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Abstract

We study the role of competitiveness, via interpersonal comparisons, in a society or a large organization. We consider a dynamic model of effort provision. Agents gain an extra utility by producing an outcome above a "comparison threshold" derived from the outcomes of their reference group in a random network. There are two different sources of competitiveness: the stringency of the comparison threshold and the weight given to relative performance in the utility function. We find that these two sources operate in opposite directions. Societies with higher competitiveness may not necessarily lead to more competitive outcomes (i.e., higher effort provision). Finally, we show that the effects of an increase in the density and variance of the network crucially depend on the stringency of the comparison threshold.
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1. Introduction

"*Always be the best and be far above others*", demanded Pelean, father of Achilles the Achaean, to his son when sending him to fight at Troy (Homer, Iliad 11.783). The ancient Greeks highly appreciated a competitive spirit, as reflected by the concept of *agon* (see, for instance,

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Knox, 1999; Colaguori, 2012). Such a spirit, based on interpersonal comparisons and relative performance, is still present in modern Western societies to a greater or lesser extent.

In this paper, we present a stylized dynamic model of effort provision to study the role of interpersonal comparisons in a society or in a large organization. In particular, agents' sole decision is how much (costly) effort to exert in a certain activity over time (e.g., studying for exams). This decision then leads to observable individual outcomes (e.g., the grades on such exams), which we assume to be concave on efforts. Agents may gain an extra utility from relative performance, depending on how much better their individual outcome is relative to that of their reference group in a random network. In particular, we introduce heterogeneity in our model by assigning to each agent a degree indicating the number of agents observed by her before making a decision, i.e., the size of her reference group.

Competitiveness is driven by two different sources: the *comparison threshold* and the *intensity of comparisons*. On the one hand, the *comparison threshold* is defined by a summary statistic of the reference group output. We consider three different scenarios. The "high comparison threshold" scenario corresponds to societies or organizations that, in line with our opening quotation of the *Iliad*, encourage individuals to "*be the best*". In this case, agents obtain an extra utility if their outcome is above the *maximum* outcome in their reference group. The "mild comparison threshold" scenario corresponds to a situation in which it suffices to "*be good enough*". In this case, agents obtain an extra utility if their outcome is above the *average* outcome of their peers. Finally, in the "low comparison threshold" case the behavioral rule is "*do not be the worst*", and individuals obtain an extra utility if their outcome is above the *minimum* outcome of their peers. On the other hand, the *intensity of comparisons* is determined by the weight given to relative performance in the agent's utility function.

We define a dynamic process in which, given their degree, agents repeatedly sample from the population their reference groups over time, and best-respond to the observed outcomes. The (comparison) network, so determined, specifies who is influenced by whom at different time periods. In this context, we analyze the long-run prediction (stationary state) of such dynamics for each competitiveness scenario. We also characterize how agents' degree affects their choice of effort. Consequently, we evaluate how the level of competitiveness in a society and the properties of the network affect aggregate outcomes.

Our results are as follows. We first show that agents' best response choices are dichotomous (Lemma 1), i.e., all agents optimally choose one of two effort levels, labeled the low effort and the high effort, when revising their actions. In fact, efforts are (weakly) strategic substitutes, i.e., the incentives for an agent to exert the high effort (weakly) decrease with the number of neighbors exerting the high effort. We then show that, for any comparison threshold and intensity of comparisons, there is a unique stationary state of the dynamics, which is globally stable (Proposition 1).

We find that in the stationary state, the probability of choosing the high effort crucially depends on the comparison threshold of the society (Proposition 2). In particular, with the high comparison threshold, the worst-connected agents are the ones that exert the highest levels of effort. In contrast, with the low threshold the opposite occurs. The result for the mild threshold is in-between. In such a case, it's possible that the probability of choosing the high effort is the same for all agents, regardless of their degree, or even that it varies non-monotonically with respect to degree.

We then analyze how the *aggregate outcome*, or *production*, in the stationary state depends on each of the two sources determining the level of competitiveness of the society. Indeed, in many real-world situations, societies and organizations evaluate their performance in terms of their aggregate outcome (rather than other measures that might be used to combine utilities).[1] First, we find that societies with a lower comparison threshold produce greater (aggregate) outcomes (Proposition 3). The intuition behind this finding is that, with a high comparison threshold agents may be discouraged from being prominent, since they compare themselves only with the "best", in contrast to other (less stringent) societies. A somewhat opposite result holds regarding the intensity of comparisons. Namely, societies with a higher intensity of comparisons lead to higher aggregate outcomes (Proposition 4). The overall effect of higher competitiveness depends on which of these two confronting forces prevails.

We then turn to analyze how changes in the degree distribution affect aggregate outcomes (Proposition 5). To this aim, we focus on the two polar cases, i.e., the high and low comparison thresholds. We find that in the former case the number of agents choosing the high effort is inversely related with the density of the network. In contrast, in the latter case a denser network increases the number of agents choosing the high effort. On the other hand, an increase in the variance of the network operates in the opposite direction to the density: with the high (low) comparison threshold a higher variance increases (decreases) the number of agents choosing the high effort.

Finally, we discuss some variations of our model. First, we study an alternative setting in which, rather than achieving an extra utility if being above others, agents incur in a disutility if their outcome is below the reference point, and also comment on the possibility of inequality averse agents. We find that regardless of the comparison threshold, there is a continuum of stationary states of the dynamics with all agents, independent on their degree, coordinating on the same level of effort (Proposition 6). Second, we consider the case in which agents are heterogeneous in terms of the cost of exerting effort. We find that there is a higher variation of observed efforts, but still, a lower comparison threshold can produce a greater outcome (Proposition 7).

There are several real-life examples in which agents compare themselves with their peers, a noteworthy one being that of education. Indeed, providing students with information about relative performance has an impact on their achievements at different stages of their academic formation (see, e.g., Azmat and Iriberri, 2010; Azmat et al., 2019; Murphy and Weinhardt, 2020). The educational psychology literature distinguishes between two ways of introducing social comparisons to students (see Darnon et al., 2007): The *performance-approach goals* focus on the aspiration to outperform others, whereas the *performance-avoidance goals* focus on the desire to avoid performing more poorly than others do. The first case is directly related to our benchmark model, in which individuals gain an extra utility from relative performance, whereas the second one relates to the discussed alternative scenario in which agents incur in a disutility. Empirical and experimental findings suggest that it is more effective to orientate students' motivation toward the aspiration to outperform others, rather than promoting the avoidance of (relative) poor performance.[2] In this sense, the theoretical results of our benchmark model can be useful to inform the design of educational policies. In particular, they might shed light on how the level of competitiveness across peers (susceptible of being modulated by educators) and the sizes of the

[1] For example, the OECD Programme for International Student Assessment (PISA) ranks countries according to the extent to which students have acquired some of the competencies proved essential to participate in the labor market and society, regardless of the students' effort costs.

[2] As pointed out by Darnon et al. (2007), several studies show that performance-avoidance goals are generally associated with low interest and poor performance by students, whereas performance-approach goals have positive effects, for example, in college classes. These two alternative motivational strategies can be induced using a different framing to encourage students for each case (see Elliot et al., 2005).

working groups or classes (correlated with the students' degrees) may affect students' efforts and achievements.

The remainder of the paper is organized as follows. In Section 2 we review the closest related literature. In Section 3 we describe the model. In Section 4 we present the results. We discuss variations of our model in Section 5. Finally, in Section 6 we conclude.

2. Related literature

There is an abundant body of literature that highlights the importance of reference points for understanding preferences. For instance, Kőszegi and Rabin (2006) assume a person's reference point is her rational expectations held in the recent past about current outcomes. They provide some implications of this framework for consumer and labor-supply decisions. More recent studies have considered that instead of expectations, the reference point could be derived from social comparisons, a premise that has been supported empirically. For instance, it has been documented that individuals feel worse off when others around them earn more (Luttmer, 2005) and that social comparisons have implications for personal well-being (White et al., 2006).

From a theoretical viewpoint, Roels and Su (2014) analyze a model in which all agents belong to the same reference group. They focus on the role of a social planner that can potentially adjust the amount of information provided to agents (e.g., the full distribution of outcomes versus the average outcome) to pursue some preestablished objectives. An important additional perspective in the analysis of social comparisons is the existence of individual-specific reference points derived from the location of agents in a social network. In this regard, network considerations have been included into models of conspicuous consumption (Ghiglino and Goyal, 2010) and status seeking (Immorlica et al., 2018).

We add a novel view to the network perspective, in which societies and organizations are compared in terms of their level of competitiveness. Rather than focusing on a static setting, we consider a dynamic model in a random network context. This framework has been largely used to study diffusion processes (see, e.g., Pastor-Satorrás and Vespignani, 2001; López-Pintado 2008, 2012; Galeotti and Rogers, 2013; Jackson and López-Pintado, 2013). Additionally, several studies have considered general adoption rules in networks (characterized by local summary statistics) through games of incomplete information (see, e.g., Jackson and Yariv, 2007; Galeotti et al., 2010; Feri and Pin, 2020). To this respect, some features of the stationary state of our dynamics resemble the Bayesian equilibrium predictions of the models of general network games with incomplete information.[3]

Finally, our model also shares some characteristics with contest games (see for a survey Corchón and Serena, 2018). Indeed, our framework might be interpreted as a set of overlapped "local contests" in each neighborhood, in which the sizes of the prizes are effort-dependent. Other recent papers that explore different dimensions of the relationship between contests and networks are Franke and Öztürk (2015) and Bozbay and Vesperoni (2018).

[3] One of the main methodological differences between our approach and these models is that, in our context, agents variate efforts best responding to different neighbors over time in the *interim* (when the realized networks are already at place). In the incomplete information case, the best responses are computed *ex-ante* and typically involve the use of mixed strategies. In Appendix C we explore the robustness of our main findings under this alternative approach.

3. The model

We consider a continuum of agents $N = [0, 1]$, who choose actions (effort choices) over time. Each agent $i \in N$ is characterized by her connectivity or degree. In particular, let $d_i \geq 1$ be the degree of agent i (which is exogenous and constant over time). Time is modelled continuously, with $t \in [0, \infty)$, and arrivals for revision opportunities follow a Poisson process with rate $\lambda \geq 0$.[4] If an individual, say i, receives the opportunity to revise her action at time t, i.e., an effort choice $e_i(t) \in [0, \infty)$, the agent randomly draws a set of d_i links to the population and observes the choices of the agents she gets linked to, i.e., her current d_i neighbors, denoted by $N_i(t)$.[5] Then, agent i selects a myopic-best response to the observed choices of her neighbors, given a payoff function that will be formally defined below. Note that our formulation implies that links do not need to be reciprocated, meaning that the fact that one individual pays attention to another does not imply that the reverse holds.

The population is described by a degree probability distribution. Formally, for each $d \geq 1$, let $P(d)$ denote the fraction of agents in the population with degree d. For instance, if $P(d)$ is a homogeneous distribution, then all agents have the same degree (i.e., $P(\bar{d}) = 1$, for some $\bar{d} \geq 1$), if $P(d)$ follows a Poisson distribution then all individuals have a similar degree, and if $P(d)$ is a Scale-free distribution then there is a significant fraction of individuals with a much larger degree than the average. We can identify a (comparison) network as a combination of the degree distribution $P(d)$ and the sampling process described above.

We shall now define the payoff function. Each effort choice $e \in [0, \infty)$ is transformed into a certain observable outcome (e.g., the grade obtained in an exam). We assume that all agents have the same technology function $f(e)$, which provides an output (i.e., outcome or production) to every level of effort. In addition, $f : [0, +\infty) \to \mathbb{R}$ is continuous, (strictly) increasing and (strictly) concave. Moreover, agents have a constant marginal cost $c > 0$ of exerting effort.[6] The payoff obtained by an individual corresponds to her output net of costs, plus an extra utility from relative performance that materializes only when her output exceeds a reference point, which depends on the behavior of her neighbors.[7] In particular, the relative performance component of the utility function is characterized by the *intensity of comparisons* $\theta > 0$ and the *comparison threshold* R, which is a statistic that resumes the neighbors' outcomes. Formally, for each $i \in N$, the reference point R_i is derived from R, i.e.,

$$R_i = R(\{f(e_j)\}_{j \in N_i}), \tag{1}$$

where we omit the time index for expositional simplicity. The utility or payoff function is, therefore, described as follows:

$$\pi_i(e_i, e_{-i}) = \begin{cases} f(e_i) - c \cdot e_i + \theta \cdot (f(e_i) - R_i) \text{ if } f(e_i) \geq R_i \\ f(e_i) - c \cdot e_i \text{ otherwise.} \end{cases} \tag{2}$$

The relative performance term, $\theta \cdot (f(e_i) - R_i)$, measures the excess of agent i's output with respect to her reference point, weighted by θ. There are two key elements included in this term:

[4] That is, at each point in time t, with exogenous instantaneous probability λdt each agent receives a revision opportunity to choose her action afresh. Conversely, with probability $1 - \lambda dt$, the agent keeps selecting the same action.

[5] Note that, at each arrival time the agent revising her action draws a new set of neighbors from the overall population and thus, the model basically corresponds to a random sampling process with heterogeneous agents.

[6] In Section 5 we extend this framework to heterogeneous populations in terms of effort costs.

[7] See Section 5 for the discussion of more general payoff structures. In particular, the study of the counterpart to our benchmark case, namely, situations where individuals receive a disutility whenever they are below the reference point.

On the one hand, θ, the intensity (or weight) of comparisons, which determines the weight of the non-standard part of the payoff function. On the other hand, different specifications of the statistic R reflect different comparison thresholds (which may be inherent to different societies/organizations, or even induced by a social planner). For any given value of the intensity of comparisons $\theta \geq 0$, we consider three stylized comparison thresholds:

(i) The first one, which we label the "*high comparison threshold*" (or threshold h), reflects the highest comparisons level. In such a case an agent only enjoys the extra utility from relative performance if she is the best (in terms of outcomes) among her neighbors. Thus,

$$R^h(\{f(e_j)\}_{j\in N_i}) = \max_{j\in N_i}\{f(e_j)\}.$$

(ii) The second one, which we label the "*mild comparison threshold*" (or threshold m), reflects an intermediate comparisons level. In such a case an agent only enjoys the extra utility from relative performance if she is good enough, in particular, if she is above (in terms of outcomes) the average of her neighbors. Thus,

$$R^m(\{f(e_j)\}_{j\in N_i}) = \frac{1}{d_i}\sum_{j\in N_i} f(e_j).$$

(iii) The third one, which we label the "*low comparison threshold*" (or threshold l), reflects the lowest comparisons level. In such a case an agent only enjoys the extra utility from relative performance if she is not the worst (in terms of outcomes) among her neighbors. Thus,

$$R^l(\{f(e_j)\}_{j\in N_i}) = \min_{j\in N_i}\{f(e_j)\}.$$

Hence, the competitive culture of a society is defined by a duple $(\theta, R) \in [0, +\infty) \times \{R^h, R^m, R^l\}$. Notice that, for a fixed specification of R, an increase in θ raises the intensity of comparisons. In addition, for any given value of θ, R^h represents the most stringent comparison threshold, R^m an intermediate threshold, and finally, R^l represents the softest one.

4. Results

Given the preliminaries of the model, our objective is to describe the evolution of effort choices over time and, consequently, to characterize the stationary (and stable) states of such dynamics. To this aim, we begin by deriving the best response function.

4.1. The best response function

We define two different effort levels, which turn out to be key for our analysis: the low effort level, $e_L = \arg\max_{e\geq 0} f(e) - c \cdot e$, and the high effort level, $e_H = \arg\max_{e\geq 0} f(e) - c\cdot e + \theta \cdot f(e)$. Moreover, let us define a cut-off value r satisfying $f(e_L) - c\cdot e_L = f(e_H) - c\cdot e_H + \theta \cdot (f(e_H) - r)$. It directly follows that:

$$f'(e_L) = c \tag{3}$$

$$f'(e_H) = \frac{c}{1+\theta} \tag{4}$$

$$r = \frac{(1+\theta)f(e_H) - f(e_L) - c(e_H - e_L)}{\theta} \tag{5}$$

Notice that, due to the concavity of f, $e_L < e_H$. The best response function is derived in the following lemma.

Lemma 1. *For an agent i with reference point R_i the best response function is*

$$BR_i = \begin{cases} e_H & \text{if } R_i < r \\ \{e_H, e_L\} & \text{if } R_i = r \\ e_L & \text{if } R_i > r \end{cases} \tag{6}$$

where $f(e_L) < r < f(e_H)$.

The proof of Lemma 1 is in Appendix A. In order to interpret it, notice that agents face a trade-off when deciding their (optimal) effort level. On the one hand, agents may target an effort level that is high enough to guarantee them a high extra utility from relative performance; on the other hand, by selecting such a high effort, the standard part of the utility (absent of social comparisons) decreases. Consequently, if the reference point is above a certain cut-off (i.e., $R_i > r$), agents do not find it worthy to aim for the extra utility from relative performance and thus choose the low effort (i.e., e_L is selected). In contrast, for lower reference points (i.e., $R_i < r$), benefiting from such an extra utility is worthwhile and thus choosing the high effort is optimal (i.e., e_H is selected). Finally, if $R_i = r$ agents are indifferent about both effort levels (e_L or e_H), and hereafter, we assume that, in such a case, they choose among them randomly with uniform probability.

This best response function implies that efforts are (weakly) strategic substitutes, i.e., the incentives for an agent to exert the high effort (weakly) decreases with the number of neighbors exerting the high effort. This implies, in particular, that, with the high comparison threshold, an agent will exert the high effort if and only if all her neighbors are choosing the low effort, whereas with the low comparison threshold, an agent will exert the high effort if and only if at least one of her neighbors is choosing the low effort. Finally, with the mild one, there exists a cut-off value which depends on the fundamentals of the model (i.e., f, c and θ) satisfying that if the number of neighbors choosing the low effort is above such a value, then choosing the high effort becomes the agent's best response.

We observe that the cut-off value r may yield a simple expression. For instance, if $f(e) = \sqrt{e}$, then it is straightforward to show that $r = \frac{f(e_L)+f(e_H)}{2}$, which is, precisely the middle point between the low and high output levels.

4.2. The stationary states

We shall now analyze the dynamics and characterize the stationary (and stable) states. Recall that at any time t, each agent revises her strategy at a rate $\lambda \geq 0$ and chooses an effort level that is a myopic best response to her current neighbors' choices, with the payoff function defined by (1)-(2). Given the best response function (6), in order to calculate the stationary state we can focus on the discrete case where all agents choose between two levels of effort: e_L and e_H. In particular, for each $d \geq 1$, let us denote by $\rho_d(t)$ the fraction of agents with degree d that are choosing e_H at time t, and by $\rho(t) = \sum_{d\geq 1} P(d)\rho_d(t)$ the overall fraction in the population. Hence, the probability that any given link points to an agent choosing e_H at time t is given by $\rho(t)$ and, therefore,[8]

$$\frac{\partial \rho_d(t)}{\partial t} = \lambda \cdot (1-\rho_d) \cdot \phi_{RH}(\rho, d) - \lambda \cdot \rho_d \cdot \phi_{RL}(\rho, d), \tag{7}$$

[8] See Appendix B for a description of how this deterministic dynamics represents a good approximation to the corresponding stochastic dynamics in which the population is finite but large enough.

where $\phi_{RL}(\rho, d)$ ($\phi_{RH}(\rho, d)$) is the probability of choosing the low (high) effort for an agent with degree d, given the comparison threshold R. In words, equation (7) states that, among those agents with degree d, the variation of the proportion of agents choosing the high effort is given by the proportion of agents choosing the low effort that switch to the high effort at time t minus the proportion of agents choosing the high effort that switch to the low effort at time t. Hence,

$$\frac{\partial \rho_d(t)}{\partial t} = \lambda(\phi_{RH}(\rho, d) - \rho_d), \tag{8}$$

since $\phi_{RL}(\rho, d) + \phi_{RH}(\rho, d) = 1$.

Regarding the overall fraction of agents choosing the high effort in the population, we get $\frac{\partial \rho(t)}{\partial t} = \sum_{d \geq 1} P(d) \frac{\partial \rho_d(t)}{\partial t}$ and, by equation (8),

$$\frac{\partial \rho(t)}{\partial t} = \lambda \cdot (\sum_{d \geq 1} P(d) \cdot \phi_{RH}(\rho(t), d) - \rho). \tag{9}$$

In a stationary state, $\{\rho_d^*\}_{d \geq 1}$, it must hold that $\frac{\partial \rho_d(t)}{\partial t} = 0$ for all $d \geq 1$. Hence, $\frac{\partial \rho(t)}{\partial t} = 0$ as well, and, therefore, the following equation characterizes the stationary state value of ρ given P and R;

$$\rho^* = H_{P,R}(\rho^*), \tag{10}$$

where $H_{P,R}(\rho) \equiv \sum_{d \geq 1} P(d) \cdot \phi_{RH}(\rho, d)$. The following proposition specifies the existence of a unique stationary state of the dynamics which is globally stable, a result that allows for a clean comparative statics analysis. In addition to verifying the existence of a globally stable state, this result characterizes it as the solution of a fixed point equation.

Proposition 1. *For any given $P(d)$, $\theta \geq 0$ and $R \in \{R^h, R^m, R^l\}$, there exists a unique stationary state of the dynamics, $\{\rho_d^*\}_{d \geq 1}$, which is globally stable.*

Proof. We first show that, for any possible $P(d)$ and $R \in \{R^h, R^m, R^l\}$, there exists a unique equilibrium value $\rho^* \in (0, 1)$, solution of condition (10), by proving that (i) $H_{P,R}(\rho)$ is continuous and decreasing in $\rho \in [0, 1]$, (ii) $H_{P,R}(0) = 1$ and (iii) $H_{P,R}(1) = 0$. Notice that, when $R = R^h$ and $R = R^l$, conditions (i)-(iii) hold, since $\phi_{R^h H}(\rho, d) = (1 - \rho)^d$ and $\phi_{R^l H}(\rho, d) = 1 - \rho^d$. The analysis of the case $R = R^m$ is more demanding since the reference point is not necessarily $f(e_H)$ nor $f(e_L)$. In particular, let x be the fraction of neighbors that must be choosing e_H in order for an agent to be indifferent between e_L or e_H. Then, given (6),

$$x f(e_H) + (1 - x) f(e_L) = r = \frac{(1 + \theta) f(e_H) - f(e_L) - c(e_H - e_L)}{\theta},$$

and, thus,

$$x = \frac{(1 + \theta) f(e_H) - (1 + \theta) f(e_L) - c(e_H - e_L)}{\theta(f(e_H) - f(e_L))}. \tag{11}$$

The concavity of f implies that

$$\frac{f(e_H) - f(e_L)}{e_H - e_L} > \frac{c}{1 + \theta} = f'(e_H).$$

Hence, $x > 0$. Moreover, $x < 1$ since, by the definition of e_L,

$$f(e_H) - ce_H < f(e_L) - ce_L.$$

Let $\lfloor xd \rfloor$ be the integer part of xd. Consequently,

$$\phi_{R^m H}(\rho, d) = \begin{cases} \sum_{a=0}^{\lfloor xd \rfloor} \binom{d}{a} \rho^a (1-\rho)^{d-a} & \text{if } xd \text{ is not an integer} \\ \sum_{a=0}^{xd-1} \binom{d}{a} \rho^a (1-\rho^*)^{d-a} + \frac{1}{2}\binom{d}{xd} \rho^{xd}(1-\rho)^{d-xd} & \text{otherwise.} \end{cases} \tag{12}$$

It is straightforward to show that (i) $H_{P,R^m}(\rho)$ is continuous and decreasing in $\rho \in [0, 1]$, (ii) $H_{P,R^m}(0) = 1$ and (iii) $H_{P,R^m}(1) = 0$, which proves the existence and uniqueness of the stationary state for this case as well.

To conclude, we show that the stationary state ρ^* is globally stable. Note that due to de properties of $H_{P,R}(\rho)$ and equation (9), for each possible value $\rho_0 < \rho^*$, $\frac{\partial \rho(t)}{\partial t} > 0$, whereas if $\rho_0 > \rho^*$, $\frac{\partial \rho(t)}{\partial t} < 0$. This proves the convergence to the stationary state of the dynamics starting from any possible initial condition and thus, its global stability. □

As part of the proof of Proposition 1 we have shown that the transitions probabilities to the high effort in the polar cases are quite straightforward. That is, $\phi_{R^h H}(\rho, d) = (1-\rho)^d$ and $\phi_{R^l H}(\rho, d) = 1 - \rho^d$, for the high and the low comparison threshold scenarios, respectively. Notice that in the former case, an agent chooses the high effort if and only if all neighbors are selecting the low efforts which occurs with probability $(1-\rho)^d$. On the contrary, in the latter case, the condition is that at least one neighbor is selecting the low effort which occurs with the complementary probability $1 - (\rho)^d$. For the mild comparison threshold case, the transition probability is given in equation (12). Following on these results, the next proposition, describes the choices of efforts of individuals with different degrees in the stationary state, for each one of our scenarios.

Proposition 2. *For any given $P(d)$, $\theta \geq 0$ and $R \in \{R^h, R^m, R^l\}$, the (unique) stationary state is such that:*

(i) with the high comparison threshold (R^h), the probability of choosing the high effort strictly decreases with degree.

(ii) with the low comparison threshold (R^l), the probability of choosing the high effort strictly increases with degree.

(iii) with the mild comparison threshold (R^m), the probability of choosing the high effort may be independent of degree, or even non-monotonic.

Proof. Note that, in the stationary state, $\frac{\partial \rho_d(t)}{\partial t} = 0$ for all d. Thus, substituting in equation (8),

$$\rho_d^* = \phi_{RH}(\rho^*, d). \tag{13}$$

Consequently, following the proof of Proposition 1, in the case $R = R^h$ ($R = R^l$), since $\rho_d^* = (1-\rho^*)^d$ ($\rho_d^* = 1 - (\rho^*)^d$), the fraction of agents choosing the high effort in equilibrium decreases (increases) with respect to the degree.

In the case $R = R^m$, we use formulation (12) above to show that ρ_d^* may not depend on degree, or even vary non-monotonically. For instance, in the particular case where $f(e) = \sqrt{e}$ equation (11) implies that $x = \frac{1}{2}$. Thus, applying the development of the Binomial of Newton and simple combinatorial properties, we find that $\rho_d^* = \frac{1}{2}$ for all $d \geq 1$. This shows that, in this case, the probability of choosing the high effort is independent on degree.[9] To conclude the proof, we provide an example where the probability of choosing the high effort is non-monotonic with respect to degree. For instance, if $f(e) = \ln e$ then, after simple algebraic computations, we deduce from equation (11) that $\frac{1}{2} < x$. In particular, x is a continuous function of θ satisfying that $\lim_{\theta \to 0} x = \frac{1}{2}$. Hence, for sufficiently low θ we can assure that $\frac{1}{2} < x < \frac{2}{3}$. In such a case, $\rho_1^* = 1 - \rho^* < \rho_2^* = 1 - (\rho^*)^2 > \rho_3^* = (1 - \rho^*)^2(1 + 2\rho^*)$, which proves the non-monotonicity of $\{\rho_d^*\}_{d \geq 1}$. □

In Proposition 2 we find that the probability that an agent chooses the high effort crucially depends on her degree. In particular, the higher the degree the lower the probability of exerting the high effort with the high comparison threshold, whereas the opposite holds with the low comparison threshold case. With the mild comparison threshold, there exist technologies inducing equilibrium effort levels that are the same for all individuals, regardless of their degree, and other technologies yielding non-monotonic outcomes.

4.3. Comparative statics: competitiveness

In this section, we analyze, given a degree distribution, how the aggregate effort provision (or aggregate outcomes) in the stationary state varies in terms of the two sources of competitiveness: the comparison threshold and the intensity of comparisons. We focus first on the study of how different comparison thresholds lead to different outcomes, given a fixed intensity of comparisons θ. Then, we fix the comparison threshold and analyze how the comparative weight (or intensity) affects the equilibrium outcomes.

Proposition 3. *For any given degree distribution $P(d)$ and intensity of comparisons $\theta \geq 0$, both the fraction of agents choosing the high effort and the aggregate (effort and) outcome in the stationary state are decreasing in the comparison threshold.*

Proof. It is easy to show, by the direct examination of the binomial distribution with parameters ρ and d (probability of linking with an agent choosing the high effort and number of links, respectively) that

$$\phi_{R^h H}(\rho, d) = (1-\rho)^d \leq \phi_{R^m H}(\rho, d) \leq (1 - (\rho)^d) = \phi_{R^l H}(\rho, d) \tag{14}$$

for all $\rho \in (0, 1)$ and $d \geq 1$, given the value of $\phi_{R^m H}(\rho, d)$ derived in (12).

This implies, by the definition of $H_{P,R}(\rho)$ that

$$H_{P,R^h}(\rho) \leq H_{P,R^m}(\rho) \leq H_{P,R^l}(\rho) \tag{15}$$

for all $\rho \in (0, 1)$ and $d \geq 1$, which implies that, given the equilibrium condition (10) then ρ^* is higher when $R = R^l$ than when $R = R^m$, and it is higher when $R = R^m$ than when $R = R^h$.

It can also be shown that the inequalities in condition (14) and, consequently, in condition (15) may be strict. In particular, it holds that $\rho^* \geq \frac{1}{2}$ when $R = R^l$ (as $\frac{1}{2} \leq H_{P,R^l}(\frac{1}{2})$ for any given

[9] Notice that, given the development of the Binomial of Newton, $(1+1)^d = \sum_{a \geq 0}^{d} \binom{d}{a}$, and since $\binom{d}{a} = \binom{d}{d-a}$ for all $0 \leq a \leq d$ then $2^{d-1} = \frac{(1+1)^d}{2} = \sum_{a \geq 0}^{\lfloor \frac{d}{2} \rfloor} \binom{d}{a}$ if d is odd (and thus the number of sums in the binomial is even) and $2^{d-1} = \sum_{a \geq 0}^{\frac{d}{2}-1} \binom{d}{a} + \frac{1}{2}\binom{d}{\frac{d}{2}}$ if d is even (and thus the number of sums in the binomial is odd).

$P(d)$) and $\rho^* \leq \frac{1}{2}$ when $R = R^h$ (as $\frac{1}{2} \geq H_{P,R^h}(\frac{1}{2})$ for any given $P(d)$), being the inequalities strict if $P(1) < 1$. Moreover, if the technology function is such that $x = \frac{1}{2}$ (as, e.g., if $f(e) = \sqrt{e}$), then $\rho^* = \frac{1}{2}$ when $R = R^m$ which lies strictly in between the predictions found for the two polar scenarios ($R = R^h$ and $R = R^l$).

Moreover, since the value of the high and low efforts do not depend on the comparison threshold (see equations (3) and (4)), due to the first part of this proof, the aggregate (effort and) outcome in the stationary state is also decreasing in the comparison threshold.[10] □

Proposition 3 shows that, provided other characteristics remain equal (e.g., the degree distribution and the intensity of comparisons), the more stringent the comparison threshold is in a society, the lower the fraction of agents choosing the high effort. Consequently, in equilibrium more stringent comparisons imply less aggregate effort (and outcome). The reason for this finding is that enjoying an extra utility from relative performance is more challenging in societies with a higher comparison threshold. As a consequence, in such cases agents become discouraged to exert effort.[11]

Next, we analyze the effects of an increase in the intensity of comparisons, keeping the comparison threshold fixed.

Proposition 4. *For any given $P(d)$, with the high and low comparison thresholds (R^h and R^l), the fraction of agents choosing the high effort in the stationary state is constant as the intensity of comparisons θ increases, whereas it is increasing with the mild threshold (R^m). Moreover, for each $R \in \{R^h, R^m, R^l\}$, the aggregate (effort and) outcome increases with the intensity of comparisons θ.*

Proof. With the comparison thresholds R^h and R^l, from equation (10) we get, respectively, $\rho^* = \sum_{d\geq 1} P(d) \cdot (1-\rho^*)^d$ and $\rho^* = \sum_{d\geq 1} P(d) \cdot (1-(\rho^*)^d)$. Hence, the fraction of agents choosing e_H in the stationary state is invariant with respect to θ.

With the comparison threshold R^m, from equation (5), the relationship between the cut-off value r and θ is

$$\begin{aligned}\frac{\partial r}{\partial \theta} &= \frac{\theta(f(e_H) + (1+\theta)f'(e_H)\frac{\partial e_H}{\partial \theta} - c\frac{\partial e_H}{\partial \theta}) - ((1+\theta)f(e_H) - f(e_L) - c(e_H - e_L))}{\theta^2} \\ &= \frac{-f(e_H) + f(e_L) + c(e_H - e_L)}{\theta^2},\end{aligned}$$

where we use the fact that, by equation (3), $(1+\theta)f'(e_H) = c$. Hence, $\frac{\partial r}{\partial \theta} \geq 0$ if and only if $c \geq \frac{f(e_H)-f(e_L)}{e_H-e_L}$ which, given equation (4), is equivalent to $f'(e_L) \geq \frac{f(e_H)-f(e_L)}{e_H-e_L}$. Since due to the concavity of $f(\cdot)$ the latter inequality holds, we conclude that r increases with θ. Hence, the facts that $xf(e_H) + (1-x)f(e_L) = r$ and $f(e_H) > f(e_L)$ imply that x (defined in equation (11))

[10] Note that, by aggregate effort (outcome) we mean $\rho e_H + (1-\rho)e_L$ ($\rho f(e_H) + (1-\rho)f(e_L)$).

[11] To understand better the logic behind this result, consider a culture in which players could compare themselves to an unrealistically high reference point $R = \infty$ (in the spirit of our opening quotation from the Iliad, this extreme situation could represent the case in which individuals compare themselves to Greek Goods rather than other humans). Given that the threshold is not attainable, the players would optimally ignore the competitive part in their utility functions altogether. On the other hand, if the benchmark reference point is very low, all agents take part in the "rat race" to obtain the extra utility derived from relative performance.

is also increasing. Therefore, the fraction of agents choosing e_H with the mild comparative threshold increases with θ (see equation (12)).

Finally, from equations (4) and (3), it directly follows that e_L does not depend on θ, whereas e_H is increasing in θ. Thus, for each $R \in \{R^h, R^m, R^l\}$, the aggregate (effort and) outcome increases with θ. □

Note that the proof of the first part of Proposition 4 (regarding the fraction of agents choosing e_H in the stationary state) for the two polar settings is immediate, since the behavioral rules describing the dynamics do not depend on the value of θ and, thus, the prediction regarding the number of agents choosing each effort is independent on θ. Regarding the case of the mild comparison threshold we show that r, given by equation (5), is increasing in θ, which subsequently implies that the number of agents choosing e_H is also increasing in θ in this scenario.

Regarding the second part of the proposition (aggregate outcome and effort), note that the low effort level is the same regardless of the intensity of comparisons, whereas the high effort level increases with the intensity of comparisons (see equations (3) and (4)). Thus, although the number of agents exerting high effort is constant with the low and high comparison thresholds, the aggregate effort and output increases with θ. With the mild comparison threshold, as θ increases, not only the level of effort provided is higher but also the number of agents choosing e_H.

To summarize, we find that if the society is more competitive in terms of intensity, e.g., agents put more weight to social comparisons in their utility functions, then in equilibrium more aggregate effort is provided, which implies that an ex-ante intensive comparative culture in this respect leads to higher ex-post competitive outcomes.

Note that, if the two sources of competitiveness of the society/organization discussed in the paper (comparison threshold and intensity of comparisons) change in the same direction, the consequences on aggregate efforts would crucially depend on which of the two confronting effects described above prevails. To clarify this issue let us consider the following specific example: Assume, for simplicity, that the technology function is $f(e) = \sqrt{e}$ and that we compare two societies A and B, where society A has the low comparison threshold ($R_A = R^l$), whereas society B has the high comparison threshold ($R_B = R^h$), i.e., in society A (B) the reference point is the minimum (maximum) outcome in the reference group. Moreover, let θ_A and θ_B be the intensity of comparisons in society A and B, respectively, and assume that society B presents a higher intensity of comparisons than society A, i.e., $\theta_B > \theta_A$. Given Proposition 2 (and its proof), the equilibrium fraction of agents choosing high effort must be higher in society B than in society A, and these fractions are independent on the intensity level. In general, however, one can specify that, for any given value ρ^*, it holds that the aggregate outcome corresponds with

$$\rho^* \cdot f(e_H) + (1 - \rho^*) \cdot f(e_L).$$

Since for the technology considered we know that $f(e_H) = \frac{1+\theta}{2c}$ and $f(e_L) = \frac{1}{2c}$, in society A the aggregate outcome must be equal to

$$\rho_A^* \cdot \frac{1 + \theta_A}{2c} + (1 - \rho_A^*) \cdot \frac{1}{2c},$$

whereas in society B it must be equal to

$$\rho_B^* \cdot \frac{1 + \theta_B}{2c} + (1 - \rho_B^*) \cdot \frac{1}{2c}.$$

After simple algebraic computations one can show that the aggregate outcome is higher in society B than in society A if and only if

$$\frac{\theta_B}{\theta_A} \geq \frac{\rho_A^*}{\rho_B^*},$$

or, in words, only if the intensity of comparisons in society B with respect to A is sufficiently high. For the sake of illustration, if we consider the stylized case in which all agents in both societies have degree 2, it is straightforward to show, using equation (10), that the aggregate outcome is higher in society B than in society A if and only if $\frac{\theta_B}{\theta_A} \geq 1.62$.

4.4. Comparative statics: degree distribution

To continue, we take as given the competitive level of the society and explore how changes in the degree distribution affect our predictions regarding the stationary state. We start by commenting on the case with the mild comparison threshold in the following remark.

Remark 1. If the technology function is $f(e) = \sqrt{e}$ then, with the mild comparison threshold (R^m) half of the population exerts the high effort in equilibrium, regardless of the degree distribution. That is, $\rho^* = \frac{1}{2}$ for any possible $P(d)$.

This result follows directly from the fact that, if $f(e) = \sqrt{e}$, then $\rho_d^{m*} = \frac{1}{2}$ for any $d \geq 1$, as shown in the proof for part (iii) of Proposition 2. Since, with the mild comparison threshold, the aggregate outcome may indeed be invariant to the degree distribution, in order to get sharp predictions, the following result focusses on the two polar scenarios.

Proposition 5. *For any given $P(d)$, and θ, the following holds:*

(i) With the high (low) comparison threshold, a first-order stochastic dominance shift of $P(d)$ decreases (increases) the fraction of agents choosing the high effort and the aggregate outcome in the stationary state.

(ii) With the high (low) comparison threshold, a mean-preserving spread of $P(d)$ increases (decreases) the fraction of agents exerting the high effort and the aggregate outcome in the stationary state.

Proof. Part (i) of the proof follows from the fact that $\phi_{R^l H}(\rho, d) = 1 - \rho^d$ is an increasing function of degree whereas $\phi_{R^h H}(\rho, d) = (1-\rho)^d$ is decreasing. Part (ii) is obtained as a consequence of $\phi_{R^l H}(\rho, d) = 1 - \rho^d$ being a concave function of degree and $\phi_{R^h H}(\rho, d) = (1-\rho)^d$ being convex. Notice that if $\widetilde{P}(d)$ (first-order stochastic) dominates $P(d)$ then, for any increasing function $u(d)$, we have that

$$\sum\nolimits_{d\geq 1} u(d)P(d) \leq \sum\nolimits_{d\geq 1} u(d)\widetilde{P}(d),$$

where the opposite inequality holds if the function $u(d)$ is decreasing. Moreover, if $\widetilde{P}$ is a mean preserving spread of P, then for any concave function $u(d)$

$$\sum\nolimits_{d\geq 1} u(d)\widetilde{P}(d) \leq \sum\nolimits_{d\geq 1} u(d)P(d),$$

where the opposite inequality holds if the function $u(d)$ is convex. Taking $u(d) = \phi_{RH}(\rho, d)$ and condition (10) we complete the proof. □

Proposition 5 follows from the fact that $\phi_{RH}(\rho, d)$ is increasing and concave (decreasing and convex) in degree with the high (low) comparative threshold. A first-order stochastic dominance shift of the degree distribution implies, in particular, an increase in the density of the network, whereas a mean-preserving spread leads to more dispersion in the network in terms of degrees. Thus, the first part of Proposition 5 suggests that in denser networks, the difference in effort provision between societies with low and high comparison thresholds is exacerbated. On the other hand, the second part of the proposition indicates that in more dispersed networks, these discrepancies are reduced.

5. Discussion

In this section we analyze the implications of some of the features of our model by considering two variations of it. First, we study an alternative model in which agents do not gain utility from being above others, but rather get a disutility from being below them, and discuss on the possibility of introducing inequality aversion considerations in the model. Then, we extend our approach in order to account for heterogeneous costs (abilities) of exerting effort.[12]

5.1. *Alternative payoff functions*

We shall first study a variation of the model in which, rather than achieving an extra utility if being above others, agents incur in a disutility if their outcome is below the reference point (i.e., it could be conceived as the "*not below others*" counterpart of our model). Then we discuss about the possibility of considering inequality averse agents (Fehr and Schmidt, 1999).

Regarding the "not below others" case, assume that the payoff function is now described as follows:

$$\pi_i(e_i, e_{-i}) = \begin{cases} f(e_i) - c \cdot e_i \text{ if } f(e_i) > R_i \\ f(e_i) - c \cdot e_i + \theta \cdot (f(e_i) - R_i) \text{ if } f(e_i) \leq R_i \end{cases}, \tag{16}$$

where $\theta > 0$ measures the weight or intensity of the non-standard (comparative) component of the payoff function, i.e., of the loss experienced by falling behind the reference point.[13]

Alike in the benchmark framework, we denote by e_H the high effort level deduced from condition $f'(e_H) = \frac{c}{1+\theta}$ which maximizes the payoff obtained when the comparative component is active (i.e., the lower part of $\pi_i(e_i, e_{-i})$). We denote by e_L the low effort level obtained from condition $f'(e_L) = c$ which maximizes the payoff obtained when the comparative component is absent (i.e., the upper part of $\pi_i(e_i, e_{-i})$).

We then use the same dynamic specification as in the benchmark model. That is, at any time t, each agent revises her strategy at a rate $\lambda \geq 0$ and chooses an effort level that is a myopic best response to her neighbors' choices. The stationary states of the dynamics are characterized in the next result.

Proposition 6. *For any given $P(d)$, θ and $R \in \{R^h, R^m, R^l\}$, an effort profile is a stationary state $\{e_i^*\}_{i \in N}$ of the best response dynamics if and only if, for each $i, j \in N$, $e_i^* = e_j^*$, and $e_i^* \in [e_L, e_H]$.*

[12] Moreover, in Appendix C we compare the stationary state derived from our dynamic approach to the mixed strategy Bayesian Nash equilibrium obtained in the counterpart static (incomplete information) context of our model (e.g., Jackson and Yariv, 2007; Galeotti et al., 2010; Feri and Pin, 2020). We find that although our main result qualitatively follows in the (static) counterpart case, there are quantitative differences.

[13] Alternatively, we could construct a model in which agents care both about being above the reference point, weighted by parameter θ_a, and being below it, weighted by parameter θ_b. It can be shown that our results in our benchmark model qualitatively follow if $\theta_a > \theta_b$, and that the results in the current section follow if $\theta_b > \theta_a$.

The proof of Proposition 6 is in Appendix A. We find that, in contrast to the results for our benchmark model, when agents mainly care about not being below the reference point, the stationary states satisfy that all agents coordinate on the same effort level. Moreover, there are infinitely many stationary states of the dynamics, which prevents a clean comparative statics analysis in this case.

The main issue that drives the sharp difference between the results for our baseline model, with payoff function given by (2), and those of the counterpart case studied in this section is the following. In our baseline model efforts are (weakly) strategic substitutes, since the incentives for an agent to exert high effort (weakly) decrease with the number of neighbors exerting high effort, which induces a unique equilibrium. Differently, the payoff function given by (16) yields a game in which efforts are (weakly) strategic complements, i.e., the (marginal) payoff to an agent by raising her effort (weakly) increases when her neighbors increase their efforts, and this coordination motive causes multiple equilibria in which agents conform to a common effort level.

We shall now briefly discuss on the possibility of inequality averse agents. To this aim, given any payoff profile $\pi = (\pi_i)_{i\in N}$, consider a utility function a là Fehr and Schmidt (1999), $v_i(\pi) = \pi_i - \alpha \frac{1}{d_i}\sum_{j\in N_i} \max\{\pi_j - \pi_i, 0\} - \beta \cdot \frac{1}{d_i}\sum_{j\in N_i} \max\{\pi_i - \pi_j, 0\}$, where $\min\{\alpha, 1\} \geq \beta \geq 0$. This formulation introduces a coordination motive in the game, similar to that induced by the variation of our baseline game just studied (with the payoff function given by (16)). Thus, likewise, inequality aversion considerations could generate multiple equilibria. This would give rise to quite different results to those obtained in our baseline model (with the payoff function given by (2)). In Fehr and Schmidt's (1999) formulation agents dislike inequality (in both directions, albeit they dislike the inequality in their direction more), whereas in our baseline model, players derive an extra gain from the disparity as long as they are ahead. Although in many real life situations inequality aversion may be the main force driving behavior, these are mainly cases in which payoffs are monetary, and there are other contexts in which ahead seeking behavior prevails.[14]

5.2. *Heterogeneous costs*

Our benchmark model introduces heterogeneity only regarding the agents' degree. In this section, we examine an extension in which agents also have heterogeneous costs of exerting effort. For concreteness, let us consider two types of agents. For an agent of type $j \in \{1, 2\}$, let $c_j > 0$ be her (marginal) cost of exerting effort, where we assume $c_2 \geq (1+\theta)c_1$.[15] Thus, type 1 agents have higher ability (i.e., lower cost) than type 2 agents.

Following analogous arguments to the ones used in the benchmark model, we find that in equilibrium, for each type of agent j, there are two possible effort levels exerted: the high effort level, denoted by e_{Hj} such that $f'(e_{Hj}) = \frac{c_j}{1+\theta}$, and the low effort level e_{Lj} such that $f'(e_{Lj}) = c_j$. Notice that, for each $j \in \{1, 2\}$, due to the concavity of f, $e_{Hj} > e_{Lj}$. Moreover, since $c_2 \geq (1+\theta)c_1$, it also follows that $e_{L1} > e_{H2}$.

We assume that there exists a fraction $\alpha \in (0, 1)$ of high ability agents (i.e., a fraction $1 - \alpha$ of low ability agents) in the population, distributed randomly in the network. Let $\rho_j(t)$ be the

[14] For instance, as pointed out by Roels and Su (2014), based on the cross-cultural study for elementary school students developed by Stevenson et al. (1990), "*in some educational systems, students strive to be at the top of their class. (...) A student who achieved a perfect score on a test may derive pleasure out of doing better than other students who scored less; the same perfect score yields less utility if the entire class has the same achievement*".

[15] For the sake of exposition, we shall focus on the case in which the difference in abilities (or costs) across types is high enough, which significantly simplifies the analysis as compared to the case of low differentiation.

fraction of the population of type j agents choosing effort e_{Hj} at time t. The following result compares the aggregate outcomes with high and low comparison thresholds.

Proposition 7. *Consider the heterogeneous cost model with $c_2 \geq (1+\theta)c_1$. For any given degree distribution $P(d)$ and intensity of comparisons $\theta \geq 0$, the expected aggregate (effort and) outcome in the stationary state is lower with the high comparison threshold than with the low comparison threshold.*

The proof of Proposition 7 is in Appendix A. We mainly find that, by adding some heterogeneity regarding the ability of agents, a larger set of efforts can be observed in equilibrium. Nevertheless, we show that the one of the main insights of the paper remains true, namely, the high comparison threshold leads to less aggregate (effort and) outcome than the low comparison threshold.

6. Conclusion

We have presented a model that aims to better understand how the competitive culture of societies and large organizations, defined in terms of interpersonal comparisons and reference points, may affect aggregate production. We consider two ways of describing competitiveness, and find that their effects on aggregate outcomes operate in opposite directions. Regarding the comparison threshold defining the reference point (low, mild or high), we obtain that, in order to maximize outcomes, a low threshold is optimal since more stringent thresholds may lead to agents' discouragement and, therefore, lower effort provisions. We find the inverse result with respect to the intensity of comparisons. Namely, more competitiveness in this respect lead to higher aggregate outcomes. Our results suggest that the cultural background within companies regarding these two sources of competitiveness might be, among others, a relevant determinant of their success.

We also study random network effects with a focus on the degree distribution, a large-scale property of the network that can be easily estimated given the growing availability of data. We find that with a low comparison threshold, aggregate production increases with the density of the network, whereas the opposite occurs with a high comparison threshold. This result implies that in an era of globalization, in which social networks are progressively becoming denser, the differences between our predictions for the high and low comparison thresholds become larger (in favor of the low one). However, our results also suggest that these differences would be alleviated if the social networks became more heterogeneous in terms of connectivity.

Our findings may be relevant not only from a positive perspective (i.e., for predicting outcomes in particular settings), but also from a normative one, since a principal could manipulate the agents' reference point in order to induce a certain behavior. For instance, to the best of our knowledge, in the educational context discussed in the Introduction there is no systematic empirical study comparing the effect of the specific information about peers provided to students. This might be a reasonable way of inducing different comparison thresholds simply through information.[16] In this respect, our results suggest that providing students with information based on the poorest performances like, e.g., a low percentile (low comparison threshold) can lead to

[16] In the cases analyzed by Azmat et al. (2019) and Murphy and Weinhardt (2020) the information provided is the ranking position, whereas in the case studied by Azmat and Iriberri (2010) the information provided is the average grade in the class.

more effort than reporting the average (average comparison threshold) which, in turn, can yield more effort than revealing the best outcome (high comparison threshold). In any instance, if the educator/planner cannot control the competitiveness of the student population, but she is able to identify whether they mainly compare themselves with either the best, the average or the poorest outcomes, our results can help to distribute the students in classes or working groups. For example, if the students are very competitive and tend to compare themselves with the best, it seems convenient to form small groups (associated to smaller degree). The reverse holds for populations with a low level of competitiveness. By controlling the nature and size of the reference point, our study can thus orientate the design of future laboratory and field experiments, and inform educational policies.

It is worth noting that our theoretical findings may also help the design of policies in other relevant settings apart from education. Indeed, recent advances in applied behavioral economics suggest that providing individuals with relative information about others is effective in changing behaviors like, e.g., electricity consumption (Schultz et al., 2007; Allcott, 2011; Allcott and Rogers, 2014) or water consumption (Ferraro and Price, 2013; Datta et al., 2015). For instance, Schlultz et al. (2007) and Allcott (2011), among others, analyze the effects of Home Energy Reports, one-page letters that compare a household's energy use to that of its neighbors (postal codes) and provide energy conservation tips. In particular, they provide the average consumption and the consumption of the most efficient neighbors. The reports also include smiley/sad faces when the individual's consumption is below/above that of her neighbors. The main finding is that providing such a relative information makes individuals adopt a more responsible use of electricity. Hence, in this setting, controlling for the nature and size of the reference point is also possible and useful. For instance, if the home energy reports provide information on the most efficient neighbors (somehow inducing the high comparison threshold), it might be helpful to limit neighborhood sizes by, e.g., providing the information at the street (rather than at the postal code) level.

We believe that this work is just a step towards a much broader area of study. There are several directions in which our analysis could be extended. For instance, the random network assumption constitutes an initial approximation to the more complex and structured real networks. In this sense, the analysis of networks with some level of homophily or clustering may add new insights to our current study. Finally, we have introduced interpersonal considerations via an extra utility (or disutility) from relative performance which is added to the utility function using a multiplicative constant, representing the intensity of comparisons. Further work considering alternative specifications for the payoff function, like the ones proposed in prospect theory, can enrich our understanding of the interplay between social networks, peer comparisons and production.

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Appendix A. Proofs of Lemma 1 and Propositions 6 and 7

Proof of Lemma 1. Consider first a reference point for agent i, R_i, such that $f(e_L) \leq R_i \leq f(e_H)$. The agent's choice reduces to selecting the high effort e_H if aiming to receive the extra utility from relative performance, or the low effort e_L otherwise. Thus, given the payoffs defined by (1) and (2) the best response would be e_H if and only if

$$f(e_L) - ce_L < f(e_H) - ce_H + \theta(f(e_H) - R_i)$$

or, analogously,

$$\frac{(1+\theta)f(e_H) - f(e_L) - c(e_H - e_L)}{\theta} = r < R_i,$$

where we use equation (5). Notice that $f(e_L) < r$, since

$$r - f(e_L) = \frac{(1+\theta)f(e_H) - (1+\theta)f(e_L) - c(e_H - e_L)}{\theta}$$

is positive due to the concavity of f and the definition of e_H given by (4). That is,

$$\frac{f(e_H) - f(e_L)}{e_H - e_L} > \frac{c}{1+\theta} = f'(e_H).$$

Notice that $r < f(e_H)$, since

$$r - f(e_H) = \frac{f(e_H) - f(e_L) - c(e_H - e_L)}{\theta}$$

is negative as $(f(e_H) - ce_H) - (f(e_L) - ce_L) < 0$ by the definition of e_L.

Assume now that $R_i < f(e_L)$. The agent's choice reduces to selecting the high effort e_H, if wanting to receive the extra utility from relative performance, or the effort $\widehat{e}_i$ such that $f(\widehat{e}_i) = R_i$, otherwise. Thus, the best response would be e_H since

$$\begin{aligned} f(\widehat{e}_i) - c\widehat{e}_i &< f(e_L) - ce_L = f(e_H) - ce_H + \theta(f(e_H) - r) \\ &< f(e_H) - ce_H + \theta(f(e_H) - R_i) \end{aligned}$$

To conclude, assume that $f(e_H) < R_i$. Obtaining the extra utility from relative performance here is not optimal and thus the agent would have to settle with the effort level e_L, which maximizes profits conditional on not receiving the extra utility. □

Proof of Proposition 6. In this case, for an agent i with reference point R_i the best response function is

$$BR_i = \begin{cases} e_L & \text{if } R_i \leq f(e_L) \\ \widehat{e}_i & \text{if } f(e_L) < R_i < f(e_H) \\ e_H & \text{if } R_i \geq f(e_H) \end{cases} \tag{17}$$

where $f(\widehat{e}_i) = R_i$ and $f(e_L) < f(e_H)$.

To show this we consider first a reference point for agent i, R_i, such that $f(e_L) \leq R_i \leq f(e_H)$. Notice that, if the agent restricts her choice to outputs above the reference point, her best response would be the effort level $\widehat{e}_i$ such that $f(\widehat{e}_i) = R_i$ given that her payoff function (i.e., the upper part of function (16)) is decreasing with respect to effort in such range. Moreover, if instead the agent restricts her choice to outputs below the reference point, her best response would also be $\widehat{e}_i$

since her payoff function (i.e., the lower part of function (16)) is increasing with respect to effort in such range.

We assume now that $R_i < f(e_L)$. Hence, the optimal effort level conditional on outputs being above the reference point would be e_L since it maximizes the agent's payoffs in such range (i.e., the upper part of function (16)). If instead the agent maximizes payoffs conditional on choosing an effort level with an output below the reference point given that the payoff function is increasing in such range (recall that $f(e_L) < f(e_H)$) then the optimal choice would be the effort level $\widehat{e_i}$. Since both parts of the payoff function coincide at such level of effort then e_L would provide higher payoffs and therefore would be chosen. Finally, if it is the case that $f(e_H) < R_i$ then an analogous proof applies. In particular, the optimal effort level conditional on outputs being below the reference point would be e_H since it is the effort level that maximizes payoffs in such range (i.e., the lower part of function (16)). If instead the agents maximizes payoffs conditional on outputs being above the reference point given that the payoff function is decreasing in such range then the optimal choice would be the effort level $\widehat{e_i}$. Again, since at this effort both parts of the payoff function coincide then e_H would be chosen.

In summary, the best response (17) induces agents to coordinate with the reference point. Thus, there is a continuum of effort levels which range in the interval $[e_L, e_H]$ that could potentially be sustained in equilibrium. In particular, given (17) it follows that, for some finite t, the dynamic process will eventually lead to a situation where, for all $i \in N$, $e_i(t) \in [e_L, e_H]$. Moreover, it is straightforward to see that if all agents choose the same effort level within such interval, this state would be stationary. Reversely, let us show that an asymmetric effort profile cannot be sustained as an equilibrium. Consider an effort profile $\{e_i^*\}_{i\in N=[0,1]}$ which is asymmetric. Then, let $k \in N$ be such that $e_k^* = \max_{i\in N}\{e_i^*\}$. Given that not all agents can be choosing such maximum effort level, there is a positive probability that at some point in time the reference point observed by k is lower than the maximum output both with the mild and the low comparison thresholds, regardless of k's degree. If so, this would imply that at such time step agent k would deviate from e_k^* which contradicts the equilibrium assumption. With the high comparison threshold, a similar argument applies if we consider an agent j such that $e_j^* = \min_{i\in N}\{e_i^*\}$. Consequently, this state cannot be stationary for this case either. □

Proof of Proposition 7. For each type of agent $j \in T = \{1, 2\}$ the low effort level is defined as $e_{Lj} = \arg\max_{e\geq 0} f(e) - c_j \cdot e$, and the high effort level is defined as $e_{Hj} = \arg\max_{e\geq 0} f(e) - c_j \cdot e + \theta \cdot f(e)$. Finally, the threshold r_j satisfies that $f(e_{Lj}) - c_j \cdot e_{Lj} = f(e_{Hj}) - c_j \cdot e_{Hj} + \theta \cdot (f(e_{Hj}) - r_j)$. It directly follows that, for each $j \in \{1, 2\}$:

$$f'(e_{Lj}) = c_j \tag{18}$$

$$f'(e_{Hj}) = \frac{c_j}{1+\theta} \tag{19}$$

$$r_j = \frac{(1+\theta)f(e_{Hj}) - f(e_{Lj}) - c_j(e_{Hj} - e_{Lj})}{\theta} \tag{20}$$

and, using arguments analogous to those developed in the proof of Lemma 1 for the homogeneous case, it follows that the best response function for an agent i of type j with reference point R_{ij} is

$$BR_{ij} = \begin{cases} e_{Hj} & \text{if } R_{ij} < r_j \\ \{e_{Hj}, e_{Lj}\} & \text{if } R_{ij} = r_j \\ e_{Lj} & \text{if } R_{ij} > r_j \end{cases} \tag{21}$$

where $f(e_{Lj}) < r_j < f(e_{Hj})$.

Considering the best response dynamics and using (21), it is straightforward to see that eventually, after all agents have revised their choices at least once, each agent of type 1 (respectively 2) is choosing either e_{L1} or e_{H1} (respectively e_{L2} or e_{H2}), as given by (18) and (19). For each $j \in \{1,2\}$, let $\rho_j(t)$ be the fraction of the population choosing effort e_{Hj} at time t, conditional on being of type j. We show next that the overall effort (or the overall outcome) provided with the high comparison threshold is lower than in the case of the low comparison threshold. Since the effort levels coincide in both cases, the differences in results must be due to a difference in the fraction of agents choosing each effort in the stationary state of the dynamics. In particular, to prove the result, it would be enough to show that the high efforts are chosen by each type (i.e., e_{H1} and e_{H2}) more often with the low comparison threshold than with the high comparison threshold. Let $\rho_{jd}(t)$ be the fraction of the population choosing effort e_{Hj} at time t, conditional on being of type j and having degree d, where we assume that the degrees and types are independently distributed in the population. The mean field dynamics equation is the following:

$$\frac{\partial \rho_{1d}(t)}{\partial t} = -\lambda \cdot \rho_{1d} \cdot \phi_{RL1}(\rho_1, \rho_2, d) + \lambda \cdot (1-\rho_{1d}) \cdot \phi_{RH1}(\rho_1, \rho_2, d)$$
$$\frac{\partial \rho_{2d}(t)}{\partial t} = -\lambda \cdot \rho_{2d} \cdot \phi_{RL2}(\rho_1, \rho_2, d) + \lambda \cdot (1-\rho_{2d}) \cdot \phi_{RH2}(\rho_1, \rho_2, d)$$

where, for each $j \in \{1,2\}$ and $k \in \{H, L\}$, $\phi_{Rkj}(\rho_1, \rho_2, d)$ is the probability that an individual of type $j \in \{1,2\}$ and with degree d chooses effort e_{kj}. Given that $\phi_{RHj}(\rho_1, \rho_2, d) + \phi_{RLj}(\rho_1, \rho_2, d) = 1$ for all j and d, in the stationary state (i.e., when $\frac{\partial \rho_{jd}(t)}{\partial t} = 0$ for all j and d) we have that:

$$\rho_{1d} = \phi_{RH1}(\rho_1, \rho_2, d)$$
$$\rho_{2d} = \phi_{RH2}(\rho_1, \rho_2, d)$$

Since we assume $c_2 > (1+\theta)c_1$, $e_{L1} > e_{H2}$ and thus $f(e_{L1}) > f(e_{H2})$. In words, high ability agents (i.e., type 1) are always exerting more effort (and generating more output) than low ability agents. Moreover, for each type $j \in \{1,2\}$, $f(e_{Lj}) < r_j < f(e_{Hj})$ which implies that $r_1 > f(e_{H2})$ and $r_2 < f(e_{L1})$. In what follows, in order to refer to $\rho_{jd}(t)$ ($\rho_j(t)$) for the cases of the comparison thresholds R^h and R^l, we will use the notation $\rho^h_{jd}(t)$ and $\rho^l_{jd}(t)$ ($\rho^h_j(t)$ and $\rho^l_j(t)$), respectively. The following expressions can be derived for the two polar comparison thresholds (high and low).

In the case $R = R^h$ (maximum), it is straightforward to show that:

$$\rho^h_{1d} = (1-\alpha\rho^h_1)^d$$
$$\rho^h_{2d} = ((1-\alpha)(1-\rho^h_2))^d$$

and, thus,

$$\rho^h_1 = \sum_{d\geq 1} P(d)(1-\alpha\rho^h_1)^d$$
$$\rho^h_2 = \sum_{d\geq 1} P(d)((1-\alpha)(1-\rho^h_2)^d$$

In the case $R = R^l$ (minimum), we have that:

$$\rho^l_{1d} = 1-(\alpha\rho^l_1)^d$$
$$\rho^l_{2d} = 1-(\alpha+(1-\alpha)\rho^l_2)^d$$

and, thus,

$$\rho_1^l = \sum_{d\geq 1} P(d)(1-(\alpha\rho_1^l)^d)$$
$$\rho_2^l = \sum_{d\geq 1} P(d)(1-(\alpha+(1-\alpha)\rho_2^l)^d).$$

Notice that $\rho_1^l > \rho_1^h$ because $1-(\alpha x)^d \geq (1-\alpha x)^d$ for all $\alpha \in (0,1)$ and all $x \in (0,1)$. Also, $\rho_2^l > \rho_2^h$ because $1-(\alpha+(1-\alpha)x)^d \geq ((1-\alpha)(1-x))^d$ for all $\alpha \in (0,1)$ and all $x \in (0,1)$.

The (expected) aggregate efforts and outcomes associated to the stationary state in the high and low comparative structures are, respectively, given by

$$\begin{aligned}
\mathit{Effort}^h &= (1-\alpha)((1-\rho_2^h)e_{L2}+\rho_2^h e_{H2})+\alpha((1-\rho_1^h)e_{L1}+\rho_1^h e_{H1})\\
\mathit{Outcome}^h &= (1-\alpha)((1-\rho_2^h)f(e_{L2})+\rho_2^h f(e_{H2}))+\alpha((1-\rho_1^h)f(e_{L1})+\rho_1^h f(e_{H1}))\\
\mathit{Effort}^l &= (1-\alpha)((1-\rho_2^l)e_{L2}+\rho_2^l e_{H2})+\alpha((1-\rho_1^l)e_{L1}+\rho_1^l e_{H1})\\
\mathit{Outcome}^l &= (1-\alpha)((1-\rho_2^l)f(e_{L2})+\rho_2^l f(e_{H2}))+\alpha((1-\rho_1^l)f(e_{L1})+\rho_1^l f(e_{H1})),
\end{aligned}$$

Hence, since $\rho_1^l > \rho_1^h$ and $\rho_2^l > \rho_2^h$, it follows that, for all $\alpha \in (0,1)$, $\mathit{Effort}^l > \mathit{Effort}^h$ and $\mathit{Ouitcome}^l > \mathit{Outcome}^h$. □

Appendix B. Mean field dynamics

A mean field dynamics is given by a system of ordinary differential equations that reflects expected motion. This approach is justified by the assumption that there is such a large population that the randomness implicit in the finite system can be essentially ignored within any finite time horizon (see Vega-Redondo, 2007, for an elaboration of mean field theory and its applications).

In this section we pose that our model and results, based on the deterministic dynamics (8)-(10) represent a good approximation of a stochastic model with a discrete population of size n (with n large enough), with agents choosing among the discrete set of actions $\{e_H, e_L\}$. Given a degree distribution $P(d)$, for each degree d, we have $n_d = nP(d)$ individuals of degree d. For analytical convenience, we shall further assume that there exists a bound $\hat{d}$, with $\hat{d}$ arbitrarily large but finite, such that for all $d > \hat{d}$, $P(d) = 0$.[17]

The stochastic process of interest is the population state $X^n(t) = \{X_d^n(t)\}_{d\in\{1,\ldots,\hat{d}\}}$, where $X_d^n(t)$ is the fraction of agents with degree d in the population who choose action e_H. The set of all possible vectors $X^n = \{X_d^n\}_{d\in\{1,\ldots,\hat{d}\}}$ constitute the state space Ω. Then, the mean field dynamics is defined as the continuous time ordinal differential equations on Ω whose induced paths satisfy, for each d,

$$\frac{\partial X_d^n(t)}{\partial t} = \lim_{\Delta t\to 0} \frac{E[\Delta X_d^n \mid X^n(t) = X^n]}{\Delta t} \tag{22}$$

where $E[\Delta X_d^n \mid X^n(t) = X^n]$ represents the conditional expected change in X_d^n during the infinitesimal time interval $(t, t+\Delta t)$. Given the continuous population considered in the text, the dynamics of $\rho_d(t)$ as defined by equation (8) essentially coincides with (22) in the limit when $n \to \infty$.

[17] By bounding the support of the degree distribution, we shall be able to adapt the existing results by Benaïm and Weibull (2003, 2009), which presume that the vector field is finite-dimensional.

In the following, we rely on the work by Benaïm and Weibull (2003, 2009), to argue that our deterministic dynamics (8) offers a good approximation for the evolution of the stochastic process $X^n(t)$. To this aim, we apply their Lemma 1 (Benaïm and Weibull, 2003, page 880).[18,19] The crucial ingredient of the current model that allows us to do so is that, whenever an adjustment event takes place, it can only involve (with full probability) a bounded change in the choices of a finite number of agents. By applying Lemma 1 of Benaïm and Weibull (2003), for any time horizon T, and any given $\varepsilon, \delta > 0$, there exists some lower bound $\bar{n}$ on population size such that if $n \geq \bar{n}$, then

$$\Pr\left[\max_{0\leq t\leq T}\left\|X^n(t) - \{\rho_d(t)\}_{d\in\{1,\ldots,\hat{d}\}}\right\| \geq \varepsilon\right] \leq \delta,$$

where $\|.\|$ stands for the sup norm. Note that, in the main text Benaïm and Weibull (2003) considers the case in which at each time step in a set of discretized transition times exactly one agent is selected uniformly at random, independently chosen at each time, to revise her action. However, in Section 6 they argue that all the qualitative results remain valid if the discrete-time process is replaced by a continuous-time one whose transition times are generated by a Poisson process with constant intensity, as it is our case (see the "Proof of Lemma 1 for Poisson Processes" in the Appendix of Benaïm and Weibull, 2003, page 901).

Appendix C. A comparison with the static approach

In this section we consider the static incomplete information counterpart of our model. We show through a simple example that the stationary state derived from our dynamic approach does not need to coincide with the mixed strategy Bayesian Nash equilibrium of the static model. To this end, for the sake of concreteness, we shall focus on our two polar comparison threshold models (R^h and R^l) and on the simplest case of a homogeneous degree distributions (i.e., $P(\bar{d}) = 1$, for a certain $\bar{d} \geq 2$).

We claim that the (mixed strategy) equilibrium of the static game reproduces *qualitatively* the main result obtained in our dynamics model. Namely, more (aggregate) effort (and outcome) is provided in the low comparison threshold case R^l than in the high comparison threshold case R^h (Proposition 3). Moreover, comparing across different (homogeneous) degree distributions parametrized by $\bar{d}$ (i.e., with $P(\bar{d}) = 1$), the higher $\bar{d}$, the lower (the higher) the probability of choosing the high effort in the mixed strategy equilibrium of the static game when $R = R^h$ (when $R = R^l$), which coincides with the prediction given by Proposition 5 for our dynamics model.

However, the results of the two (dynamic and static) approaches *quantitatively* differ. Focusing on the case $f(e) = \sqrt{e}$, we claim that

$$\sigma^h < \rho^h < \rho^l < \sigma^l,$$

where, for each $i \in \{h, l\}$, ρ^i is the fraction of agents choosing the high effort in the stationary state of the dynamics with comparison threshold R^i, and σ^i is the probability of choosing the high effort in the mixed strategy equilibrium of the static game also for R^i.[20] Note that, the

[18] The framework studied by Benaïm and Weibull (2003) considers that switching probabilities are defined independently of population size. In this respect, see also Lemma 2 in Benaïm and Weibull (2009), which relaxes this assumption.

[19] Note that Benaïm and Weibull's (2003) use notation ξ (instead of ρ) to refer to the deterministic approximation solution at time t.

[20] The case $\bar{d} = 1$ is discarded since it would yield $\sigma^h = \rho^h = \rho^l = \sigma^l = 1/2$.

disparities in the results for the low and high comparison thresholds are augmented in the static framework.

In the following steps we prove these claims.

I) Stationary state of the dynamics for $R = R^h$ (ρ^h)

Consider the high comparative threshold $R = R^h$. The best response dynamics is such that an individual chooses the high effort only if no other neighbor has done so. Hence,

$$\frac{\partial \rho}{\partial t} = \lambda((1-\rho)^{\bar{d}} - \rho),$$

and, in the stationary state,

$$\rho^h = (1-\rho^h)^{\bar{d}}. \tag{23}$$

Equation (23) provides an implicit characterization of the (stable) stationary state which is independent on the primitives of the model such as $f(\cdot)$, θ and c. Notice that, the unique solution of the previous equation is decreasing in $\bar{d}$ (as also shown in part (i) of Proposition 5).

II) Mixed strategies equilibrium in the static game when $R = R^h$ (σ^h)

Agents simultaneously choose efforts knowing the degree distribution (hence, also their degree $\bar{d}$), but not who their neighbors are. The (symmetric) mixed strategy equilibrium is described by the probability σ that an agent chooses the high effort. The strategy profile where all agents are making such choice is an equilibrium if and only if any given agent, say $i \in N = [0, 1]$ is indifferent between choosing e_H or e_L when all her ($\bar{d}$) neighbors are choosing σ.

If the choice is e_H the expected profits would be

$$\begin{aligned}\pi_i(e_H, \sigma_{-i}) &= (1-(1-\sigma)^{\bar{d}})(f(e_H) - ce_H) \\ &\quad +(1-\sigma)^{\bar{d}}(f(e_H) - ce_H + \theta(f(e_H) - f(e_L)) \\ &= f(e_H) - ce_H + (1-\sigma)^{\bar{d}}\theta(f(e_H) - f(e_L)).\end{aligned}$$

If the choice is e_L the expected profits would be

$$\pi_i(e_L, \sigma_{-i}) = f(e_L) - ce_L.$$

Hence, agent i is indifferent between the two strategies if and only if:

$$f(e_H) - ce_H + (1-\sigma)^{\bar{d}}\theta(f(e_H) - f(e_L)) = f(e_L) - ce_L,$$

which yields

$$(1-\sigma)^{\bar{d}} = z,$$

with

$$z \equiv \frac{f(e_L) - ce_L - (f(e_H) - ce_H)}{\theta(f(e_H) - f(e_L))} \tag{24}$$

We claim that $0 < z < 1$. We have that $z > 0$ because (i) its denominator is positive since $f(e_H) > f(e_L)$ (ii) its numerator is positive because $f(e_H) - ce_H < f(e_L) - ce_L$, as e_L is the effort which maximizes profits when there is no extra utility from relative performance. To see that $z < 1$, note that

$$f(e_L) - ce_L - (f(e_H) - ce_H) < \theta(f(e_H) - f(e_L)),$$

since e_H maximizes profits in case of enjoying the extra utility from relative performance, that is, the expression

$$f(e) - ce + \theta(f(e) - R)$$

is maximized at e_H for any reference point R (i.e., also for $R = f(e_L)$) as long as $f(e) \geq R$.

Then, given z defined in (24), the equilibrium condition with $R = R^h$ is

$$\sigma^h = 1 - z^{1/\bar{d}}. \tag{25}$$

satisfying

$$\frac{\partial \sigma^h}{\partial \bar{d}} = -z^{1/\bar{d}}(-1/\bar{d}^2)\ln z < 0,$$

since $z < 1$.

III) Comparison of ρ^h and σ^h when $f(e) = \sqrt{e}$

As shown by equations (23) and (24)-(25), in general ρ^h and σ^h do not need to coincide. In the following, we focus on the case $f(e) = \sqrt{e}$. In such a case, given (3)-(4), $e_L = (\frac{1}{2c})^2$ and $e_H = (\frac{1+\theta}{2c})^2$. Substituting in (24), we get

$$z = \frac{\frac{1}{2c} - c(\frac{1}{2c})^2 - (\frac{1+\theta}{2c} - c(\frac{1+\theta}{2c})^2)}{\theta(\frac{1+\theta}{2c} - \frac{1}{2c})} = \frac{1}{2}. \tag{26}$$

Hence, by (25),

$$\sigma^h = 1 - \left(\frac{1}{2}\right)^{1/\bar{d}} < \frac{1}{2},$$

as $\bar{d} \geq 2$ and, therefore,

$$(1 - \sigma^h)^{\bar{d}} = \frac{1}{2}.$$

This implies that the fixed point equation given by (23) when substituting σ^h yields the following inequality, $\sigma^h < \frac{1}{2} = (1 - \sigma^h)^{\bar{d}}$ which, in turn, implies that $\sigma^h < \rho^h$.

IV) Stationary state of the dynamics for $R = R^l$ (ρ^l)

Consider the low comparative threshold $R = R^l$. The best response dynamics is such that an individual chooses the high effort if there is at least one neighbor that has not done so. Hence,

$$\frac{\partial \rho}{\partial t} = \lambda(1 - \rho^d - \rho),$$

and, in the stationary state,

$$\rho^l = 1 - (\rho^l)^{\bar{d}}. \tag{27}$$

Equation (27) provides an implicit characterization of the (stable) stationary state which is independent on the primitives of the model such as $f(\cdot)$, θ and c. Notice that, the unique solution of the previous equation is increasing in $\bar{d}$ (following also part (i) of Proposition 5).

Moreover, since for each $\rho \in (0, 1)$ and $\bar{d} \geq 2$, $(1 - \rho)^{\bar{d}} < 1 - \rho^{\bar{d}}$, by (23) and (27), it follows that $\rho^h < \rho^l$ (as predicted in general by Proposition 3).

V) Mixed strategies equilibrium in the static game when $R = R^l$ (σ^l)

With the low comparative threshold $R = R^l$, given that the remaining players play a mixed strategy σ, the expected profits associated to the choice e_H would be

$$\pi_i(e_H, \sigma_{-i}) = \sigma^{\bar{d}}(f(e_H) - ce_H) + (1-\sigma^{\bar{d}})(f(e_H) - ce_H + \theta(f(e_H) - f(e_L))$$
$$= f(e_H) - ce_H + (1-\sigma^{\bar{d}})\theta(f(e_H) - f(e_L)).$$

If the choice is e_L the expected profits would be

$$\pi_i(e_L, \sigma_{-i}) = f(e_L) - ce_L.$$

Hence, agent i is indifferent between the two strategies if and only if:

$$f(e_H) - ce_H + (1-\sigma^{\bar{d}})\theta(f(e_H) - f(e_L)) = f(e_L) - ce_L,$$

which yields

$$1 - \sigma^{\bar{d}} = z,$$

with z defined in (24).

Then, the equilibrium condition with $R = R^l$ is

$$\sigma^l = (1-z)^{1/\bar{d}}. \tag{28}$$

Notice that, since $z \in (0, 1)$,

$$\frac{\partial \sigma^h}{\partial \bar{d}} = (1-z)^{1/\bar{d}} \ln(1-z)(-1/\bar{d}^2) > 0.$$

VI) Comparison of ρ^l and σ^l when $f(e) = \sqrt{e}$

As shown by equations (27) and (24)-(28), in general ρ^l and σ^l do not need to coincide. In the following, we focus on the case $f(e) = \sqrt{e}$. In such a case, by (26) we know that $z = 1/2$. Hence, by (28),

$$\sigma^l = \left(\frac{1}{2}\right)^{1/\bar{d}} > \frac{1}{2},$$

as $\bar{d} \geq 2$ and, therefore,

$$1 - (\sigma^l)^{\bar{d}} = \frac{1}{2}.$$

This implies that the fixed point equation given by (27) when substituting σ^l yields the following inequality, $\sigma^l > \frac{1}{2} = 1 - (\sigma^l)^{\bar{d}}$ which, in turn, implies that $\rho^l < \sigma^l$.

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