4^{th} *URSI AT-RASC, Gran Canaria, 19 – 24 May 2024*

Analytical Method to Model Spatiotemporal Metasurfaces

Mario Pérez-Escribano(1), Salvador Moreno-Rodríguez(2), Antonio Alex-Amor(3),
Juan Valenzuela-Valdés(2), Pablo Padilla(2), and Carlos Molero(2)
(1) Telecommunication Research Institute (TELMA), Universidad de Málaga,
E.T.S. Ingeniería de Telecomunicación, 29010 Málaga, Spain
(2) Department of Signal Theory, Telematics and Communications,
Research Centre for Information and Communication Technologies (CITIC-UGR), University of Granada, Granada, Spain
(3) University of Pennsylvania, Department of Electrical and Systems Engineering,
Philadelphia, Pennsylvania 19104, United States

Abstract

This work introduces a theoretical framework to develop fully-analytical and multi-modal equivalent circuits that are applicable to spatiotemporal metasurfaces. Here, time is considered a periodic variable. Thus, classical models for purely spatial structures are generalized to cover space-time scenarios. This paper explores various periodic metastructures from a circuital perspective, considering just time periodicity or space-time periodicities. The performance of the analytical equivalent circuits is validated through an external FDTD code. The results suggest that our analytical model is a powerful tool to describe and design novel and advanced space-time microwave and photonic systems.

1 Introduction

The study and development of space-time systems have become a hot topic in the electromagnetics and optic fields since a few years ago [1–3]. Special attention has been devoted to metasurfaces/metamaterials governed by periodic modulations and/or embedded in moving media [4–6]. The range of applicability of space-time metasurfaces is vast: direction-of-arrival (DOA) estimation [7], non-reciprocal systems [8], novel conceptions of beamformers and frequency mixers [9, 10], digital and programmable surfaces [11], antireflection temporal coatings [12], microwave circuit and antenna engineering [13, 14], gain amplifiers [15] or Doppler clocking [16], among others. However, the complex electromagnetic analysis of these structures represents the main drawback for them to be studied, designed, and developed. To date, most popular commercial software lacks specific modules applied to analyze time-modulated systems.

The use of circuit-model approaches [17] has proven to be a good solution for analyzing space-time metasurfaces under periodic modulations [18]. Circuit models have a long history of resolving wave-guiding problems in microwave and millimeter-wave regimes. Very original and popular solutions are found in [19], where many waveguide discontinuities are modeled via lumped elements. The subsequent emergence of frequency selective surfaces (FSSs) in terms of periodic structures [20], and the corresponding analysis techniques focused on the unit cell [21] have extended the range of application of circuit models, covering scenarios defined by *waveguides* surrounded by periodic boundary conditions (PBC) [22]. The extension to space-time systems demands circuit models capable of reproducing a multi-modal response since plenty of harmonics carrying energy along several spatial directions appear naturally.

This paper describes a technique to derive equivalent circuits for time-varying and space-time-varying metasurfaces. The methodology is inspired by the multi-modal and fully-analytical circuit derivations for spatially-periodic structures in [23–25]. The inclusion of time as a new periodic variable has been successfully tested in [18,26], where an incident plane wave feeds a simple time-varying screen. This paper extends the method to two-dimensional spatiotemporal screens (3 different periodicities occur in the problem). The method is applied and validated in a space-time 1D grating excited by an external plane wave.

2 Analytical method

Let us consider an extremely thin (negligible thickness) space-time metasurface standing in free space. This structure is externally excited by the incidence of a plane wave with period T_0 ($\omega_0 = 2\pi/T_0$). To generalize, the metasurface is assumed to be spatially periodic along the transverse directions $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$, controlled by p_x and p_y respectively. In addition, the structure is periodically modulated in time with periodicity value T_s ($\omega_s = 2\pi/T_s$). Given the problem's periodic nature, the structure's analysis is reduced to the analysis of the unit cell, which is equivalent to analyzing a discontinuity waveguide problem bounded by periodic boundary conditions.

Figure 1(a) sketches the unit cell in lateral perspective (YZ-plane). Two regions are henceforth distinguished: left-hand side region, or *region (1)*, from which the incident wave

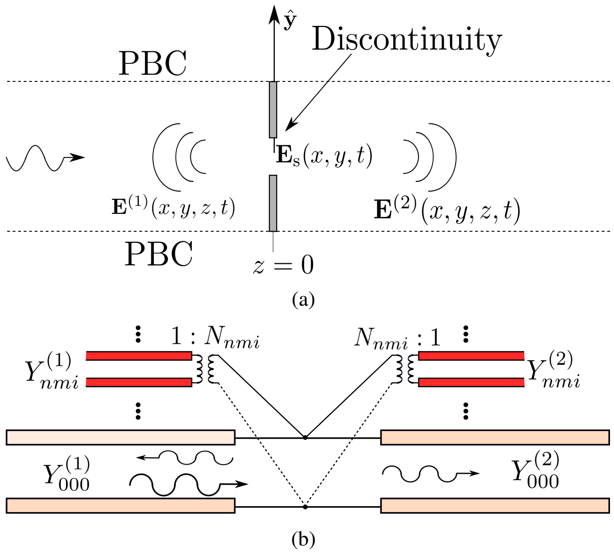


Figure 1. (a): Waveguide discontinuity problem (unit cell) of the space-time periodic structure. Lateral view (YZ-plane). (b): Circuit model representation of the unit cell. Each space-temporal harmonic is considered in terms of a single transmission line. Just a single higher-order harmonic is considered, though the number is theoretically infinite.

impinges, and right-hand side region, or *region (2)*, where the transmitted wave propagates. The discontinuity, where the metasurface stands ($z=0$), separates regions (1) and (2).

The interaction of the incident wave with the discontinuity induces diffraction, which is described in terms of Floquet harmonics. That means that the electromagnetic field at both sides of the discontinuity, $\mathbf{E}(x,y,z,t)^{(1)/(2)}$ and $\mathbf{H}(x,y,z,t)^{(1)/(2)}$, can be expressed in terms of a Floquet-Bloch expansion of harmonics as

$$\mathbf{E}(x,y,z,t)^{(1)/(2)} = \sum_m \sum_n \sum_i E_{nmi}^{(1)/(2)} \mathbf{e}_{nmi} \quad (1)$$

$$\mathbf{H}(x,y,z,t)^{(1)/(2)} = \sum_m \sum_n \sum_i Y_{nmi} E_{nmi}^{(1)/(2)} \mathbf{e}_{nmi}, \quad (2)$$

being n,m,i integer numbers. Each of the indexes n,m,i denotes a single periodicity of the problem (p_x, p_y and T_s). The longitudinal direction z is not periodic. The coefficients $E_{nmi}^{(1)/(2)}$ refer to the amplitude of a (n,m,i)-th space-time harmonic, while $\mathbf{e}_{nmi}$ denotes its vector form. The admittance term Y_{nmi} of a (n,m,i)-th harmonic may be of TM or TE nature. The amplitude of the incident harmonic ($n=m=i=0$) is composed by the contribution of the incident- and reflection-coefficient in region (1) $E_{000}^{(1)} = 1+R$, and the transmission coefficient in region (2) $E_{000}^{(2)} = T$.

The main key aspect for the circuit derivation is the knowledge of the electric-field profile $\mathbf{E}_s(x,y,t)$ at the space-time discontinuity ($z=0$), which allows for the following electric-field continuity condition

$$\mathbf{E}(x,y,z=0,t)^{(1)/(2)} = \mathbf{E}_s(x,y,t), \quad (3)$$

that after some manipulations leads to

$$E_{nmi}^{(1)/(2)} = E_{000} \frac{\int_{p_x}\int_{p_y}\int_{T_s} \mathbf{E}_s(x,y,t)\cdot[\mathbf{e}_{nmi}]^* dx dy dt}{\int_{p_x}\int_{p_y}\int_{T_s} \mathbf{E}_s(x,y,t)\cdot[\mathbf{e}_{000}]^* dx dy dt}, \quad (4)$$

rewritten as

$$E_{nmi}^{(1)/(2)} = E_{000}^{(1)} N_{nmi}\,. \quad (5)$$

N_{nmi} is interpreted as a complex transformer with turn ratio $1:N_{nmi}$ from a circuital point of view. It is straightforward to demonstrate that $E_{nmi}^{(1)} = E_{nmi}^{(2)}$, thus we can henceforth remove the superscripts $(1)/(2)$ and directly write E_{nmi}.

By now imposing the instantaneous continuity of the Poynting vector across the space-time discontinuity

$$\mathbf{H}^{(1)}(x,y,z,t)\times\mathbf{E}_s(x,y,t) = \mathbf{H}^{(2)}(x,y,z,t)\times\mathbf{E}_s(x,y,t), \quad (6)$$

and after some manipulations, the reflection coefficient R is finally expressed as

$$R = \frac{Y_{000}^{(1)} - Y_{000}^{(2)} - \sum_{n,m,i} |N_{nmi}|^2 (Y_{nmi}^{(1)} + Y_{nmi}^{(2)})}{Y_{000}^{(1)} + Y_{000}^{(2)} + \sum_{n,m,i} |N_{nmi}|^2 (Y_{nmi}^{(1)} + Y_{nmi}^{(2)})}\,. \quad (7)$$

The expression in (7) describes a circuit topology based on an infinite connection in parallel of transmission lines with admittances $Y_{nmi}^{(1)/(2)}$. Each of these transmission lines is actually related to a different and individual space-time Floquet harmonic. A sketch of the model is illustrated in Fig. 1(b).

3 Validation

In order to validate the equivalent circuit, two different systems will be considered. The first case is the simplest one, consisting of an infinitely extended metallic plane (perfect conductor is assumed), which appears and disappears periodically in time (see inset in Fig. 2). In the periodic cycle, the time in which the screen remains in the metallic state is given by DT_s, while $(1-D)T_s$ denotes the time in which the screen keeps vanished, with $0 \le D < 1$, D is the duty cycle. The structure is excited by a plane wave impinging normally and with frequency $\omega_0 = 2\pi \cdot 20 \cdot 10^9\,\mathrm{rad\,s^{-1}}$ ($T_0 = 1/[20\cdot 10^9]\,\mathrm{s}$). The time period of the screen is twice the period of the incident wave; namely, $T_s = 2T_0$. In this case, the nonexistence of spatial periodicity reduces the analysis to a 1D-periodic (time-only) problem. The circuit elements, as the transformers N_{nmi} or the harmonic amplitudes E_{nmi} can be perfectly described by temporal index, N_i, E_i. The incident wave excites *infinite* temporal harmonics, whose amplitude is mainly governed by the ratio T_s/T_0 and the duty cycle D. Fig. 2 shows the amplitude of the first

51 harmonics with orders ranging from $i = -25$ to $i = 25$. As can be seen, a few of them have similar amplitude, remarking the fundamental one ($i = 0$) and the harmonic with order $i = -4$. These results have been validated with an inhouse FDTD code intentionally developed for this purpose. The agreement between the results given by our model and CST is very good, confirming the circuit approach as a useful design tool.

A second case concerns a space-time grating alternating between its natural state and a fully metal screen, as shown in the inset of Fig. 3(a). The grating is spatially periodic along the y direction. Its periodicity is defined as $p_y = p$. The width of the grating slit is defined by the parameter w. The structure is excited by the incidence of a plane wave impinging normally. Again, the time modulation of the screen is $T_s = 2T_0$. The problem now has two periods; thus, the circuit elements can be described by two indexes: m and i. Notice that the index m refers to the spatial modulation, whereas i denotes the temporal one. When the incident wave illuminates the grating, infinite Floquet harmonics are excited at the space-time discontinuity. Fig. 3 shows the amplitude of some of these harmonics. We focus on those with orders m for $i = 0$ in Fig. 3(a), and the inverse case considering orders given by $m = 0$, $\forall i$. In both cases, comparisons with FDTD have been carried out to check the validity of the predictions given by the circuit model. As can be observed, a good agreement is again obtained. As a result of the findings, the present analytical framework positions as an interesting tool for the analysis and design of spatiotemporal metastructures.

Acknowledgements

This work has been supported by grant PID2020-112545RB-C54 funded by MCIN/AEI/10.13039/501100011033 and by the European Union NextGenerationEU/PRTR. It has also been supported by grants PDC2022-133900-I00, TED2021-129938B-I00, and TED2021-131699B-I00, and by Ministerio de Universidades and the European Union NextGenerationEU, under Programa Margarita Salas, and by MCIN/AEI/10.13039/501100011033 and the European Union NextGenerationEU/PRTR under grant IJC2020-043599-I.

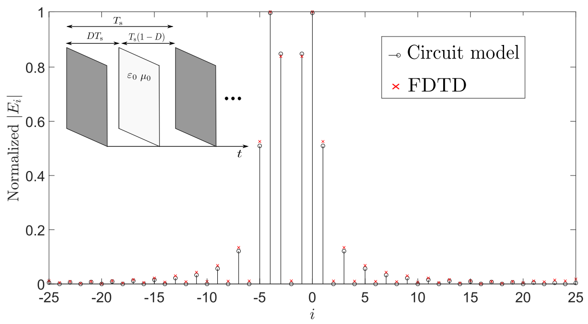
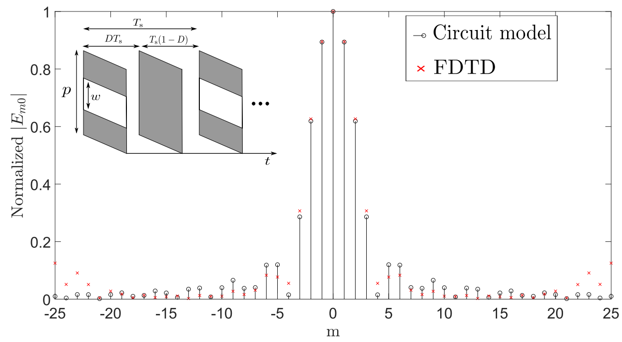
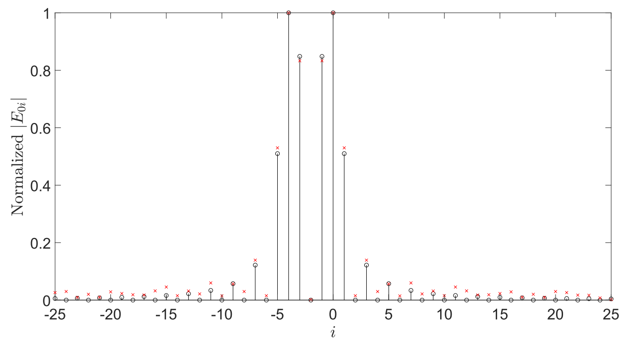


Figure 2. Representation of the amplitude associated with the excited harmonics E_i. The number is restricted to harmonics with order from $i = -25$ to $i = 25$. The metallic screen appears/dissapears with a frequency $\omega_s = \omega_0/2$. the angular frequency of the incident wave is $\omega_0 = 2\pi 20 \cdot 10^9\, s^{-1}$. The duty cycle for this case is $D = 0.5$. In the inset, grey elements represent metals.



(a)



(b)

Figure 3. Representation of the amplitude associated with the excited spacetemporal harmonics E_{mi}. Now, a metallic grating alternates, with a frequency $\omega_0 = \omega_s/2$, between its natural state and a metallic screen. The duty cycle for this second case is again $D = 0.5$. In the inset, grey elements represent metals. (a): Spatiotemporal harmonics with $i = 0$. (b): Spatiotemporal harmonics with m = 0. Structure parameters: $p = 10\,\text{mm}$, $w = 3\,\text{mm}$.

References

[1] N. Engheta, "Four-dimensional optics using time-varying metamaterials," Science, vol. 379, no. 6638, pp. 1190-1191, 2023.

[2] C. Caloz and Z. -L. Deck-Léger, "Spacetime Metamaterials—Part I: General Concepts," in IEEE Transactions on Antennas and Propagation, vol. 68, no. 3, pp. 1569-1582, March 2020.

[3] G. Ptitcyn, M. S. Mirmoosa, A. Sotoodehfar and S. A. Tretyakov, "A Tutorial on the Basics of Time-Varying Electromagnetic Systems and Circuits: Historic overview and basic concepts of time-modulation,"

IEEE Antennas and Propagation Magazine, vol. 65, no. 4, pp. 10-20, 2023.

[4] E. Galiffi *et al.*, “Photonics of time-varying media," Advanced Photonics, vol. 4, 014002, 2022.

[5] A. Alex-Amor, C. Molero, M. G. Silveirinha, "Analysis of Metallic Space-Time Gratings using Lorentz Transformations," Physical Review Applied, vol. 20, p. 014063, 2023.

[6] A. Bahrami, Z.-L. Deck-Léger, and C. Caloz, "Electrodynamics of Accelerated-Modulation Space-Time Metamaterials," Physical Review Applied, vol. 19, p. 054044, 2023.

[7] X. Fang et al., "Accurate Direction–of–Arrival Estimation Method Based on Space–Time Modulated Metasurface," in IEEE Transactions on Antennas and Propagation, vol. 70, no. 11, pp. 10951-10964, Nov. 2022, doi: 10.1109/TAP.2022.3184556.

[8] D. L. Sounas, A. Alù, "Non-reciprocal photonics based on time modulation," Nature Photonics, vol. 11, pp. 774–783, 2017.

[9] S. Taravati and G. V. Eleftheriades, "Generalized Space-Time-Periodic Diffraction Gratings: Theory and Applications," Physical Review Applied, vol. 12, p. 024026, 2019.

[10] S. Moreno-Rodríguez, A. Alex-Amor, P. Padilla, J. F. Valenzuela-Valdés and C. Molero, "Theory and Design of Space-Time Metallic Metasurfaces for Wireless Communications," ArXiv preprint arXiv:2312.16491.

[11] M. Q. Qi, Tie Jun Cui, X. Wan, J. Zhao, and Q. Cheng, “Coding metamaterials, digital metamaterials, and programmable metamaterials,” Light: Science and Applications, vol. 3, 2014

[12] V. Pacheco-Peña and N. Engheta, "Antireflection temporal coatings," Optica, vol. 7, pp. 323-331, 2020.

[13] A. Alvarez-Melcon, X. Wu, J. Zang, X. Liu and J. S. Gomez-Diaz, "Coupling Matrix Representation of Nonreciprocal Filters Based on Time-Modulated Resonators," IEEE Transactions on Microwave Theory and Techniques, vol. 67, no. 12, pp. 4751-4763, 2019.

[14] M. H. Mostafa, N. Ha-Van, P. Jayathurathnage, X. Wang, G. Ptitcyn, S. A. Tretyakov, "Antenna bandwidth engineering through time-varying resistance," Applied Physics Letters, vol. 22, p. 171703, 2023.

[15] J. B. Pendry, E. Galiffi, and P. A. Huidobro, "Gain in time-dependent media: a new mechanism," J. Opt. Soc. Am. B, vol. 38, no. 11, pp. 3360-3366, 2021

[16] B. Liu, H. Giddens, Y. Li, Y. He, S. Wong, and Y. Hao, "Design and experimental demonstration of Doppler cloak from spatiotemporally modulated metamaterials based on rotational Doppler effect," Optics Express 28, 3745-3755 (2020)

[17] C. G. Montgomery, R. H. Dicke and E. M. Purcell, *Microwave circuits*, McGraw-Hill Book Company, 1948.

[18] A. Alex-Amor, S. Moreno-Rodríguez, P. Padilla, J. F. Valenzuela-Valdés and C. Molero, "Diffraction Phenomena in Time-Varying Metal-Based Metasurfaces," Physical Review Applied, vol. 19, p. 044014, 2023.

[19] N. Marcuvitz, *Waveguide handbook*, McGraw-Hill Book Company, 1951.

[20] B. A. Munk, *Frequency Selective Surfaces: Theory and Design*, John Wiley, 2000.

[21] R. Mittra, C. H. Chan and T. Cwik, "Techniques for analyzing frequency selective surfaces-a review," in Proceedings of the IEEE, vol. 76, no. 12, pp. 1593-1615, Dec. 1988.

[22] G. Floquet, "Sur les équations différentielles linéaires à cofficients périodiques," in Ann. Sci. Ec. Norm. Sup., vol. 12, pp. 47-88, 1883.

[23] R. Rodriguez-Berral, C. Molero, F. Medina and F. Mesa, "Analytical Wideband Model for Strip/Slit Gratings Loaded With Dielectric Slabs," in IEEE Transactions on Microwave Theory and Techniques, vol. 60, no. 12, pp. 3908-3918, Dec. 2012.

[24] R. Rodríguez-Berral, F. Mesa and F. Medina, "Analytical Multimodal Network Approach for 2-D Arrays of Planar Patches/Apertures Embedded in a Layered Medium," in IEEE Transactions on Antennas and Propagation, vol. 63, no. 5, pp. 1969-1984, May 2015.

[25] C. Molero, A. Alex-Amor, F. Mesa, Á. Palomares-Caballero and P. Padilla, "Cross-Polarization Control in FSSs by Means of an Equivalent Circuit Approach," in IEEE Access, vol. 9, pp. 99513-99525, 2021.

[26] S. Moreno-Rodríguez, A. Alex-Amor, P. Padilla, J. F. Valenzuela-Valdés and C. Molero, "Time-Periodic Metallic Metamaterials Defined by Floquet Circuits," in IEEE Access, vol. 11, pp. 116665-116673, 2023.