\partial \rho} \gtreqless 0$ if $t_n - t_0 \gtreqless (\theta^* - 1)(t_0 + t^*)$ in the non-selective setting, whereas $\frac{\partial t^*}{\partial \rho} \lesseqgtr 0$ if $t_n - t_0 \gtreqless (\theta^* - 1)(t_0 + t^*)$ in the selective educational environment.

iv. *There exists a certain threshold,* $K \in \mathbb{R}^+$*, such that when* $\rho < K$*, then* $\frac{\partial t^*}{\partial P_0} < 0$ *in the non-selective educational scenario and* $\frac{\partial t^*}{\partial P_0} > 0$ *in the selective educational.*

v. *When* $\rho > K$*,* $\frac{\partial t^*}{\partial P_0} \gtreqless 0$ if $t_n - t_0 \gtreqless \left(\theta^* \frac{\lambda^*}{\delta \rho} + \theta^* - 1\right)(t_0 + t^*)$ in the non-selective educational system, and $\frac{\partial t^*}{\partial P_0} \lesseqgtr 0$ if $t_n - t_0 \gtreqless \left(\theta^* \frac{\lambda^*}{\delta \rho} + \theta^* - 1\right)(t_0 + t^*)$ in the selective setting.

This proposition confirms all of our qualitative results with risk-neutrality with two exceptions. In particular, now the effects of a change in ρ or a change in P_0 when ρ is sufficiently high depend on the dispersion of the prior distribution of worker´s types, which can be measured by $t_n - t_0$, and on the relative value of future and present incomes and the relative value of future income among educated and uneducated workers in equilibrium. When ρ or P_0 goes up with a high value of ρ, then the future price of the product will increase and as a result, the worker´s future wage with or without education will go up. We focus on a risk-averse worker in order to illustrate our results, but the intuition of the effects of changes in ρ or P_0 is similar when the worker is risk-lover.

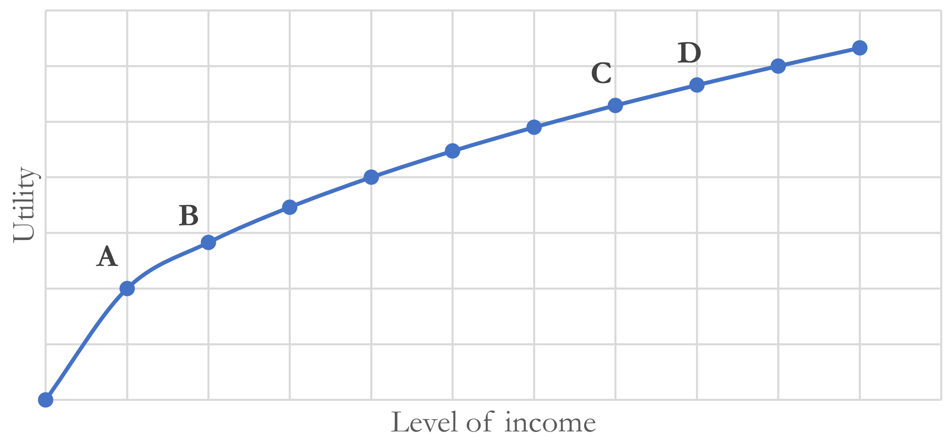

Figure 1. Risk Averse Worker

In our model, the wage in period one is uncertain because of the random component of the future price, ε. Moreover, the uncertain component of the future wage is greater among educated than among uneducated types in our separating equilibrium because $\left|\frac{t^*+t_n}{2}\varepsilon\right| > \left|\frac{t_0+t^*}{2}\varepsilon\right|$ $\forall\varepsilon \in [\overline{\varepsilon}-\Delta, \overline{\varepsilon}+\Delta]$, which implies that there an additional risk associated with education. When the worker is risk averse, she must receive a greater expected income (expected wage minus cost of education) in period 1 with education than without it. Otherwise, the worker would not give up the current wage in order to take the additional risk associated with education. Imagine that the income received by the uneducated worker in period one is given by the first coordinate of point A in Figure 1, whereas the income received by the educated worker is the first coordinate of point C. As the wage in period 1 is the monetary value of the expected physical productivity, it is greater among educated ($\frac{t^*+t_n}{2}P_1$) than among non-educated types ($\frac{t_0+t^*}{2}P_1$) in the separating equilibrium considered and the difference between those wages is proportional to $t_n - t_0$. For this reason, a greater price in period one will increase the wage among educated types by more than the wage among non-educated types in equilibrium. This effect strengthens the worker´s incentives to choose the high level of education. Additionally, as the expected income in period one is greater among educated than among non-educated workers in equilibrium, the same increase in the wage in period one will increase the non-educated worker´s utility level by more than that of the educated worker because the utility function is concave. In figure 1, if we increase the uneducated worker´s wage by a certain amount, we move from point A to point B and the increase in the utility level is greater than that generated by the same increase in the educated worker´s wage by moving from point C to point D. This effect gives the worker stronger incentives to choose the low level of education. Therefore, more worker´s types choose the high level of education (t^* decreases) when the first effect dominates the second ($t_n - t_0 > (\theta^* - 1)(t_0 + t^*)$ or $t_n - t_0 > \left(\theta^* \frac{\lambda^*}{\delta\rho} + \theta^* - 1\right)(t_0 + t^*)$), whereas fewer types invest in education (t^* increases) when the second effect outweighs the first ($t_n - t_0 < (\theta^* - 1)(t_0 + t^*)$ or $t_n - t_0 < \left(\theta^* \frac{\lambda^*}{\delta\rho} + \theta^* - 1\right)(t_0 + t^*)$) in the selective scenario. The opposite occurs in the non-selective setting for similar reasons to those shown in the previous section. In an equilibrium with underpaid educated workers, the first effect always dominates the second because an increase in the future wage causes a greater increase in the expected utility among educated workers by more than among non-educated workers.

There is only one difference between the effect of a change in ρ and the effect of a change in P_0. In particular, when ρ changes, there is only a change in the wage of period one, but when P_0 changes, there is also a change in the wage of period zero. If the uneducated worker´s marginal utility from an additional income in period zero is equal to zero, then $\lambda^* = 0$, in which case only the increase in the wage in period 1 matters and the effect of a change in ρ is the same as the effect of a change in the price in period zero. On the contrary, if λ^* is sufficiently high, it means that the worker values additional money today much more than money in the future, in which case a greater price in period zero will lead to stronger incentives to choose the low level of education in order to receive a wage today and as a result, fewer types will invest in education (t^* increases) in the selective educational system. Once again, the opposite occurs in the non-selective scenario for the same reasons as those shown in the previous section with risk neutrality.

6 Robustness to the specification of the prior distribution of worker´s types

In previous sections, we assumed that the worker´s type was drawn from a uniform distribution. In this section, we analyze the effect of specifying other types of distribution on the results obtained. In order to see the extent to which our results can be generalized, we will use the general relationship between the sender´s incentives to signal and the prior distribution of sender´s types obtained by Adriani and Sonderegger (2019) and Jewit (2004).

Now, let $g: T \rightarrow [0,1]$ denote the prior density function from which the sender´s type is drawn at the beginning of the signaling game we are describing. In our model, the signal can only take two values and the sender´s incentives to choose the high value is given by the following expression in the separating equilibrium considered:

$$\phi(t^*) = E(t|t > t^*) - E(t|t < t^*) \quad (4)$$

Where t^* is the worker´s type who is indifferent between the low and high values of the signal. In this setting, $\phi(t^*) > 0$ measures the worker´s incentive to invest in education.

We summarize the results obtained by Adriani and Sonderegger (2019) and Jewitt (2004) in the next lemma.

LEMMA 1. $\phi'(t^*)$ will depend on the shape of $g(t)$ as follows:

I. If g is an everywhere increasing (decreasing) function, then ϕ will be decreasing (increasing) everywhere.

II. *If* g *is strictly increasing and then decreasing (unimodal), then there exists* $t_m \in [t_0, t_n]$ *such that* ϕ *is strictly decreasing when* $t < t_m$ *and strictly increasing when* $t > t_m$. *Moreover, if* g *is symmetric, then* t_m *coincides with the mode of* g.

III. *If* g *is strictly decreasing and then increasing, then there exists* $t_M \in [t_0, t_n]$ *such that* ϕ *is strictly increasing when* $t < t_M$ *and strictly decreasing when* $t > t_M$. *Moreover, if* g *is symmetric, then* t_M *coincides with the anty-mode of* g.

The proof of this lemma can be found in Adriani and Sonderegger[4] (2019).

In this setting, when the worker is risk-neutral, the type who is indifferent between investing or not investing in education, t^*, will be given by:

$$U[E(t|t < t^*)P_0] + \delta U\big[E(t|t < t^*)\big(\rho P_0 + E(\varepsilon)\big)\big] = \delta U\big[E(t|t > t^*)\big(\rho P_0 + E(\varepsilon)\big) - c(t^*, e_1)\big] \quad (5)$$

Once again, we consider both scenarios previously described.

Non-selective Educational System. We assume that the following conditions will be satisfied:

Assumption 5. $U(t_0 P_0) + \delta U\big[t_0\big(\rho P_0 + E(\varepsilon)\big)\big] < \delta U\big[E(t)\big(\rho P_0 + E(\varepsilon)\big) - c(t_0, e_1)\big]$.

Assumption 6. $U[E(t)P_0] + \delta U\big[E(t)\big(\rho P_0 + E(\varepsilon)\big)\big] > \delta U\big[t_n\big(\rho P_0 + E(\varepsilon)\big) - c(t_n, e_1)\big]$.

Assumption 7. $\left|\frac{\partial c(t^*, e_1)}{\partial t^*}\right| < \frac{P_0}{\delta}\frac{\partial E(t|t < t^*)}{\partial t^*} - \frac{\partial \phi(t^*)}{\partial t^*}\big(\rho P_0 + E(\varepsilon)\big)\ \forall t^* \in [t_0, t_n]$.

Selective Educational System. We assume that the following conditions will be satisfied:

Assumption 8. $U(t_0 P_0) + \delta U\big[t_0\big(\rho P_0 + E(\varepsilon)\big)\big] > \delta U\big[E(t)\big(\rho P_0 + E(\varepsilon)\big) - c(t_0, e_1)\big]$.

Assumption 9. $U[E(t)P_0] + \delta U\big[E(t)\big(\rho P_0 + E(\varepsilon)\big)\big] < \delta U\big[t_n\big(\rho P_0 + E(\varepsilon)\big) - c(t_n, e_1)\big]$.

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[4] See their lemmas 1 and 2.

Assumption 10. $\left|\frac{\partial c(t^*,e_1)}{\partial t^*}\right| > \frac{P_0}{\delta}\frac{\partial E(t|t<t^*)}{\partial t^*} - \frac{\partial \phi(t^*)}{\partial t^*}\left(\rho P_0 + E(\varepsilon)\right) \forall t^* \in [t_0, t_n]$.

As in the previous section, we also assume that the worker´s utility from the minimum level of consumption in period 1 is positive, that is, $U(t_0 P_0) > 0$.

Now, we can determine the effect of a change in the parameters of the model on the worker´s incentives to invest in education.

PROPOSITION 4. For any prior distribution of sender´s types, t^* *increases with* δ *and with* ρ*, and decreases with* e_1 *in the non-selective scenario and* t^* *decreases with* δ *and with* ρ*, and increases with* e_1 *in the selective setting. Furthermore, there exists a certain threshold,* $K \in \mathbb{R}^+$*, such that* t^* *decreases (increases) with* P_0 *when* $\rho < K$ *(*$\rho > K$*) in the non-selective set-up, but* t^* *increases (decreases) with* P_0 *when* $\rho < K$ *(*$\rho > K$*) in the selective scenario.*

This proposition shows that our results are robust to the specification of the prior distribution of the worker´s types as long as our assumptions of the non-selective and selective scenarios are satisfied with that specification. Then, it will be interesting to determine the type of prior distribution functions of the worker´s productivities under which our scenarios arise.

Specifically, when the prior distribution of worker´s types is concentrated on the highest types or when $g(\cdot)$ is increasing ($\phi(\cdot)$ decreases everywhere as shown in lemma 1), then the non-selective scenario is more likely to appear. On the contrary, when the prior distribution of worker´s types is concentrated on the lowest types or when $g(\cdot)$ is decreasing ($\phi(\cdot)$ increases everywhere as shown in lemma 1), then the selective scenario is more likely to arise. Therefore, some sufficient conditions for each scenario are shown in our next lemma.

LEMMA 2. There exists $c_1 \in \mathbb{R}^+$ *such that the non-selective scenario will arise providing that* $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < c_1 \; \forall t \in [t_0, t_n]$ *and* $g(\cdot)$ *is increasing everywhere. Likewise, there exists* $c_2 \in \mathbb{R}^+$ *such* *that the selective scenario will arise as long as* $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > c_2 \; \forall t \in [t_0, t_n]$ *and* $g(\cdot)$ *is decreasing everywhere.*

This lemma shows that the non-selective scenario will be more likely to arise in those labor markets where most workers are highly productive and the educational system is universal. In fact, when the prior density function of worker´s types is increasing everywhere, an increase in the indifferent type leads to a greater expected wage among uneducated types in our separating equilibrium because the probability of the highest types in the pool of non-educated types is greater. As a result, when the indifferent type goes up, the expected wage among non-educated types goes up by more and the opportunity cost of education plays a more important role, which is what happens in the non-selective scenario. On the contrary, the selective environment will arise as long as most workers are less productive and the cost of education decreases with the worker´s type by more. In this case, a greater indifferent type increases the expected wage among non-educated types in equilibrium by less because the probability of the highest type in the pool of educated types is lower. For this reason, a rise in the indifferent type increases the opportunity cost of education by less and the signaling role of education is strengthened, which is what happens in the selective environment.

7 Conclusions

In this article, we analyze the effects of a change in the labor market conditions on people´s incentives to invest in education. In order to meet this goal, we extend Spence´s model of signaling by considering a two-period game in which a worker with private information on her productivity in the labor market has to decide whether she invests in education or not in the first period. Unlike the classical model, we assume that the worker cannot participate in the labor market during the investment period. Under these assumptions, we study the worker´s incentives to study in two different educational systems. First, we consider a universal system in which the rate at which the cost of education decreases with the worker´s ability is sufficiently low, and even low-ability workers can complete the high level of education by incurring a slightly higher cost. Second, we analyze a selective educational system in which the cost of education decreases with the worker´s ability significantly and low-ability workers find it prohibitively expensive to complete the high level of education.

In the universal educational system, the wage lost during the investment period is a key component of the cost of education. In this context, an equilibrium arises in which only those workers with an ability greater than a certain threshold invest in education. In this equilibrium, the wage received by uneducated workers is sufficiently high because sufficiently high types of worker do not invest in education. As a result, the wage lost by educated workers during the investment period is so high that low-ability workers do not have incentives to choose the high level of education. Consequently, the employer will be able to identify those workers with the highest productivity by using the educational credentials even when the cost of education slightly decreases with the worker´s ability.

Additionally, in the selective educational system, the signaling equilibrium will arise for different reasons. Now, high-ability workers will use a high level of education as a signal of their lower educational cost and higher productivity. As expected, when the worker is more patient, when prospects of the labor market conditions in the future improve, or when the cost of education goes down, the incentives to invest in education become stronger in this scenario. Interestingly, the opposite occurs in the universal educational system.

We also identify some prior distributions of the worker´s abilities under which the results obtained in the non-selective or the selective scenario are more likely to arise. Specifically, a prior distribution of worker´s productivities sufficiently concentrated on the highest types will lead to the effects observed in the non-selective scenario when the cost of education does not decrease with the worker´s ability by too much. Similarly, the results obtained in the selective scenario are more likely to appear in a labor market dominated by low-productivity workers when the cost of education decreases significantly with the worker´s ability.

Finally, we found that the relationship between the labor market conditions and workers´ incentives to invest in education are the same when the worker is not risk neutral or when the prior distribution of workers´ abilities is not uniform.

Appendix

Proof of Proposition 1. First of all, we rewrite equation (1) as the following function of the indifferent worker´s type, t^*:

$$F(t^*,\delta,P_0,\rho,e_1) = U\left(\frac{t_0+t^*}{2}P_0\right) + \delta\left\{\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left(\frac{t_0+t^*}{2}(\rho P_0+\varepsilon)\right) f(\varepsilon)d\varepsilon - \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left[\frac{t^*+t_n}{2}(\rho P_0+\varepsilon) - c(t^*,e_1)\right] f(\varepsilon)d\varepsilon\right\} = 0 \quad \text{(A.1)}$$

Where $F: [t_0,t_n]\times[0,1]\times[0,+\infty)\times(0,+\infty)\times[0,+\infty)\to\mathbb{R}$ is the real valued function defined in equation (A.1). If we derive this function with respect to each variable, we obtain the following expressions:

$$\frac{\partial F}{\partial t^*} = \kappa\left[\frac{P_0}{2} + \delta c_1(t^*,e_1)\right] \quad \text{(A.2)}$$

$$\frac{\partial F}{\partial \delta} = \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left(\frac{t_0+t^*}{2}(\rho P_0+\varepsilon)\right) f(\varepsilon)d\varepsilon - \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U\left[\frac{t^*+t_n}{2}(\rho P_0+\varepsilon) - c(t^*,e_1)\right] f(\varepsilon)d\varepsilon \quad \text{(A.3)}$$

$$\frac{\partial F}{\partial P_0} = \kappa\left(\frac{t_0+t^*}{2} - \delta\frac{t_n-t_0}{2}\rho\right) \quad \text{(A.4)}$$

$$\frac{\partial F}{\partial \rho} = -\delta\kappa P_0\frac{t_n-t_0}{2} \quad \text{(A.5)}$$

$$\frac{\partial F}{\partial e_1} = \delta\kappa c_2(t^*,e_1) \quad \text{(A.6)}$$

Where $\kappa = U'(x)\ \forall x\in\mathbb{R}$, $c_1(t^*,e_1) = \frac{\partial c(t^*,e_1)}{\partial t^*}$ and $c_2(t^*,e_1) = \frac{\partial c(t^*,e_1)}{\partial e_1}$.

It is easy to see that $\frac{\partial F(t)}{\partial t} > 0 \; \forall t \in [t_0, t_n]$, $F(t_0) < 0$ and $F(t_n) > 0$ under assumptions 1, 2 and 3, which implies that there exists a unique $t^* \in [t_0, t_n]$ such that $F(t^*) = 0$. Likewise, $\frac{\partial F(t)}{\partial t} < 0 \; \forall t \in [t_0, t_n]$, $F(t_0) > 0$ and $F(t_n) < 0$ under assumptions 1´, 2´ and 3´, which implies that there exists a unique $t^* \in [t_0, t_n]$ such that $F(t^*) = 0$. Therefore, in the non-selective and selective educational systems, there is a unique $t^* \in [t_0, t_n]$ such that a separating equilibrium arises in which those worker´s types lower than t^* choose e_0 and those types greater than t^* choose e_1.

Additionally, since $U(0) = 0$, $U'(\cdot) > 0$ and $\frac{t_0 + t^*}{2} P_0 > 0 \; \forall t^* \in [t_0, t_n]$, then $U\left(\frac{t_0 + t^*}{2} P_0\right) > 0$. As a result, $\frac{\partial F}{\partial \delta} < 0$ so that equation (A.1) is satisfied.

Finally, as shown by equations (A.5) and (A.6), $\frac{\partial F}{\partial \rho} < 0$ and $\frac{\partial F}{\partial e_1} > 0$. Thus, if we use the implicit function theorem, we obtain the desired results under assumption 3:

$$\frac{\partial t^*}{\partial \delta} = -\frac{\frac{\partial F}{\partial \delta}}{\frac{\partial F}{\partial t^*}} > 0 \qquad \text{(A.7)}$$

$$\frac{\partial t^*}{\partial \rho} = -\frac{\frac{\partial F}{\partial \rho}}{\frac{\partial F}{\partial t^*}} > 0 \qquad \text{(A.8)}$$

$$\frac{\partial t^*}{\partial e_1} = -\frac{\frac{\partial F}{\partial e_1}}{\frac{\partial F}{\partial t^*}} < 0 \qquad \text{(A.9)}$$

Under assumption 3´, these inequalities are reversed because $\frac{\partial F}{\partial t^*} < 0$ and this completes the proof of proposition 1. QED.

Proof of Proposition 2. From equation (A.4), we see that $\frac{\partial F}{\partial P_0} > 0$ when $\rho = 0$ and $\frac{\partial F}{\partial P_0}$ decreases with ρ. Then, there will be a threshold, $K \in \mathbb{R}^+$, such that $\frac{\partial F}{\partial P_0} > 0$ when $\rho < K$, whereas

$\frac{\partial F}{\partial P_0} < 0$ when $\rho > K$. Once again, we use the implicit function theorem in order to obtain the desired results. In particular, under assumptions 1, 2 and 3, we conclude that $\frac{\partial t^*}{\partial P_0} < 0$ when $\rho < K$ and $\frac{\partial t^*}{\partial P_0} > 0$ when $\rho > K$. Likewise, under assumptions 1´, 2´ and 3´, we obtain that $\frac{\partial t^*}{\partial P_0} > 0$ when $\rho < K$ and $\frac{\partial t^*}{\partial P_0} < 0$ when $\rho > K$. QED.

Proof of Proposition 3. Once again, we use the worker´s indifference equation as shown in (A.1):

In our setting, $\frac{\partial F}{\partial t^*} > 0$ under assumption 4 and $\frac{\partial F}{\partial t^*} < 0$ under assumption 4´. Furthermore, as $U\left(\frac{t_0+t^*}{2}P_0\right) > 0$, the indifference condition (A.1) holds as long as:

$$\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta}\left\{U\left(\frac{t_0+t^*}{2}(\rho P_0+\varepsilon)\right) - U\left[\frac{t^*+t_n}{2}(\rho P_0+\varepsilon) - c(t^*,e_1)\right]\right\} f(\varepsilon)\, d\varepsilon < 0 \qquad \text{(A.10)}$$

This implies that $\frac{\partial F}{\partial \delta} < 0$. Inequality (A.10) also implies that there must exist at least one value of $\varepsilon \in [\bar{\varepsilon}-\Delta, \bar{\varepsilon}+\Delta]$ such that $U\left(\frac{t_0+t^*}{2}(\rho P_0+\varepsilon)\right) < U\left[\frac{t^*+t_n}{2}(\rho P_0+\varepsilon) - c(t^*,e_1)\right]$. In fact, if $U\left(\frac{t_0+t^*}{2}(\rho P_0+\varepsilon)\right) \geq U\left[\frac{t^*+t_n}{2}(\rho P_0+\varepsilon) - c(t^*,e_1)\right]$ for all values of $\varepsilon \in [\bar{\varepsilon}-\Delta, \bar{\varepsilon}+\Delta]$, then inequality (A.10) cannot be satisfied. Let us call $\varepsilon^* \in [\bar{\varepsilon}-\Delta, \bar{\varepsilon}+\Delta]$ the minimum value of ε for which $U\left(\frac{t_0+t^*}{2}(\rho P_0+\varepsilon)\right) \leq U\left[\frac{t^*+t_n}{2}(\rho P_0+\varepsilon) - c(t^*,e_1)\right]$, in which case:

$$U\left(\frac{t_0+t^*}{2}(\rho P_0+\varepsilon^*)\right) \leq U\left[\frac{t^*+t_n}{2}(\rho P_0+\varepsilon^*) - c(t^*,e_1)\right] \qquad \text{(A.11)}$$

As the utility function is strictly increasing, this implies that $\frac{t_0+t^*}{2}(\rho P_0+\varepsilon^*) \leq \frac{t^*+t_n}{2}(\rho P_0+\varepsilon^*) - c(t^*,e_1)$. As ε^* is the minimum value of ε for which this inequality is satisfied, we conclude that $\frac{t_0+t^*}{2}(\rho P_0+\varepsilon^*) = \frac{t^*+t_n}{2}(\rho P_0+\varepsilon^*) - c(t^*,e_1)$, which is equivalent to $c(t^*, e_1) = \frac{t_n - t_0}{2}(\rho P_0 + \varepsilon^*)$. Hence, we have proven the following inequalities:

$$U\left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right) \geq U\left[\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right] \ \forall \varepsilon \in [\overline{\varepsilon} - \Delta, \varepsilon^*] \qquad \text{(A.12)}$$

$$U\left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right) \leq U\left[\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right] \ \forall \varepsilon \in [\varepsilon^*, \overline{\varepsilon} + \Delta] \qquad \text{(A.13)}$$

Using inequality (A.10), we obtain:

$$\int_{\overline{\varepsilon}-\Delta}^{\varepsilon^*} \left\{U\left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right) - U\left[\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right]\right\} f(\varepsilon)\, d\varepsilon +$$

$$\int_{\varepsilon^*}^{\overline{\varepsilon}+\Delta} \left\{U\left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right) - U\left[\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right]\right\} f(\varepsilon)\, d\varepsilon < 0 \qquad \text{(A.14)}$$

This is tantamount to:

$$\int_{\overline{\varepsilon}-\Delta}^{\varepsilon^*} \left\{U\left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right) - U\left[\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right]\right\} f(\varepsilon)\, d\varepsilon <$$

$$\int_{\varepsilon^*}^{\overline{\varepsilon}+\Delta} \left\{U\left[\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right] - U\left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right)\right\} f(\varepsilon)\, d\varepsilon \qquad \text{(A.15)}$$

Next, it is straightforward to see that $\frac{\partial F}{\partial e_1} > 0$. Then, using the implicit function theorem, we obtain that $\frac{\partial t^*}{\partial \delta} > 0$ and $\frac{\partial t^*}{\partial e_1} < 0$ under assumption 4 (non-selective educational system), whereas $\frac{\partial t^*}{\partial \delta} < 0$ and $\frac{\partial t^*}{\partial e_1} > 0$ under assumption 4´ (selective educational system). This completes the proof of part i of proposition 3.

In the next stage of our proof, we analyze the effect of a change in ρ by obtaining the following derivative:

$$\frac{\partial F}{\partial \rho} = \delta P_0 \left\{\frac{t_0+t^*}{2} \int_{\overline{\varepsilon}-\Delta}^{\overline{\varepsilon}+\Delta} U'\left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right) f(\varepsilon) d\varepsilon - \frac{t^*+t_n}{2} \int_{\overline{\varepsilon}-\Delta}^{\overline{\varepsilon}+\Delta} U'\left(\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right) f(\varepsilon) d\varepsilon\right\} \qquad \text{(A.16)}$$

Then, $\frac{\partial F}{\partial \rho} \lesseqqgtr 0$ if, and only if the following condition holds:

$$\frac{t^*+t_n}{t_0+t^*} \gtreqless \frac{\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U'\left(\frac{t_0+t^*}{2}(\rho P_0+\varepsilon)\right) f(\varepsilon)d\varepsilon}{\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U'\left(\frac{t^*+t_n}{2}(\rho P_0+\varepsilon)-c(t^*,e_1)\right) f(\varepsilon)d\varepsilon} = \theta^* \tag{A.17}$$

When $\theta^* < 1$, $\frac{t^*+t_n}{t_0+t^*} > 1 > \theta^*$ and $\frac{\partial F}{\partial \rho} < 0$. As $\frac{\partial F}{\partial t^*} > 0$ in the non-selective educational system, then $\frac{\partial t^*}{\partial \rho} > 0$ when $\theta^* < 1$. Likewise, as $\frac{\partial F}{\partial t^*} < 0$ in the selective educational system, then $\frac{\partial t^*}{\partial \rho} < 0$ when $\theta^* < 1$. This proves part ii of proposition 3.

Since $\frac{t^*+t_n}{t_0+t^*} = 1 + \frac{t_n-t_0}{t_0+t^*}$, we can rewrite (A.17) as:

$$t_n - t_0 \gtreqless (\theta^* - 1)(t_0 + t^*) \tag{A.18}$$

Then, when $\theta^* > 1$, then $\frac{\partial t^*}{\partial \rho} \gtreqless 0$ if $t_n - t_0 \gtreqless (\theta^* - 1)(t_0 + t^*)$ in the non-selective environment, whereas $\frac{\partial t^*}{\partial \rho} \lesseqqgtr 0$ if $t_n - t_0 \gtreqless (\theta^* - 1)(t_0 + t^*)$ in the selective educational system. Then, we have proven part iii of proposition 3.

Now, we study the effect of a change in P_0 on the worker´s incentives to invest in education. To this end, we obtain the following derivative:

$$\frac{\partial F}{\partial P_0} = U'\left(\frac{t_0+t^*}{2}P_0\right)\frac{t_0+t^*}{2} + \delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U'\left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right)\rho\frac{t_0+t^*}{2} f(\varepsilon)d\varepsilon -$$
$$\delta \int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U'\left(\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right)\rho\frac{t^*+t_n}{2} f(\varepsilon)d\varepsilon \tag{A.19}$$

We can rewrite this derivative in the following way:

$$\frac{\partial F}{\partial P_0} = U'\left(\frac{t_0+t^*}{2}P_0\right)\frac{t_0+t^*}{2} + \delta\rho\left\{\frac{t_0+t^*}{2}\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U'\left(\frac{t_0+t^*}{2}(\rho P_0 + \varepsilon)\right) f(\varepsilon)d\varepsilon -\right.$$
$$\left.\frac{t^*+t_n}{2}\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U'\left(\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right) f(\varepsilon)d\varepsilon\right\} \tag{A.20}$$

If $\rho = 0$, then $\frac{\partial F}{\partial P_0} = U'\left(\frac{t_0+t^*}{2}P_0\right)\frac{t_0+t^*}{2} > 0$. As the right-hand side of equation (A.20) is a continuous function with respect to ρ, there is a threshold $K > 0$ such that $\frac{\partial F}{\partial P_0} > 0$ when $\rho < K$. Then, $\frac{\partial t^*}{\partial P_0} < 0$ under assumption 4 and $\frac{\partial t^*}{\partial P_0} > 0$ under assumption 4´ when $\rho < K$. Then, we have proven part iv of proposition 3.

In order to study the sign of $\frac{\partial F}{\partial P_0}$ when $\rho > K$, we divide the right-hand side of (A.20) by $\int_{\bar{\varepsilon}-\Delta}^{\bar{\varepsilon}+\Delta} U'\left(\frac{t^*+t_n}{2}(\rho P_0 + \varepsilon) - c(t^*, e_1)\right) f(\varepsilon) d\varepsilon$ and obtain:

$$\frac{\partial F}{\partial P_0} = \frac{t_0+t^*}{2}\theta^*(\lambda^* + \delta\rho) - \delta\rho\frac{t^*+t_n}{2} \tag{A.21}$$

Then, $\frac{\partial F}{\partial P_0} \lesseqgtr 0$ if, and only if:

$$\frac{t^*+t_n}{t_0+t^*} \gtreqless \frac{\theta^*(\lambda^*+\delta\rho)}{\delta\rho} \tag{A.22}$$

This is equivalent to:

$$t_n - t_0 \gtreqless \left(\theta^*\frac{\lambda^*}{\delta\rho} + \theta^* - 1\right)(t_0 + t^*) \tag{A.23}$$

Once again, as $\frac{\partial F}{\partial t^*} > 0$ in the non-selective educational system, then $\frac{\partial t^*}{\partial P_0} \gtreqless 0$ if $t_n - t_0 \gtreqless \left(\theta^*\frac{\lambda^*}{\delta\rho} + \theta^* - 1\right)(t_0 + t^*)$, and as $\frac{\partial F}{\partial t^*} < 0$ in the selective educational system, then $\frac{\partial t^*}{\partial P_0} \lesseqgtr 0$ if $t_n - t_0 \gtreqless \left(\theta^*\frac{\lambda^*}{\delta\rho} + \theta^* - 1\right)(t_0 + t^*)$. This proves part v and completes the proof of proposition 3. QED.

Proof of Proposition 4. To start with, we rewrite the indifference condition included in equation (5) as:

$$F[t^*, \delta, P_0, \rho, e_1] = U[E(t|t < t^*)P_0] + \delta U\big[E(t|t < t^*)\big(\rho P_0 + E(\varepsilon)\big)\big] -$$

$$\delta U\big[E(t|t > t^*)\big(\rho P_0 + E(\varepsilon)\big) - c(t^*, e_1)\big] = 0 \qquad \text{(A.24)}$$

If we derive this function with respect to the indifferent type, we obtain:

$$\frac{\partial F}{\partial t^*} = \kappa P_0 \frac{\partial E(t|t < t^*)}{\partial t^*} - \delta\kappa \frac{\partial \phi(t^*)}{\partial t^*}\big(\rho P_0 + E(\varepsilon)\big) + \delta\kappa \frac{\partial c(t^*, e_1)}{\partial t^*} \qquad \text{(A.25)}$$

Recall that $\kappa = \frac{\partial U(x)}{\partial x}$, which is constant because the worker is risk-neutral. Under assumption 7 (assumption 10), $\frac{\partial F}{\partial t^*} > 0$ ($\frac{\partial F}{\partial t^*} < 0$).

Similarly, we obtain the following derivatives:

$$\frac{\partial F}{\partial \delta} = U\big[E(t|t < t^*)\big(\rho P_0 + E(\varepsilon)\big)\big] - U\big[E(t|t > t^*)\big(\rho P_0 + E(\varepsilon)\big) - c(t^*, e_1)\big] \qquad \text{(A.26)}$$

$$\frac{\partial F}{\partial \rho} = -\delta\kappa P_0 \phi(t^*) \qquad \text{(A.27)}$$

$$\frac{\partial F}{\partial P_0} = \kappa[E(t|t < t^*) - \delta\phi(t^*)\rho] \qquad \text{(A.28)}$$

$$\frac{\partial F}{\partial e_1} = \delta\kappa c_2(t^*, e_1) \qquad \text{(A.29)}$$

Now, we determine the signs of these derivatives. Recall that the indifference condition included in equation (5) can be expressed as:

$$U(E(t|t < t^*)P_0) = \delta\big\{U\big[E(t|t > t^*)\big(\rho P_0 + E(\varepsilon)\big) - c(t^*, e_1)\big] -$$

$$U\big[E(t|t < t^*)\big(\rho P_0 + E(\varepsilon)\big)\big]\big\} \qquad \text{(A.30)}$$

The left-hand side of this equation is positive because $U(E(t|t < t^*)P_0) \geq U(t_0 P_0) > 0$. Therefore, the right-hand side of the equation is also positive, which implies that $\frac{\partial F}{\partial \delta} < 0$. It is also clear that $\frac{\partial F}{\partial \rho} < 0$ and $\frac{\partial F}{\partial e_1} > 0$. By using the implicit function theorem, we conclude that $\frac{\partial t^*}{\partial \delta} > 0$, $\frac{\partial t^*}{\partial \rho} > 0$ and $\frac{\partial t^*}{\partial e_1} < 0$ in the non-selective scenario, whereas $\frac{\partial t^*}{\partial \delta} < 0$, $\frac{\partial t^*}{\partial \rho} < 0$

and $\frac{\partial t^*}{\partial e_1} > 0$ in the selective scenario. Finally, $\frac{\partial F}{\partial P_0}$ in equation (A.28) is positive when $\rho = 0$ and strictly decreases with ρ. Hence, there exists $K \in \mathbb{R}^+$ such that $\frac{\partial F}{\partial P_0} \gtreqless 0$ when $\rho \lesseqgtr K$, which implies that $\frac{\partial t^*}{\partial P_0} \lesseqgtr 0$ when $\rho \lesseqgtr K$ in the non-selective scenario and $\frac{\partial t^*}{\partial P_0} \gtreqless 0$ when $\rho \lesseqgtr K$ in the selective scenario. QED.

Proof of Lemma 2. We need to analyze the sign of $\frac{\partial F}{\partial t^*}$, which is shown in equation (A.25).

As shown by lemma 1, when f is an everywhere increasing function, then $\frac{\partial \phi(t^*)}{\partial t^*} < 0\ \forall t \in [t_0, t_n]$. As a result, equation (A.25) shows that $\frac{\partial F}{\partial t^*} > 0$ when $\left|\frac{\partial c(t^*,e_1)}{\partial t^*}\right| = 0$. Since $\frac{\partial F}{\partial t^*}$ decreases with $\left|\frac{\partial c(t^*,e_1)}{\partial t^*}\right|$, there exists $c_1 \in \mathbb{R}^+$ such that $\frac{\partial F}{\partial t^*} > 0$ when $\left|\frac{\partial c(t^*,e_1)}{\partial t^*}\right| < c_1$.

Hence, we have proven that $\frac{\partial F}{\partial t^*} > 0$ as long as g is an everywhere increasing function and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| < c_1\ \forall t \in [t_0, t_n]$, which is the first part of lemma 2.

Similarly, lemma 1 shows that $\frac{\partial \phi(t^*)}{\partial t^*} > 0\ \forall t \in [t_0, t_n]$ when g is an everywhere decreasing function. From equation (A.25), we can guarantee that $\frac{\partial F}{\partial t^*} < 0$ when $\left|\frac{\partial c(t^*,e_1)}{\partial t^*}\right| > \max_{t^* \in [t_0,t_n]} \left\{\frac{P_0}{\delta} \frac{\partial E(t|t < t^*)}{\partial t^*}\right\} = c_2$.

Therefore, we have proven that $\frac{\partial F}{\partial t^*} < 0$ as long as g is an everywhere decreasing function and $\left|\frac{\partial c(t,e_1)}{\partial t}\right| > c_2\ \forall t \in [t_0, t_n]$, which is the second part of lemma 2 and this completes the proof. QED.

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