Horizontal product differentiation and policy adjustment in the presence of abatement subsidies and emission taxes

Abstract

There are important examples of countries which have implemented policies to promote pollution abatement activities in sectors characterized by some degree of product differentiation. This paper examines the role of product differentiation on optimal policy and industry emissions in a Cournot oligopoly model in the presence of abatement efforts, abatement subsidies and emission taxes. The analysis indicates that as products become more differentiated the government can afford a tax increase due to the presence of subsidies and abatement efforts. Additionally, highly differentiated industries may experience a rapid increase in emissions and so policies such as R&D may be needed to tackle higher emissions. The government adjusts optimal policy as industries become more or less pollution-intensive, and the extent of the adjustment varies across industries characterized by different degrees of product differentiation. The analysis is potentially relevant in industries where firms are taking steps to differentiate their products in order to capture particular market niches, and lower production and abatement costs.

Keywords: horizontal product differentiation; emission taxes; environmental subsidy; Cournot

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1 Introduction

There are a number of countries which have recently implemented policies to promote pollution abatement activities. The policies set by the EU and UK to promote carbon capture and storage technology in the energy sector serve as an example of such effort.[1] This sector, among others, may be characterized by some degree of product differentiation and is subject to funding schemes (sometimes in the form of a pollution abatement subsidy) as well as an emission tax.[2] Despite the presence of horizontal product differentiation and various tax-subsidy schemes in the real-world, the literature has provided little analysis on the relation between product differentiation, optimal emission taxes and subsidies, and industry emissions.[3] This paper examines the role of the degree of horizontal product differentiation on the characterization of optimal environmental policy where firms behave à la Cournot in the presence of abatement efforts, emission taxes and abatement subsidies. The effect on industry emissions via the policy adjustment as products become more differentiated is also analyzed.

The literature has examined different aspects of differentiated markets, an important element in the design of environmental policy (e.g., Fujiwara 2009; Espínola-Arrendondo and Zhao 2012; Lambertini 2013). One important strand of this literature looks at the role of vertically differentiated markets in the context of optimal environmental policy (e.g., Poyago-Theotoky and Teerasuwannajak 2002; Bansal and Gangopadhyay 2003; Rodríguez-Ibeas, 2006), a different strand at

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[1] See the UK Department of Energy http://www.decc.gov.uk/en/content/cms/what_we_do/uk_supply /energy_mix/ccs/ccs.aspx and a recent EU initiative to promote CCS, the "NER 300 call" at http://ec.europa.eu/clima/funding/ner300/index_en.htm. See also PEW Center of Global Climate Change, http://www.pewclimate.org/technology/factsheet/ccs. Information taken from Massachusetts Institute of Technology (MIT), "The Future of Coal: Options for a Carbon-Constrained World", 2007.

[2] Other sectors by NACE classification subject to an energy tax in the EU include NACE Rev. 1.1 Sections A to K and N and Divisions 90, 92 and 93. The complete data set on environmental taxes in the EU, including pollution taxes, is available at http://epp.eurostat.ec.europa.eu/portal/page/portal/product_details/dataset?p_product_code=ENV_AC_TAXIND . These data show that some of the sectors mentioned, which have applied for NER 300 funding and which include the power generation, iron and steel production and cement sectors, see http://ec.europa.eu/clima/funding/ner300/00001/summary_en.pdf , are also subject to a pollution tax. In terms of product differentiation, other industries which may be relevant to the present analysis and CCS technology include the chemicals and petroleum refineries; see Anderson and Newell (2003) for examples.

[3] Real-world examples of industries which can be associated with product differentiation include the chemicals industry whereas the pulp and paper industry exemplifies the case of homogeneous goods (Fujiwara 2009, p. 240). In addition, Thollander and Ottosson (2008) explain the CO2 tax (see p. 26) firms in the pulp and paper industry in Sweden face as well as some of the potential investment subsidies for energy-efficiency technologies (to reduce energy costs associated to production costs) firms may use to reduce energy and therefore production costs (see pp. 30-31).

the role of emission fees (e.g., Poyago-Theotoky 2003; Fujiwara 2009) and subsidies in the context of technology diffusion (e.g., McGinty and de Vries 2009) in the presence of horizontally differentiated markets. A third strand, analyzes optimal environmental policy where firms compete for the production of a homogeneous good assuming away abatement technologies (e.g., Simpson 1995; Levine 1985); and Ebert (1992) and Katsoulacos and Xepapadeas (1995) consider abatement technologies and assume homogeneous goods. The contribution of the present work is at the intersection of these strands.[4]

In the context of horizontal product differentiation Poyago-Theotoky (2003) shows, in Cournot competition where firms choose environmental R&D strategically, and are subject to an emission tax and R&D subsidy, that the optimal tax is less than marginal damages and the subsidy may be positive or negative in the short-run; the author however but does not consider the case of free-entry, an aspect considered in section 2.2.[5] Moreover, Lahiri and Symeonidis (2007), and Gautier (2013) examine policy reform in an international context in the presence of horizontal product differentiation. However, none of these works analyzes the adjustment of policy arising from changes in the degree of product differentiation and the resulting effect on industry emissions. Katsoulacos and Xepapadeas (1995) examine the role of free-entry in the presence of an emission tax, and Lee (1999), Ebert (1992) and Simpson (1995) are important analyses on optimal taxation under Cournot competition where the number of firms is exogenous; none of these works allow for the possibility of product differentiation. Conrad and Wang (1993) consider an emission tax and abatement subsidy, but do not consider product differentiation or welfare analysis. This branch of the literature, to the best of my knowledge, has not examined in a systematic way the effect of policy adjustment on emissions, which is one important contribution of the present work.

In a formal sense the present work is closest to Fujiwara (2009). Fujiwara (2009) analyzes the role of product differentiation in a Cournot setting in the presence of emission taxes only, and

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[4] Requate (2006) and Lambertini (2013) discuss the literature on environmental policy under Cournot oligopoly.

[5] One key difference between the present and Poyago-Theotoky's model is that the effective subsidy here is an output subsidy and, consequently, the present work may be understood in this context; whereas in Poyago-Theotoky's model the subsidy is interpreted as an R&D subsidy (with the associated R&D market failure), not an output subsidy.

assumes away abatement efforts by firms; the author shows, *inter alia*, that as products become differentiated the emission tax rises *if and only if* the government cares sufficiently about the environment. The present analysis extends Fujiwara's work in two fundamental ways. First, abatement efforts are incorporated into the model and a subsidy is part of the policy scheme, two elements which are relevant in a number of industries. Second, the impact of product differentiation and pollution intensity (i.e., pollution per unit of output) on emissions via the adjustment of policy is considered, two important aspects which are not present in Fujiwara's work.

This paper builds from Fujiwara (2009) and develops a Cournot oligopoly model with product differentiation in the presence of pollution abatement technology, emission taxes and abatement subsidies, both in the case where the number of firms is exogenous and endogenous. The government and firms play a two-stage game where the government first sets policy optimally through welfare maximization, and firms then take policy as given and maximize profits by choosing the level of emissions and output in a Cournot-Nash fashion. In contrast to Fujiwara (2009) the analysis suggests that in the presence of the subsidy and pollution abatement technology, the emission tax and subsidy rise with more differentiated products in the short-run; that is, the government can afford a tax increase as products become more differentiated. In the long-run the incentives for pollution abatement arising from the tax are strong so that the tax rises and subsidy falls as products become more differentiated.

Even with the adjustment of policy industry emissions may rise with more differentiated products; it is shown that this increase in emissions is nonlinear. In particular, highly differentiated industries may experience a large increase in emissions and so policies such as R&D may be needed to tackle higher emissions. Furthermore, the government adjusts optimal policy as industries become more or less pollution-intensive, and the extent of the adjustment varies across industries characterized by different degrees of product differentiation; the analysis indicates that the government may adjust the tax and subsidy sufficiently (the adjustment is more aggressive in more differentiated markets), and as a result, industry emissions may fall. These results are important because they indicate that policymakers may need to adjust policy to changes in the degree of product differentiation, particularly in industries where firms are taking steps to differentiate their products in order to capture particular markets niches, and lower production and abatement costs (see e.g. Lambertini (2013) for a discussion on firms which differentiate themselves as environmentally conscious firms) .

To see the relevance of the analysis via the adjustment of policy consider the case where products become more differentiated. As a result, industry may experience higher levels of output and therefore higher emissions, which may induce the government to raise the tax; this adjustment of the tax results in lower emissions. If, in addition, a subsidy is part of the policy scheme, then more differentiated products may result in a higher subsidy (since more differentiated products result in more market power by each firm and a higher subsidy may offset this by encouraging production) and therefore higher emissions. The net effect on industry emissions via the policy adjustment, arising from a change in the degree of product differentiation, is therefore not clear-cut. What this example illustrates is that changes in the degree of product differentiation give rise to changes in environmental policy, thereby rendering the analysis relevant for policy making. First, more differentiated products leads to increases in output and therefore emissions, but also it increases the market power of each firm. So, policymakers, on the one hand, need to weigh environmental damages, and on the other, the effect on the market distortion.

As mentioned earlier, the extent to which pollution intensity (as measured by emissions per unit of output) alters the characterization of optimal policy is analyzed. This analysis may be motivated by a recent study published by the US Department of Commerce (DOC), which shows evidence of reductions/ increases in pollution intensities across several industries (see also Sorrell et al. (2009) for a survey on the empirical literature on the effects of energy efficiency on emissions).[6] The idea here is that an exogenous change in the pollution intensity coefficient leads to an adjustment of optimal policy such that the effect on industry emissions is ambiguous. For instance,

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[6] According to a study by the US Economics and Statistics Administration in 2010, the chemicals industry, which is one of the largest pollution intensity coefficients as measured by CO2 per unit of output, has been able to reduce its pollution intensity level between 1998 and 2006 (pp. 12-13). See US Economics and Statistics Administration (2010) "US Carbon dioxide emissions and intensities over time: A detailed accounting of industries, government and households. US Department of Commerce, April 2010. Link: http://www.esa.doc.gov/sites/default/files/reports/documents/co2reportfinal.pdf

an increase in the intensity coefficient raises emissions, which induces the government to raise the tax and so, *from this policy adjustment*, emissions may fall. But a larger intensity coefficient also induces firms to lower production (since a larger coefficient increases payments from the emission tax), which would induce the government to lower the tax thereby raising emissions from the policy adjustment. As a result, the net effect on the adjustment of policy and the resulting impact on emissions is not clear-cut. If, in addition, firms face a subsidy, then higher emissions (arising from a larger intensity coefficient) may induce the government to lower the subsidy to control pollution. The analysis indicates that the government adjusts optimal policy as industries become more or less pollution-intensive, and the extent of the adjustment varies across industries characterized by different degrees of product differentiation. The analysis suggests that emissions may fall via the adjustment of the tax and subsidy.

Inevitably, the policy implications derived from the analysis are limited by some of the assumptions about the cost functions and the strategic behavior of firms. For instance, I assume away issues of the strategic choice of environmental *R&D* since this is an aspect which has been analyzed elsewhere (e.g., Carlsson 2000; Montero 2000a, 200b; Poyago-Theotoky 2003; Eriksson 2004; Ulph and Ulph 1996, 2007). Second, by assuming away issues of strategic choice of abatement, the focus is on issues of the role of product differentiation as considered in part of the literature and so more directly comparable to the work of e.g., Fujiwara (2009). Additionally, issues about the strategic behavior of governments and the existence of several environmental policies (not just taxes and subsides), important aspects which have been examined in the literature, are also assumed away. These present potential extensions of the model as well as future lines of research.

The rest of the paper is structured as follows. The first section looks at the case of fixed number of firms and section two then looks at the free-entry case. The last section concludes.

2 The Model

In a formal sense the present model is based on Fujiwara (2009).[7] Consider an industry with $n > 1$ firms operating under Cournot oligopolistic conditions in the presence of product differentiation. Each firm i ($i = 1, 2, \ldots, n$) engages in the production of an imperfect substitute, which is sold in the domestic market, but also generates pollution and, as a result, additional costs to society. Additionally, each firm engages in pollution abatement activities. The government and firms play a two-stage game where the government sets policy optimally through social welfare maximization, and firms then take policy as given and maximize profits by simultaneously choosing the level of output and emissions. Throughout I shall analyze the symmetric equilibrium, assume interior solutions and that firms behave in a Cournot-Nash fashion.

I shall assume a cost function of the end-of-pipe to capture abatement efforts by firms as in Lahiri and Symeonidis (2007). In particular, each firm i's costs depend on output,q^i, and emissions,e^i, where the cost function is given by $\tilde{C}^i(q^i, e^i) = \hat{c}^i q^i + (\delta q^i - e^i)^2/2$ where $\hat{c}^i > 0$ is constant, pollution abatement by each firm is given by $a^i = \delta q^i - e^i$ and the constant $\delta^i > 0$ represents the pollution intensity coefficient. The first and second terms of the $\tilde{C}$ function capture, respectively, production and abatement costs. The reader is referred to Requate (2006) for a more general treatment of the cost function. Dropping the tilde to simplify notation, the cost function, $\tilde{C}$, satisfies (subscripts denote partial derivatives) $C_q = \hat{c} + \delta(\delta q - e) > 0$, $C_e = -(\delta q - e) < 0$, $C_{eq} = C_{qe} = -\delta < 0$, $C_{qq} = \delta^2 > 0$, $C_{ee} = 1 > 0$, $C_{qq}C_{ee} - C_{qe}C_{eq} = 0$.

Demand faced by each firm i comes from preferences such that $p^i = \alpha - (\beta - \gamma)q^i - \gamma \sum q^j$, where γ represents the degree of product differentiation, and $\gamma > 0$, $\beta > \gamma$.[8] Furthermore, each firm is subject to a per-unit emission tax, t, and pollution abatement subsidy, s, Therefore, subsidy payments to each firm for the amount of pollution abated are given by sa^i. Notice that the abatement subsidy influences output and emissions by each firm. Consequently, I define the effective tax,

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[7] Lambertini (2013) presents an overview of Fujiwara's model (see pp. 37-40).

[8] This structure follows Fujiwara (2009). The specific utility function comes from Cellini et al. (2004). The assumption of $\gamma > 0$ rules out the possibility of complements.

$\tau = t + s$, and the effective output subsidy, $\varsigma = s\delta^i$, for all i.

This concludes the description of the model.

2.1 Fixed Number of Firms

In this section I shall consider the case where the number of firms is exogenous. In particular, this section characterizes the equilibrium and examines how optimal policy changes with the degree of product differentiation and pollution intensity coefficient. I also analyze how total emissions may rise or fall depending on how policy adjusts to changes in the degree of product differentiation and pollution intensity coefficient.

Each firm i maximizes profits by choosing e^i and q^i in a Cournot-Nash fashion:

$$\max_{q^i e^i} \pi^i = p^i q^i - \tilde{C}^i(q^i, e^i) + \varsigma q^i - \tau e^i - f^i \tag{1}$$

where f^i denotes fixed costs. This yields two first-order conditions, which under symmetry are given by

$$p - \beta q - C_q + \varsigma = 0 \tag{2}$$

$$-C_e - \tau = 0 \tag{3}$$

where $p = \alpha - q(\beta + \gamma(n-1))$. These determine the equilibrium level of output,q^*, emissions,e^*, and therefore abatement, a^*, and have the usual interpretation. Since the number of firms is fixed, total output, $Q = nq^*$, emissions, $E = ne^*$, and abatement, $A = na^*$, are also implicitly determined. In particular,

$$q^* \quad = \quad \frac{\alpha - \hat{c} - \delta\tau + \varsigma}{2\beta + \gamma(n-1)} \;\; ; \;\; e^* = \delta q^* - \tau \;\; ; \;\; where \;\; a^* = \tau \tag{4}$$

It is easy to show that the comparative statics effects[9] of the output subsidy, ς, and tax, τ, are consistent with the literature: under the assumptions of the function $C(\cdot,\cdot)$ and dropping the

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[9] The comparative statics effect hold under a general functional form for the cost function.

asterisk gives, $\partial q/\partial\tau < 0$, $\partial q/\partial\varsigma > 0$; also, $\partial a/\partial\tau > 0$, $\partial a/\partial\varsigma = 0$. Moreover, total differentiation yields

$$\tilde{\Delta}(\partial q/\partial\gamma) = -q(n-1) < 0 \qquad \tilde{\Delta}(\partial e/\partial\gamma) = -\delta q(n-1) < 0. \tag{5}$$

where $\tilde{\Delta} = (2\beta + \gamma(n-1)) > 0$. In words, for a given tax and subsidy as products become more differentiated (decrease in γ) emissions rise; this is because with more product variety output rises thus indicating that changes in γ work exclusively via output. From 5 and using the definition of abatement, a, yields $\tilde{\Delta}(\partial a/\partial\gamma) = 0$. Next, the impact of policy on emissions is examined. In particular, differentiation of e^* yields

$$\tilde{\Delta} de = \left[-2\beta - \gamma(n-1) - \delta^2\right] d\tau + \delta d\varsigma \tag{6}$$

From (6) the tax raises marginal costs (thereby lowering output), encourages abatement thus reducing emissions, and the net effect of the tax is to lower emissions; the output subsidy lowers marginal costs thus raising output and consequently emissions.

The government simultaneously chooses the subsidy, ς, and tax,τ, through welfare maximization. In particular, the government solves

$$\max_{\tau,\varsigma} W = CS + ne\tau - nq\varsigma + n\pi - \varphi \tag{7}$$

where CS denotes consumer surplus, and the function $\varphi = \varphi(ne)$ represents the damage from pollution satisfying $\varphi' > 0$ and $\varphi'' > 0$. Optimal policy is characterized by solving (7):

$$\frac{\partial W}{\partial\tau} = n(q\beta - \varsigma)\partial q/\partial\tau - n(\tau - \varphi')\partial e/\partial\tau = 0 \tag{8}$$

$$\frac{\partial W}{\partial\varsigma} = n(q\beta - \varsigma)\partial q/\partial\varsigma - n(\tau - \varphi')\partial e/\partial\varsigma = 0 \tag{9}$$

Hence, $\tau^* = \varphi'$ and $\varsigma^* = q\beta$; that is, the optimal tax is equal to marginal damages and the optimal subsidy is positive. This result is not new and has been shown in Requate (2006, p.145). Indeed, this result is a special case of equations 4.48 and 4.49 in Requate (2006, p.145). Intuitively, the tax is positive to tackle the damage from pollution and the subsidy addresses the output distortion arising from the market imperfection. This result, however, contrasts with Fujiwara (2009) where the only policy is an emission tax which can be either positive or negative since two distortions are addressed with only one policy; the presence of the subsidy thus affords the government a positive tax. Under the assumption of an end-of-pipe function and a damage function $\varphi = (en)^2/2$ so that $\varphi' > 0$ and $\varphi'' = 1$, I obtain closed-form expressions for the tax and subsidy. In particular,

$$\bar{\tau} = \frac{n\delta(\alpha - \hat{c})}{(n+1)(\beta + \gamma(n-1)) + n\delta^2} \qquad \bar{\varsigma} = \frac{(n+1)(\alpha - \hat{c})\beta}{(n+1)(\beta + \gamma(n-1)) + n\delta^2} \tag{10}$$

Substitution of these into (4) yields the closed-form solution for output and emissions,

$$\bar{q} = \frac{(\alpha - \hat{c})(n+1)}{((n+1)(\beta + \gamma(n-1)) + n\delta^2)} \qquad \bar{e} = \delta\bar{q} - \bar{\tau} \tag{11}$$

Differentiation of (10) yields (subscripts denote partial derivatives) $\bar{\tau}_\gamma < 0$, $\bar{\varsigma}_\gamma < 0$, $\bar{\tau}_{\gamma\gamma} > 0$ and $\bar{\varsigma}_{\gamma\gamma} > 0$. In words, the output subsidy increases because with more differentiated products (decrease in γ) each firm has more market power and so an output subsidy is used to address this distortion since it encourages production. This increase in output results in higher emissions. The tax thus rises with more differentiated products because with more differentiated products output and emissions rise and so the government addresses this higher pollution by raising the tax. If the pollution intensity coefficient is small (i.e., δ is small), then the adjustment of the tax from a change in the degree of product differentiation becomes small since changes in γ work via changes in output. In Fujiwara (2009) the extent to which τ falls or rises with γ depends upon whether the government puts more weight on either reducing pollution or addressing the output distortion. This is because in Fujiwara's model there is only one policy (i.e., the emission tax) and two distortions. Therefore, the presence of the subsidy results in an unambiguous sign for $\partial\tau/\partial\gamma$. The second-order effects are discussed below.

Proposition 1. *In the case where the number of firms is fixed the tax and subsidy rise with more differentiated products.*

The relation of policy with respect to the degree of product differentiation is depicted in figure 1 for specific parameter values where the limiting case is when γ approaches β. Notice that figure 1 suggests that the rate at which the tax and subsidy adjust to changes in the degree of product differentiation depends on the initial level of the degree of product differentiation in the industry. To see why consider the case where the degree of product differentiation is initially small. Then, as γ falls (products become more differentiated) the subsidy rises rapidly in order to offset the increase in market power. To see the effect on the tax it is useful to notice that q is a decreasing and convex function of γ. As a result, output and therefore emissions rise rapidly for initially small values of γ, in which case the tax rises rapidly to offset the higher level of emissions; if on the other hand, γ is relatively large (i.e., closer to β), then the tax is less sensitive.

Proposition 2. *Let the number of firms be fixed. Then, starting from a situation where products are very differentiated ($\gamma \simeq 0$) the tax and subsidy are sensitive to small changes in the degree of product differentiation. Conversely, starting from a situation where products are not very differentiated ($\gamma \simeq \beta$) the tax and subsidy are less sensitive to changes in the degree of product differentiation.*

The subsidy is very sensitive at initially small values of γ because further reductions in γ render the market an increasingly differentiated one; as a result, the output subsidy needs to stimulate output sufficiently (i.e., the subsidy needs to increase enough) in order to offset the increase in market power that comes with more differentiated products. As for the tax, the large increase in output resulting from increasingly differentiated products and the subsidy response raise emissions; as a result, the tax increases accordingly to tackle ever higher level of emissions.

One policy question that arises from the above proposition is the following: what is the impact on total emissions once the tax and subsidy adjust to changes in the degree of product differentiation? More formally, on the one hand I want to examine the effect of product differentiation on the equilibrium level of emissions via the tax and subsidy i.e., the total change in $e = e(\tau(\gamma), \varsigma(\gamma))$, and on the other the total effect, which includes the direct effect (for given tax and subsidy) on emissions. The analysis indicates that emissions are a convex and decreasing function of γ. The analysis that follows identifies the components at play which explain this result. The motivation here is that, on the one hand, as products become more differentiated output rises and so do emissions. Also, market power by each firm increases and so the subsidy rises with more differentiated products to promote output, but this increase in the subsidy raises emissions. The tax, as a result, rises to tackle the higher level of emissions. However, it is not clear whether emissions rise or fall even as policy is set optimally as γ changes. This is because emissions fall via the tax, but rise via the subsidy and increased product differentiation.

To see the impact of proposition 1 on emissions consider (6) and the expressions for $\bar{\tau}_\gamma$, $\bar{\varsigma}_\gamma$. Then, *via the tax and subsidy*, more differentiated products lowers the level of emissions. This is because the tax effect completely offsets the subsidy effect. Additionally, since the effect of γ on emissions, e, works via changes in output, the effect via policy becomes small in the case where the industry exhibits a small intensity coefficient (i.e., δ is small). In particular, total differentiation of $e = e(\tau(\gamma), \varsigma(\gamma))$ yields

$$e_\tau \tau_\gamma + e_\varsigma \varsigma_\gamma = \frac{\delta q(n-1)}{(n+1)(\beta + \gamma(n-1) + n\delta^2} > 0 \tag{12}$$

Proposition 3. *In the case where the number of firms is exogenous, emissions fall via the tax and subsidy as products become more differentiated if and only if the effect of the subsidy is small. Additionally, the effect via the policy becomes small in an industry characterized by a small pollution intensity coefficient (i.e.,* $\delta \simeq 0$*).*

Combining these results and proposition 2 it can be shown that emissions fall more rapidly (slowly) for smaller (larger) values of γ, *via the tax and subsidy*, due to changes in the degree of product differentiation (see the Appendix for a derivation). This is because with initially smaller values of γ the tax adjusts rapidly as products become more differentiated thus indicating that the effect from the adjustment in the tax completely offsets the adjustment in the subsidy.[10]

Lemma 1. *Let the number of firms be fixed. Then, starting from small (large) values of* γ *emissions fall more rapidly (slowly) via the adjustment in policy as products become more differentiated.*

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[10] In order to find the change in emissions via the adjustment in policy in the case of an end-of pipe cost function and quadratic damage function I use the closed-form solution for emissions, $e = \delta q - a$.

Up to now the analysis has touched on the effect of product differentiation on emissions via the adjustment of policy; I shall now incorporate the direct effect of product differentiation on emissions into the analysis. In particular, total differentiation of $e = e(\tau(\gamma), \varsigma(\gamma), \gamma)$, using (12), (4) and simplifying gives

$$\begin{aligned} \chi^2 de/d\gamma &= e_\tau \tau_\gamma + e_\varsigma \varsigma_\gamma + e_\gamma = -(\alpha - \hat{c})\delta(n+1)(n-1) < 0 & (13) \\ \chi^3 d^2e/d\gamma^2 &= (\alpha - \hat{c})\delta(n+1)^2(n-1)^2 > 0 & (14) \end{aligned}$$

where $\chi = (n+1)(\beta + \gamma(n-1)) + n\delta^2 > 0$. The same result is obtained using (11). The intuition here is that the tax effect (which works via output and abatement), although strong, is completely offset by the direct and subsidy effects (which work via changes in output) and, as a result, emissions rise as products become more differentiated. It follows then that, even with a rapid adjustment in the tax, emissions rise rapidly for small initial values of γ; for large initial values of γ emissions are not very sensitive to changes in the degree of product differentiation. It is also noted that the change in emissions arising from more differentiated products becomes small as the pollution intensity coefficient becomes small; this is because the total effect of γ on e works via changes in q.

Proposition 4. *In the case where the number of firms is fixed, emissions is a decreasing and convex function of* γ*.*

The importance of this result is that it shows the relevance of the direct effect (for given tax and subsidy) on emissions. This effect is strong enough to offset the reduction in emissions arising form the tax. As a result, industry emissions rise as markets become characterized by differentiated products. This increase in emissions is less severe for markets which are initially characterized by homogeneous goods.

The closed-form solutions to the subsidy and tax in (10) facilitate the comparative static analysis of the pollution intensity coefficient. I first examine the adjustment of policy given a change in the intensity coefficient; second, the impact of the adjustment of policy on emissions is analyzed. It is important to note that in the present analysis changes in the intensity coefficient are assumed to be exogenous and so these may be due to, among others, changes in technology or input mix.[11]

The effect of an increase in the pollution intensity coefficient results in a decrease in the optimal subsidy, but the effect on the emission tax is ambiguous. On the one hand, an increase in the pollution intensity coefficient raises pollution, which induces the government to raise the tax and lower the subsidy; on the other, a larger pollution intensity coefficient results in higher marginal costs (in the form of more taxes paid by each firm), lower output and emissions, which induces the government to lower the tax to offset higher taxes paid by firms and correct the output distortion. As a result, the tax rises with the intensity coefficient *if and only if* the increase in emissions offsets the effect which takes place via higher tax payments i.e., $(n+1)\left(\beta+\gamma(n-1)\right) > n\delta^2$. Note that the induced reduction in the tax is small if the pollution intensity coefficient is small. The effects on the tax and subsidy are given by

$$\frac{\chi^2}{n(\alpha-\hat{c})}\frac{\partial\bar{\tau}}{\partial\delta} = (n+1)\left(\beta+\gamma(n-1)\right)-n\delta^2 \tag{15}$$

$$\frac{\chi^2}{n(\alpha-\hat{c})}\frac{\partial\bar{\varsigma}}{\partial\delta} = -2\delta\beta(n+1) < 0 \tag{16}$$

where $\chi = \left((n+1)\left(\beta+\gamma(n-1)\right)+n\delta^2\right) > 0$. These results touch on the issue mentioned earlier, viz., the adjustment of policy and its impact on emissions. In particular, total differentiation of $e = e(\tau(\delta), \varsigma(\delta))$ gives

$$\frac{\chi^2}{n(\alpha-\hat{c})}\frac{de}{d\delta} = \frac{\delta^2}{\tilde{\Delta}}\left(-(n+1)\beta-\gamma(n-1)+n\delta^2-2\beta\right)-(n+1)\left(\beta+\gamma(n-1)\right)\tilde{\Delta} \tag{17}$$

where $\tilde{\Delta} = 2\beta+\gamma(n-1) > 0$. An increase in the pollution intensity coefficient lowers emissions, *via changes in the tax and subsidy*, in the case where the effect via higher tax payments by firms, which induces a tax reduction by the government, is small (i.e., $\partial\bar{\tau}/\partial\delta > 0$) or if the effect via the

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[11] The data presented in DOC (2010) show some evidence of reductions in intensity coefficients in some sectors (e.g., transportation, government) being associated to increases in emissions. However, the analysis does not show data indicating the channel whereby this correlation works.

subsidy is large i.e., $2\beta > n\delta^2$. In the first case, an increase in the intensity coefficient raises the tax and lowers the subsidy, each of which leads to a reduction in emissions. In the case where the subsidy effect is large the decrease in the subsidy (and the resulting reduction in emissions) completely offsets the increase in emissions arising from the partial reduction in the tax which takes place as δ rises.

Furthermore, (15) and (16) indicate that in a pollution-moderate industry (i.e., $\delta \simeq 0$) a worsening in pollution abatement technology (increase in δ) results in an increase in the tax and, as a result, industry emissions fall via the adjustment in policy; the subsidy has a negligible effect in this case. A second important point to note is that the total effect on emissions arising from a change in the pollution intensity coefficient (the total change in $e = e(\tau(\delta), \varsigma(\delta), \delta)$) is ambiguous; in particular, the increase in emissions via the direct effect (i.e., for given tax and subsidy) may be offset as the government adjusts the tax and subsidy as the pollution intensity coefficient rises. If this latter effect is sufficiently strong, then emissions fall (see the Appendix for a derivation). More precisely, the tax adjustment partially offsets the direct effect; and if the subsidy adjustment (which lowers emissions) is strong then the direct effect is completely offset thereby reducing industry emissions. The policy implication is that as industry becomes relatively more pollution-intensive the adjustment of the tax/subsidy may be sufficient so as to achieve a reduction in emissions.

Proposition 5. *In the case where the number of firms is exogenous, as the industry becomes more pollution-intensive emissions may fall via the tax and subsidy.*

This section concludes by pointing out that the total effect on industry emissions, arising from a change in the pollution intensity coefficient, vary across different degrees of product differentiation. The extent to which emissions fall/rise (as captured by the total change in $e = e(\tau(\delta), \varsigma(\delta), \delta)$) becomes prominent for small values of γ i.e., industries characterized by more differentiated products. The relation is captured in figure 2. Intuitively, for small values of γ emissions rise by more if the direct effect is large since output rises rapidly for small values of γ. If the subsidy effect is large, then with rapid changes in output/emissions the government adjusts the subsidy sufficiently so that industry emissions fall.

2.2 Free Entry and Exit

This section looks at the case where the number of firms is endogenous. The free entry and exit of firms is characterized by the zero-profit condition, $\pi = 0$. Equations (2), (3) along with $\pi = 0$ implicitly determine the free-entry equilibrium values of output and emissions by each firm and the number of firms, n. Total differentiation of the first-order conditions and the zero-profit condition yields the effects of τ, ς and γ on q, e, n and a (see the Appendix for a derivation). In particular, $\partial n/\partial\gamma = -(n-1)/\gamma < 0$ and $\partial q/\partial\gamma = 0$ (these are consistent with Fujiwara (2009)) and $\partial e/\partial\gamma = 0$. Also, $\partial n/\partial\varsigma = 1/q\gamma > 0$, $\partial q/\partial\varsigma = 0$ and $\partial e/\partial\varsigma = 0$. The effect of the tax, τ, on output and the number of firms is given by

$$\breve{\Delta}\frac{\partial q}{\partial \tau} = -\gamma q(eC_{ee} + qC_{eq}) > 0; \qquad \frac{\partial n}{\partial \tau} = -\frac{e}{q^2\gamma} - \frac{(n-1)}{q}\frac{\partial q}{\partial \tau} \tag{18}$$

$$\breve{\Delta}\frac{\partial e}{\partial \tau} = q\gamma(2\beta q + C_{qq}q + eC_{eq} + qC_{qe}^2/C_{ee} - qC_{qe}^2/C_{ee}) > 0 \tag{19}$$

where under the end-of-pipe assumption $\breve{\Delta} = -\gamma q^2\left(C_{ee}2\beta + C_{ee}C_{qq} - C_{qe}^2\right) = -2\beta\gamma q^2 < 0$, $(eC_{ee} + qC_{eq}) = -aC_{ee} = -a < 0$, and $qC_{qq} + eC_{eq} = (q\delta - e)\delta = a\delta > 0$.[12] An increase in the emission tax lowers output per firm and emissions, but the effect on the number of firms is ambiguous. Intuitively, an increase in the emission tax, on the one hand, raises marginal production costs thereby decreasing emissions, profits and the number of firms; on the other, a higher tax induces abatement, which raises profits via lower tax payments and so the number of firms rises, and as a result, each firm produces less, thereby reducing emissions. These results illustrate the important role of abatement efforts where changes in output work via the abatement effect. The effect of the tax on n and q are not qualitatively similar to Fujiwara (2009); this is because abatement efforts are assumed away in Fujiwara's model. For completeness it is noted that results hold in the special case

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[12] The pollution intensity coefficient, δ, is given by $-C_{eq}/C_{ee}$; see Lahiri and Symeonidis(2007).

of the end-of-pipe and industry emissions, $E = ne$, fall with the tax since the increase in the number of firms via the abatement effect is completely offset by the reduction in per-firm emissions, e.

Optimal policy is determined through welfare maximization. In particular, the government solves

$$\max_{\tau,\varsigma} W = CS + ne\tau - qn\varsigma - \varphi \tag{20}$$

where profits are not present because of the free-entry condition. Optimization yields two first-order conditions which implicitly determine the optimal tax,τ^{**}, and subsidy, ς^{**}. After some simplification these yield the following expressions for τ^{**} and ς^{**}:

$$\begin{aligned} \tau^{**} &= \varphi' - \frac{q\beta q_\tau}{e_\tau} + \varsigma^{**}\frac{q_\tau}{e_\tau} \qquad (21) \\ \varsigma^{**} &= \frac{-eq\beta\partial q/\partial\tau}{q(\partial e/\partial\tau) - e(\partial q/\partial\tau)} \qquad (22) \end{aligned}$$

The first term in (21) denotes marginal damages, the second the output distortion (which is negative), and the last term the subsidy. In the special case where the cost function is of the end-of-pipe the subsidy is negative, and the tax is positive and less than marginal damages.[13] A negative subsidy controls the number of firms and emissions, and a tax below marginal damages tackles pollution but also addresses the output distortion. In contrast to Fujiwara (2009) the presence of the subsidy (which is effectively a tax) and abatement efforts afford a positive tax below marginal damages. Additionally, in the special case where $C_{qe} = 0$, the subsidy is positive and the tax exceeds marginal damages; this result is also obtained by Katsoulacos and Xepapadeas (1995) where the optimal tax exceeds marginal damages when marginal abatement costs are independent of output.

In particular, in the special case of the end-of-pipe cost function the expressions in (21) and

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[13] Formally for any ς, $W_\tau > 0$ at $\tau = 0$, which indicates a positive optimal tax under the concavity assumption of the W function.

(22) can be re-written as

$$\tau^{**} = \varphi^{'} + \varsigma^{**}\frac{q}{e} > 0 \tag{23}$$

$$\varsigma^{**} = \frac{-eq\beta q_{\tau}}{q_{\tau}a - q} < 0 \tag{24}$$

Next, the effect of product differentiation on optimal policy is examined. Total differentiation (see the Appendix for a detailed derivation) of $W_{\tau} = (\tau^{**}(\gamma), \varsigma^{**}(\gamma), \gamma) = 0$ and $W_{\varsigma} = (\tau^{**}(\gamma), \varsigma^{**}(\gamma), \gamma) = 0$ yields (subscripts denote partial derivatives)

$$\Omega\tau^{**}_{\gamma} = -W_{\varsigma\varsigma}W_{\tau\gamma} + W_{\varsigma\gamma}W_{\tau\varsigma} < 0 \tag{25}$$

$$\Omega\varsigma^{**}_{\gamma} = -W_{\tau\tau}W_{\varsigma\gamma} + W_{\tau\gamma}W_{\tau\varsigma} > 0 \tag{26}$$

where $\Omega = W_{\tau\tau}W_{\varsigma\varsigma} - W_{\tau\varsigma}W_{\varsigma\tau} > 0$ and $W_{\tau\tau} < 0$, $W_{\varsigma\varsigma} < 0$ due to the concavity of the welfare function, and $q_{\tau} < 0$, $\varsigma^{**} < 0$, $W_{\varsigma\tau} = W_{\tau\varsigma} < 0$, $W_{\varsigma\gamma} > 0$, $W_{\tau\gamma} < 0$. Intuitively, there are several effects at play here which affect the tax when products become more differentiated. Firstly, with more differentiated products the government raises the tax to address higher industry emissions, but also to induce (raise) abatement, raise the number of firms and therefore achieve gains from product variety. As for the subsidy, the government lowers it as products become more differentiated in order to control for the number of firms. Since the optimal subsidy is negative, this reduction in the subsidy simply indicates that the government decides to control the number of firms and emissions via taxation. Notice that in the presence of the subsidy (i.e., tax) and the effect of abatement via the number of firms, the government can afford a higher tax, τ, to address both higher emissions and achieve gains from product variety.[14] In the case where products are homogeneous (i.e., $\beta \simeq \gamma$) the gains from product variety decrease and, as a result, the government is more likely to raise the tax (and lower the subsidy) by less; this is analogous to the excess entry case refer to in Fujiwara (2009, p. 245).

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[14] Indeed, in the case of no abatement the effect of the tax via output, q, vanishes and, as a result, (i) the optimal subsidy is zero (see the expression in (24)), and the closed-form solution for the tax, assuming $\varphi' = en$, is given by $\tau = q(\delta(\alpha - c) - 2\beta + \gamma)/(\gamma + \delta^2)$, which is analogous to Fujiwara (2009) and therefore analogous results are obtained.

Proposition 6. *Let the cost function be of the end-of-pipe. Then, in the case where the number of firms is endogenous, the optimal emission tax rises and the subsidy falls with more differentiated products.*

Proposition 6 indicates that, *via the tax and subsidy*, emissions fall as products become more differentiated since a higher tax and lower subsidy lower total emissions. Moreover, if the direct effect of γ on the number of firms (which is negative) is small (i.e., the effect of product differentiation on emissions for given tax and subsidy), then total emissions are positively related to the degree of product differentiation, meaning that emissions fall as products become differentiated. Since the direct effect works via changes in the number of firms and, *via the direct effect*, n is a decreasing and convex function of γ (i.e., $n_\gamma < 0$, $n_{\gamma\gamma} > 0$), then for initially large values of γ (i.e., close to β) increases in n are small as products become more differentiated and, as a result, emissions do not rise as much via the direct effect.

The change in total emissions via policy is given by differentiating $E = E(\tau(\gamma), \varsigma(\gamma))$:

$$dE/d\gamma = e\left(n_\tau \tau_\gamma + n_\varsigma \varsigma_\gamma\right) + n e_\tau \tau_\gamma > 0 \tag{27}$$

where $e_\varsigma = 0$.

Lemma 2. *Let the cost function be of the end-of-pipe. Then, via changes in the tax and subsidy total emissions fall as products become more differentiated. Total emissions fall with more differentiated products if the direct effect is small ($\gamma \simeq \beta$).*

3 Conclusion

This paper examines the sensitivity of optimal policy with respect to the degree of product differentiation. The analysis suggests that environmental policy is indeed sensitive to the degree of product differentiation; emissions are also sensitive to the adjustment of policy. It is shown that in the case where the number of firms is exogenous the optimal tax and subsidy rise as products become more differentiated. In the case where the number of firms is endogenous the subsidy falls and the tax rises as products become more differentiated. Additionally, the analysis suggests that the extent to which the government adjusts policy depends, among others, on whether the industry is characterized by a high degree of product differentiation. These results are important because they indicate that policymakers, when setting policy, should be aware of the degree of product differentiation, particularly in industries where firms are taking steps to differentiate their products in order to capture particular markets niches and lower production and abatement costs.

Inevitably, the analysis has left out important aspects of environmental policy and generalization of results is limited by the partial equilibrium context of the model. For instance, environmental research and development, which currently plays an important role in environmental policy setting in the form of funding to the energy sector in the EU, is not part of the analysis. Even though the analysis touches on this issue tangentially by looking at how policy changes with an exogenous change in the pollution intensity coefficient, endogenizing environmental research and development would be an important extension to the present analysis. Another possible future line of research would be to extend the model to an international context and so issues of cross-border pollution could be analyzed. Additionally, it would be worth exploring the role of the subsidy within the context of an emission permit trading system; this set-up would be particularly relevant to the case of the European Union. In the present work the degree of horizontal product differentiation is at the center of the analysis and, therefore, the role of consumers (vertically differentiated markets) may be included thereby rendering the analysis richer and applicable to a larger array of industries.

The strategic choice of abatement represents an important extension of the model. To see this consider two firms competing under Cournot conditions where abatement is chosen strategically and product differentiation is endogenized by assuming a pollution intensity coefficient $\delta = \delta(\gamma)$ and $\delta' > 0$. In the presence of imperfect competition in the output market, the choice of abatement by, say, firm 1 has an effect on the choice of output by, say, firm 2. This is the strategic effect referred to in e.g., Montero (2002). With these in mind, differentiated products induce firm 1 to abatement more and so output by this firm rises; it rises because as more abatement takes place, emissions by firm 1 falls and so tax payments are reduced. By differentiating itself (decrease in γ) firm 1 can achieve this. As a result from the choice of abatement by firm 1, firm 2 may react by increasing output thus leading to a case where firm 1 decides to abatement less. One could still see positive abatement as a result of product differentiation, but to a lesser extent precisely because of the strategic effect. Thus, depending on the effect the strategic effect has on output and abatement, the government may increase or decrease (the effect is not clear *a priori*) the tax as products become more differentiated. The adjustment of the output subsidy will also depend on the extent to which the strategic effect is large or small and how it exacerbates the output distortion. But the adjustment of optimal policy will also depend on the choice of abatement (where abatement depends on γ) and output by a firm which do not affect the other firms' choice of abatement and output; this direct effect adds to the complexity of the model and the adjustment of policy. In order to determine how results would change, a deeper analysis would be needed.

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A Appendix

First, it is shown that emissions, *via the policy adjustment*, is an increasing and concave function of γ. Then, it is shown that when the direct effect is included emissions is a decreasing and convex function of γ.

Start with $e = \delta q - \tau$. Then differentiation w.r.t. γ gives the effect via policy:

$$\tau_\gamma \frac{\delta^2 + 2\beta + \gamma(n-1)}{2\beta + \gamma(n-1)} + \varsigma_\tau \frac{\delta}{2\beta + \gamma(n-1)}$$

Substituting τ_γ and ς_γ and simplifying gives

$$\frac{(\alpha - c)(n+1)(n-1)\delta}{\chi^2 \tilde{\Delta}} \left(n\delta^2 + (n-1)(\beta + \gamma n)\right) > 0 \tag{A.1}$$

where $\chi = n\delta^2 + (n+1)(\beta + \gamma(n-1))$ and $\tilde{\Delta} = 2\beta + \gamma(n-1)$. This shows that emissions, *via the policy adjustment*, is an increasing function of γ. Next, it is shown that emissions, *via the policy adjustment*, is a concave function of γ. In particular, differentiation of (A.1) w.r.t. γ gives

$$(n+1)\left(\beta(\beta + \gamma(n+1))(2-n)3 - 3n\delta^2\beta - 2\gamma(n-1)^2(\beta + \gamma(n+1))\right) - n\delta^2\chi < 0 \tag{A.2}$$

Next, it is shown that when the direct effect is included emissions is decreasing and convex in γ. Using $e = \delta q - \tau$ and substituting the expressions in (10), differentiation w.r.t. γ yields (13) and (14).

Next, the effects via policy and the direct of δ are derived. In particular, using $e = \delta q - \tau$ differentiation w.r.t. δ gives

$$\tau_\delta(\delta q_\tau - 1) + \delta q_\varsigma \varsigma_\delta + q + \delta q_\delta \tag{A.3}$$

where the *effect via policy* is obtained using (15) and (16):

$$\frac{(\alpha - c)n}{\chi^2 \tilde{\Delta}} \left[(n\delta^2 - (n+1)(\beta + (n-1)\gamma))(\delta^2 + 2\beta + \gamma(n-1)) - 2\beta\delta^2(n+1)\right] \tag{A.4}$$

The *direct effect* is given by

$$\frac{(\alpha-c)n}{\chi\tilde{\Delta}}\left(-n\delta^2+(n+1)(2\beta+\gamma(n-1)\right) \tag{A.5}$$

Combining (A.4) and (A.5) yields

$$\frac{(\alpha-c)n}{\chi^2\tilde{\Delta}}\left(-n\delta^2+(n+1)(\beta+\gamma(n-1)\right) \tag{A.6}$$

where the first term is the subsidy effect and the second is the direct effect.

The following is the system utilized for the comparative statics analysis in section (2.2). In particular, total differentiation of $\pi_q = 0$, $\pi_e = 0$ and $\pi = 0$ yields the following system

$$\begin{bmatrix} -2\beta-\gamma(n-1)-c_{qq} & -c_{qe} & -q\gamma \\ -c_{eq} & -c_{ee} & 0 \\ -q\gamma(n-1) & 0 & -q^2\gamma \end{bmatrix} \begin{bmatrix} dq \\ de \\ dn \end{bmatrix} = \begin{bmatrix} -d\varsigma+q(n-1)d\gamma \\ d\tau \\ ed\tau-qd\varsigma+q^2(n-1)d\gamma \end{bmatrix}$$

where the determinant of the coefficient matrix $\breve{\Delta} = -q^2\gamma\left(2\beta c_{ee}+c_{ee}c_{qq}-c_{qe}c_{eq}\right) < 0$ and the term $-q\gamma(n-1)$ is obtained using the first order condition $P-c_q+\varsigma = \beta q$ in $P-c_q+\varsigma-q(\beta+\gamma(n-1))$. Using Cramer's rule gives

$$\breve{\Delta}dq = -\gamma q\left[qc_{qe}+ec_{ee}\right]d\tau+[0]d\varsigma \tag{A.7}$$

$$\breve{\Delta}de = \gamma q\left[(2\beta+c_{qq})q+ec_{qe}\right]d\tau+[0]d\varsigma \tag{A.8}$$

$$dn = \left[\frac{-e}{\gamma q^2}-\frac{(n-1)(qc_{qe}+ec_{ee})}{q^2(2\beta c_{ee}+c_{ee}c_{qq}-c_{qe}c_{eq})}\right]d\tau+\left[\frac{q}{\gamma q^2}\right]d\varsigma \tag{A.9}$$

The effect of γ on q, e and n, for a given policy ς and τ, is given by $\breve{\Delta}dq/\gamma = 0$, $\breve{\Delta}de/\gamma = 0$, $dn/\gamma = -(n-1)/\gamma$.

Under the end-of-pipe assumption one can find the closed-form solution for q, e and n sequentially. In particular, substituting (3) into (2) and using the f.o.c in the zero-profit condition gives q^{**}; emissions is given by $e^{**} = \delta q^{**}-\tau$. To obtain n simply substitute q^{**} into $\pi_q = 0$.

$$q^{**} = \sqrt{\frac{f-t^2/2}{\beta}} \qquad n = \frac{(\alpha-\tilde{c}+\varsigma-\delta\tau)\sqrt{\beta/(f-\tau^2/2)}-2\beta+\gamma}{\gamma} \tag{A.10}$$

The change in total emission, $dE = nde + edn$, is given by

$$dE = \left[\frac{-e^2}{q^2\gamma} - \frac{\delta\tau}{2\beta q} - \frac{\tau(n-1)}{2\beta q^2} - n\right] d\tau + \left[\frac{eq}{\gamma}\right] d\varsigma \tag{A.11}$$

As a remark, in the special case of an end-of-pipe function $c(q,e) = \tilde{c}(q) + h(\delta(q) - e)$, $\tilde{c}^{'} > 0, \delta' > 0, \delta^{''} > 0, h^{'} > 0$, $h^{''} > 0$ one obtains $c_q = \tilde{c}^{'} + h^{'}\delta^{'}$, $c_{qq} = \tilde{c}'' + h^{'}\delta^{''} + h^{''}\delta'^2$, $c_e = -h^{'}$, $c_{ee} = h^{''}$, $c_{qe} = c_{eq} = -h^{''}\delta^{'}$ and $c_{qq}c_{ee} - c_{qe}^2 = 0$. Also, $qc_{qq} + ec_{qe} = q\delta^{''}h^{'} + h^{''}\delta^{'}(\delta^{'}q - e)$, $qc_{qe} + ec_{ee} = h^{''}(\delta^{'}q - e)$, $c_qc_{eq} - c_ec_{qq} = -\tilde{c}^{'}h^{''}\delta' + h^{'}(\tilde{c}^{''} + h^{''}\delta^{''})$ and $c_{qe}c_e - c_qc_{ee} = h^{''}\delta'h^{'} - h^{''}(\tilde{c}^{'} + h^{'}\delta') = -\tilde{c}^{'}h^{''}$. where $-a = -q\delta^{'} + e$.

Next, we derive the expressions in (25) and (26). First, to achieve this I assume and end-of-pipe function where $c(q) = \hat{c}q$, $h = (dq-e)^2/2$ and $\delta^{'} = d$. Then, total differentiation of $W_\tau = (\tau^{**}(\gamma), \varsigma^{**}(\gamma), \gamma) = 0$ and $W_\varsigma = (\tau^{**}(\gamma), \varsigma^{**}(\gamma), \gamma) = 0$ yields

$$\Omega\tau_\gamma^{**} = -W_{\varsigma\varsigma}W_{\tau\gamma} + W_{\varsigma\gamma}W_{\tau\varsigma} < 0 \tag{A.12}$$

$$\Omega\varsigma_\gamma^{**} = -W_{\tau\tau}W_{\varsigma\gamma} + W_{\tau\gamma}W_{\varsigma\tau} > 0 \tag{A.13}$$

where $\Omega = W_{\tau\tau}W_{\varsigma\varsigma} - W_{\tau\varsigma}W_{\varsigma\tau} > 0$. To derive the signs of the above expressions from $W = CS(Q) + ne\tau - nq\varsigma + \varphi(E)$ I obtain

$$W_\tau = -Qp_\tau + ne + (\tau - \varphi^{'})E_\tau - \varsigma Q_\tau \tag{A.14}$$

$$W_\varsigma = -Qp_\varsigma - nq + (\tau - \varphi^{'})E_\varsigma - \varsigma Q_\varsigma \tag{A.15}$$

where setting (A.14) and (A.15) equal to zero and simplification gives equations (21) and (22). Then, differentiation of (23) with respect to τ gives $(-\varsigma q_\tau + (\tau - \varphi^{'})e_\tau - \varphi^{''}eE_\tau + e)n_\varsigma = 0$. Also, $E_{\tau\varsigma} = n_\varsigma e_\tau + en_{\tau\varsigma}$, $e_{\tau\varsigma} = 0$, $n_\tau e_\varsigma = 0$; and $Q_{\tau\varsigma} = n_\varsigma q_\tau + qn_{\tau\varsigma}$, $q_{\tau\varsigma} = 0$, $q_\varsigma = 0$. Then, using $P_\tau = -q_\tau(\beta + \gamma(n-1)) + e/q - \tau(n-1)\gamma/2\beta q^2$ one obtains $W_\tau = Q\beta q_\tau + (\tau - \varphi')E_\tau - \varsigma Q_\tau$. Using (23)

W_τ simplifies to $W_\tau = Q\beta q_\tau + (\tau - \varphi')ne_\tau - \varsigma n q_\tau$. Then, differentiation of W_τ w.r.t. ς gives

$$\begin{aligned}
W_{\tau\varsigma} &= -Qp_{\tau\varsigma} - p_\tau Q_\varsigma + E_\varsigma + (\tau - \varphi^{'})E_{\tau\varsigma} - \varphi^{''}E_\tau E_\varsigma - Q_\tau - \varsigma Q_{\tau\varsigma} \\
&= Q\beta q_{\tau\varsigma} + \beta q_\tau Q_\varsigma + (\tau - \varphi')E_{\tau\varsigma} - \varsigma Q_{\tau\varsigma} - \varphi'' E_\tau E_\varsigma - Q_\tau \\
&= q\beta q_\tau n_\varsigma - Q_\tau - en_\varsigma \\
&= \frac{\tau(-\beta+\gamma)}{2\beta q\gamma} < 0 \qquad \text{(A.16)}
\end{aligned}$$

Then, the direct effect of γ (for given tax) gives $q_{\tau\gamma} = 0$, $e_{\tau\gamma} = 0$, $q_\gamma = 0$, $e_\gamma = 0$, $Q_{\tau\gamma} = q_\tau n_\gamma + qn_{\tau\gamma}$; $E_{\tau\gamma} = e_\tau n_\gamma + en_{\tau\gamma}$; and $qE_{\tau\gamma} - eQ_{\tau\gamma} = n_\gamma(q_\tau a - q) > 0$. Also, from (24) one gets $\beta q_\tau q n_\gamma + (\varsigma/e)n_\gamma(q_\tau a - q) = 0$ and differentiation of (21) with respect to τ gives $(-\varsigma q_\tau + (\tau - \varphi^{'})e_\tau - \varphi^{''}eE_\tau + e) = 0$. Then, differentiation w.r.t. γ in $W_\tau = Q\beta q_\tau + (\tau - \varphi')ne_\tau - \varsigma n q_\tau$ gives

$$\begin{aligned}
W_{\tau\gamma} &= -Qp_{\tau\gamma} - p_\tau Q_\gamma + E_\gamma + (\tau - \varphi^{'})E_{\tau\gamma} - \varphi^{''}E_\gamma E_\tau - \varsigma Q_{\tau\gamma} \\
&= Q\beta q_{\tau\gamma} + \beta q_\tau Q_\gamma + (\tau - \varphi')E_{\tau\gamma} - \varsigma Q_{\tau\gamma} - \varphi'' E_\tau E_\gamma \\
&= \beta q_\tau q n_\gamma + \frac{\varsigma q}{e} n_\gamma (q_\tau a - q) - \varphi'' E_\tau E_\gamma \\
&= -\varphi'' E_\tau E_\gamma < 0 \qquad \text{(A.17)}
\end{aligned}$$

Then, (A.15) simplifies to $W_\varsigma = (\tau - \varphi')E_\varsigma - \varsigma Q_\varsigma = n_\varsigma((\tau - \varphi')e - q\varsigma)$, where $-QP_\varsigma - nq = 0$, $q_\varsigma = 0$ and $e_\varsigma = 0$. Differentiation w.r.t. γ i.e., the direct effect of γ (for given tax) yields

$$\begin{aligned}
W_{\varsigma\gamma} &= -Qp_{\varsigma\gamma} - p_\varsigma Q_\gamma - Q_\gamma + (\tau - \varphi^{'})E_{\varsigma\gamma} - \varphi^{''}E_\varsigma E_\gamma - \varsigma Q_{\varsigma\gamma} \\
&= n_\varsigma((\tau - \varphi')e_\gamma - \varphi'' e E_\gamma - \varsigma q_\gamma) + ((\tau - \varphi')e - q\varsigma)\left(\frac{-1}{q\gamma^2}\right) \\
&= \frac{-E_\gamma e \varphi^{''}}{q\gamma} > 0 \qquad \text{(A.18)}
\end{aligned}$$

where $e_\gamma = 0$, $q_\gamma = 0$ and $(\tau - \varphi')e - q\varsigma = 0$.

Figure 1: Fixed Number of firms, Product Differentiation and Optimal Policy ($\beta = 1, \alpha = 1, \delta = 1, n = 15, \hat{c} = .5$)

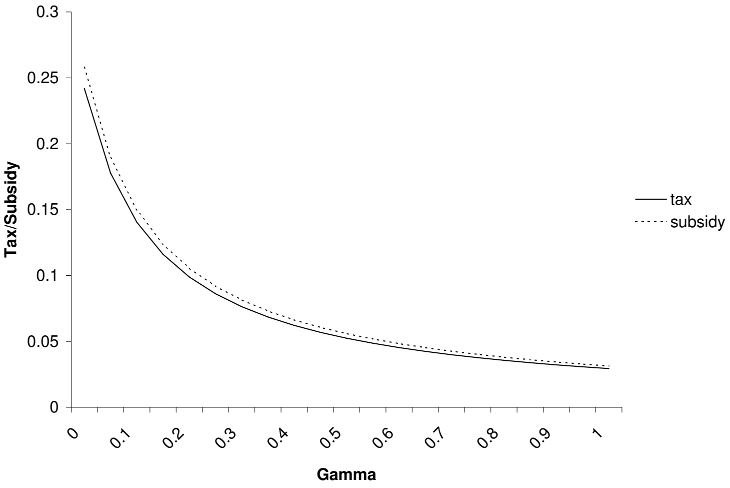

Figure 2: Fixed Number of firms, Product Differentiation, Emissions and the Pollution Intensity Coefficient