A multiobjective interval programming model to explore the trade-offs among different aspects of job satisfaction under different scenarios

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Abstract

The need for greater concern about job quality/satisfaction seems clear, due to its potential link with workers' productivity, to the extent it affects employees' quitting behavior, absenteeism, turnover, and firms' productivity. In order to guide managers and policy makers when making decisions related to future hiring of human resources, a multiobjective interval programming model, based on the results of an econometric estimation, is suggested where different (and conflicting) aspects of job satisfaction are considered. The modelling framework thus obtained allows assessing the trade-offs among the different aspects of job satisfaction under different scenarios herein given within interval ranges. Data obtained from Spain are used to carry out the model's instantiation. Possibly efficient solutions are then generated with the help of scalarizing problems relying on reference point-based methods. The solution approach herein proposed allows computing with a lower computational effort the closest "possibly" efficient solutions attainable regarding their corresponding interval ideal solutions. Overall, the results obtained highlight the trade-off between earnings and quality of life, particularly under a pessimistic scenario, with the maximization of earnings leading to the lowest value of the working times. Conversely, the lowest value obtainable for earnings is reached with the consideration of both scenarios when the maximization of the satisfaction of the quality of life seekers is attained. Finally, the trade-off between less prone to risk workers and quality of life seekers is also revealed, with the lowest job security levels found in the solution that maximizes working times.

Keywords Multiple objective programming, workers' satisfaction, econometric analysis, interval programming.

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1. Introduction

As highlighted by the European Union Treaty, one of the key issues to combat social exclusion and promote social and economic growth is to support high levels of employment and good working conditions. In this sense, the most controversial reforms, at the country level, have been those affecting labour market regulations. In general, the aim has been to generate "flexicurity", i.e. to set up stable employment relationships, avoiding an increasing rate of precarious, low-quality, low-paid work and an insecure workforce, which lead to social exclusion. This means the need to offer greater flexibility for workers, through the reorganization of working time (e.g., to enable them to reconcile work and family responsibilities) and provide salaries according to the standard of living. Regardless of the potential success of many of these measures, the need for greater concern about job quality/satisfaction seems clear, due to its potential link with workers' productivity (see, e.g. Warr, 1999), to the extent that it affects employees' quitting behaviour (Freeman, 1978), absenteeism, turnover (Green, 2010), and firms' productivity (Böckerman and Ilmakunnas, 2012). In fact, job satisfaction acts as a summary measure of the different aspects of job quality, a number of which are difficult to observe or measure. By itself, the use of satisfaction information may help to explain workers' behaviour better than data on, for example, payment and hours. Additionally, we propose job satisfaction as a subjective measure of workers' well-being because, although it is not necessarily the ideal instrument for capturing this well-being, it is the best proxy available in the dataset under scrutiny.

In this context, we develop a real data application –using data from the Spanish labour market– to optimize different aspects of job satisfaction as a proxy for job quality, in an attempt to quantify workers' individual preferences. To the extent that job satisfaction is not a single dimensional measure, different (and conflicting) aspects of job satisfaction have been taken into account, being each one translated into an objective function.

The range of variation of the estimated objective function coefficients using econometric techniques are then used to instantiate a multiobjective model with interval coefficients. In this way, further insights can be obtained, which classical econometric techniques are not able to provide. The use of such "combined" analysis and, additionally, confidence intervals stemming from the econometric analysis to build ranges of variation for the objective function coefficient values, allows to assess the trade-offs among the different aspects of job satisfaction under different scenarios herein given within interval ranges.

Based on data regarding the different aspects of job satisfaction, econometric estimates to obtain a–flexible– relationship between workers' satisfaction and an individual/contextual set of features is also provided. Particularly, we use survey data containing records on three job-related characteristics which are said to be valued by workers: earnings, job security and working times. These are all argued to be key correlates of a good job or job satisfaction. At a second stage, we will make use of multiobjective interval programming techniques to disentangle the extent to which those correlations may be affected.

The rest of the paper is organised as follows. Section 2 presents the estimation of the coefficients and the decision variables which will be used to build the multiobjective interval model suggested in Section 3. Section 4 presents an algorithm to generate the "possibly" efficient solutions to the multiobjective interval model. Sections 5 and 6 provide and contrast some illustrative results obtained with both the reference-based approach herein suggested. Finally, some conclusions are drawn and future work developments are discussed.

2. Econometric Analysis

In this section we specify and estimate the interval coefficients, which will be used to instantiate and define the objective functions and constraints of the multiobjective interval programming model suggested in the next section.

2.1. Data

The information analysed in this paper comes largely from the European Community Household Panel (ECHP) for the period 1995-2001, in which workers provide information on a wide range of personal characteristics and job attributes[1]. This survey was conducted, under Eurostat supervision, across 15 European Community state members during the period 1994-2001, although not all countries took part in all waves. We have selected the data corresponding to Spain for our study and restricted the sample to those workers working in the private sector and who are older than 25 and younger than 65 (retirement age). The reason for choosing this minimum age is that it is around

[1] The first wave of this panel survey (1994) is not considered in the analyses due to the lack of information on some of the relevant variables for the analysis.

this age that most people start looking for a job after completing their highest education level. In fact, 90% of the surveyed Spanish workers report 26 as the age when the highest level of education was completed.

Workers in the ECHP were asked to evaluate several aspects of a job, on a scale from 1 to 6, where 1 is "not satisfied at all" and 6 is "fully satisfied". The key job aspects we have analysed are: earnings, job security and working times[2]. The precise wording of the questions was: "How satisfied are you with your present job in terms of …?"[3] These categories are not exhaustive but they serve to summarize many of the job characteristics that workers find important.

With regard to the decision variables of our model, they constitute the main set or inputs referenced in the previous literature conditioning workers' satisfaction (Clark et al., 2008). They have been listed in Table A1 (Appendix). We ended up with 33 variables[4], most of which are under individual decision-makers' control. As shown in Table A1 (Appendix), there are 4 continuous variables, 1 integer and the rest are binary. For all the binary variables, a reference group which is assumed to be equal to 1 if the rest of the variables are 0 has been considered. For example, for the education level group, if $edh = eds = 0$ (or $x_2 = x_3 = 0$) the individual belongs to the reference group (first level of secondary education or lower). This fact has been taken into account in the regression analysis. Inside this group of 33 variables, 6 year dummies ($y_2, y_3, \dots, y_7$) have been used in order to take into account effects due to the year when each survey was conducted. Summary statistics distinguished by gender, for the whole set of variables incorporated in the analysis, are shown in Table A2 (Appendix).

The figures which appear in Table A2 (Appendix) disclose some well-established differences between male and female workers (Liu, 2016), which are going to be discussed below. The proportion of female workers is much lower than the one presented by male workers (36% of the sample). Therefore, patterns of women and men in the labour market are highly likely to differ. Consequently, we run separate estimates for men and women. However, men tend to have a much higher gross hourly real wage than their

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[2] In Marcenaro et al. (2010) seven aspects were considered; however, since we are aimed at contrasting the major trade-offs emerging from the assessment, we have restricted our analysis to three axes of evaluation.
[3] E.g., variable PE031: "How satisfied are you with your present job in terms of earnings?"
[4] We selected information on 33 variables that may fit in our model on satisfaction, bearing in mind that due to the aims of the paper we try to use a parsimonious specification to implement the multicriteria methodology in a clear way.

female counterparts (over 1.12 Euros higher, equivalent to 18.8%), despite the fact of having considerably lower education levels.

We also control our estimates for net family income (discounting a worker's own income). This variable is trimmed by treating income observations below 0.5% and above 99.5% as missing data in order to avoid the blurring effects of extreme values[5]. Interestingly, female employees enjoy higher net family income, reinforcing the previous argument that it is men who receive higher earnings. In respect of working hours, slightly more than 1 in 5 women currently work more than forty hours a week; however, the figure rises up to 37% for men. Likewise, supervisory or intermediate statuses are more likely among men. Being married is definitively a drawback for women when participating in the labour market, as reflected by the figures, where it can be observed that the proportion of married women is substantially lower than men. Men report slightly lower unemployment spells[6], despite having fewer formal qualifications on average. Moreover, they are exposed to much lower regional unemployment rates. It is also worth mentioning that the proportion of women working in the construction or transport sector is negligible as compared to men, this meaning that some degree of segregation across occupations between male and female employees still persists.

Due to the substantial differences between male and female workers, and to focus on the bigger subsample, in what follows we report the analysis just for men[7].

2.2. Econometric Estimates

We start the econometric analysis by estimating regression models in which our job satisfaction measures are regressed on the set of explanatory variables above reported, pooling all seven years. Satisfaction is a discrete, ordered variable categorized into one of six response codes. Thus, we first run ordered probit models, obtaining results (which are) very close to those showed by ordinary least squared estimations (OLS). For this reason, and in order to make the implementation of the Multiobjective Interval Programming approach more consistent, we decided to use the coefficients obtained by

[5] This trimming does not change significantly the composition of the sample in terms of the variables considered in our estimates. In fact, the results of our estimates do not change in a meaningful way when the sample is not trimmed.
[6] These unemployment spells are previous to the present job.
[7] The analysis for the subsample of women are available from the authors upon request.

OLS. Furthermore, the previously indicated set of variables in Table A1 (Appendix) and Table A2 (Appendix) are those which were significant at least in one objective function for men.

As previously outlined, we can proxy individuals' well-being through different categories of "job satisfaction". The level for each of these satisfaction targets results from the combination of a set of individual and contextual features, unobservable factors and a random disturbance (ε). The idea behind the OLS estimator is to minimize the latter term in order to get rid, as much as possible, of the so-called "statistical noise". If individuals are indexed by "r", and job satisfaction aspects are indexed by "s", this model can be represented by the following set of equations:

$$S_s(r) = \widehat{\alpha}^s + \widehat{\beta}_1^s ghw(r) + \widehat{\beta}_2^s edh(r) + \cdots + \widehat{\beta}_{33}^s y_7(r) + \varepsilon_s(r)$$

$$r = 1, \ldots, w; s = 1, 2, 3$$

where $S_s(r)$ is a measure of the satisfaction category s of individual r, and $ghw(r), edh(r), \ldots, y_7(r)$, a group of explanatory variables; $\varepsilon_s(r)$ is a random disturbance; $\widehat{\boldsymbol{\beta}}^s$ a vector of slope coefficients and $\widehat{\boldsymbol{\alpha}}^s$ a fixed but unknown population intercept. The size of the sample is represented by the value w. Therefore, we are assuming that each individual's job satisfaction is affected by random factors which are inherently unobservable and distributed normally. This type of parsimonious model is characterized by the parametric nature of its specification.

Table 1 shows the lower and upper bounds of the estimated coefficients on the key variables of interest (significant at 5% or 1% level). Those coefficients which are not significantly different from "0" at the standard statistical levels (10% or lower) are reported as "0". The upper and lower bounds of the coefficients correspond to the standard interval coefficients obtained from the OLS estimation; this interval is presented by $\widehat{\beta}_j \pm t_{n-k-1,\alpha/2} SE$, where SE is the standard error and "t" are the probability values corresponding to a "t" distribution. To the extent that the punctual estimate is just the central value of the distribution, considering the interval we can be 95% sure that that the true regression coefficient for each variable is between the upper and the lower bound.

Table 1. Determinants of different aspects of job satisfaction in the sample of Spanish male workers

		Satisfaction in terms of:					
		Earnings		Job security		Working times	
	Variables	Lower bound	Upper bound	Lower bound	Upper bound	Lower bound	Upper bound
Gross real hourly wage	x_1	0.105	0.126	0.011	0.032	0.024	0.046
Education level:							
Higher education	x_2	-0.237	-0.077	-0.305	-0.138	-0.268	-0.097
Secondary education	x_3	-0.213	-0.065	-0.247	-0.093	-0.242	-0.084
Net Family Income (10^3€)	x_4	-0.001	0.007	0.0003	0.009	0	0
Age	x_5	-0.015	-0.007	-0.011	-0.004	0.0001	0.008
Job seniority:							
Seniority 3-4	x_6	-0.218	-0.045	0.049	0.230	0	0
Seniority 5-9	x_7	-0.214	-0.0003	0.114	0.337	0	0
Seniority 10-14	x_8	0	0	0.132	0.465	0	0
Seniority 15+	x_9	0	0	0.131	0.598	-0.05	0.428
Working + 40 hours/week	x_{10}	0.075	0.194	0.028	0.152	-0.514	-0.387
Permanent contract	x_{11}	0.075	0.220	1.169	1.320	-0.001	0.153
Occupational status:							
Supervisory	x_{12}	0.055	0.283	0.260	0.498	0.082	0.325
Intermediate	x_{13}	0	0	0.074	0.240	0.013	0.183
Married	x_{14}	-0.231	-0.084	0	0	-0.169	-0.013
Unemployment duration	x_{15}	-0.002	0.0002	0	0	-0.002	0.0002
Worker's Health:							
Good health	x_{16}	0.210	0.626	0.118	0.551	0.203	0.646
Fair health	x_{17}	0.006	0.445	-0.066	0.391	0.102	0.570
Regional unemployment rate	x_{18}	0.0005	0.009	0	0	0	0
Industry of current job:							
Utilities and construction	x_{19}	0.028	0.183	-0.159	0.002	0.058	0.223
Hotel sales	x_{20}	0	0	0.026	0.184	-0.144	0.018
Transport	x_{21}	0	0	-0.204	0.031	-0.331	-0.090
Public education social	x_{22}	0	0	-0.015	0.389	-0.052	0.363
Other industry	x_{23}	0.048	0.347	0.028	0.339	0.008	0.326
Firm size:							
Firm size 5-19	x_{24}	-0.147	0.014	0	0	-0.174	-0.002
Firm size 20-99	x_{25}	0	0	-0.157	0.020	-0.159	0.023
Firm size 100-499	x_{26}	0	0	0	0	-0.216	0.011
Firm size 500+	x_{27}	0	0	0	0	-0.253	0.014
Year dummies:							
Year 1996	x_{28}	-0.190	0.025	-0.219	0.004	-0.212	0.017
Year 1997	x_{29}	0	0	0	0	-0.276	-0.051
Year 1998	x_{30}	0	0	0	0	-0.290	-0.062
Year 1999	x_{31}	-0.018	0.199	0	0	-0.324	-0.093
Year 2000	x_{32}	0.111	0.331	-0.022	0.208	-0.342	-0.106
Year 2001	x_{33}	0.054	0.278	-0.027	0.205	-0.352	-0.114
Constant		2.067	2.673	2.552	3.183	3.348	3.994
Observations		10,318	10,318	10,318	10,318	10,318	10,318
R-squared		0.122	0.122	0.276	0.276	0.058	0.058

Note: We have repeated observations on individuals (given that we are using a pool of waves); thus we have clustered errors across individuals.

Variables which were not significant at least at 10% have been assigned the “0” value.
Source: Authors’ own calculations.

The estimated coefficients for the earnings variable show that all the constituent parts of job satisfaction evaluated are positively and significantly correlated with a worker’s gross hourly real wage. Conversely, job satisfaction appears to decrease with the level of education, in line with some previous results (Clark and Oswald, 1996), who found greater satisfaction for the less educated in Britain in the early 1990s. Nevertheless, the literature to date has produced mixed results (see, e.g., Powdthavee et al., 2015).

The effect of net family income on job satisfaction varies depending on the job satisfaction aspect under evaluation. Concretely, men seem to be more satisfied with earnings and job security as their net family income increases, but satisfaction with working times does not seem to be affected. Similarly, they do better in terms of satisfaction with earnings and job security when they are young; however, the opposite applies in terms of satisfaction with working times[8]. Thus some trade-offs seem to be undergoing.

The number of years continuously working for the same firm holds a straightforward correlation with satisfaction in terms of job security, which seems logical as the worker will have better prospects to stay in the firm as time goes by. This is particularly relevant in Spain, where the rate of temporary employment is one of the largest in Europe. Conversely, satisfaction with earnings shows a negative sign for the first years of seniority, as a reflection of the worse salary conditions along the first stage of the working life. Thus, again we observe a trade-off.

When we examine the dummy variable of working hours, two facts deserve our attention. On one hand, those men who work more than 40 hours per week report less satisfaction with working times than the reference workers (those working 40 hours or less). On the other hand, satisfaction with earnings and job security are higher for men who work over 40 hours per week. In recent years, working hours have become an important policy issue in debates over Europe’s high rate of unemployment and overworking (mainly focused on the negative effects on workers’ health status) and on

[8] We ran several alternative specifications, one of those included the age-squared (i.e. specifying age non-linearly), but it did not work (it was not significant at the standard statistical levels).

the doubts about rationality of working times in Spain. Accordingly, it seems that working hours are a potentially useful policy instrument to change workers' satisfaction.

Turning to our findings, we see a consistent positive effect of permanent contracts –and being supervisor– on job satisfaction, as it could be expected.

However, the marital status coefficient is negative and significant when satisfaction with earnings and working times are evaluated. This may respond to the fact that marital status is linked to being the head of the household, which means more pressure to get a higher wage if married. Similarly, having a partner seems to increase the importance of the working schedule (i.e. working at nights, shifts, etc.). On the other hand, having good health increases workers' satisfaction. In the same way, higher regional unemployment rates [9] make workers more satisfied with their jobs and consequently more satisfied with earnings.

The final set of variables measures the size of the firm where the individual is currently working. Basically, big firms provide workers with lower levels of satisfaction with working times.

The significant relations between the different satisfaction levels and variables considered, which in turn provide some interesting conclusions about the structure of the Spanish labour market are, in general, quite consistent with those found in the literature. In order to help assessing the trade-offs regarding the axes of evaluation of job satisfaction of the Spanish worker under distinct scenarios and to help establish the policies which can be carried out to increase employees' satisfaction, a multiobjective interval model is suggested in the next section.

3. The multiobjective interval model

The model presented in Table 1 enabled us to express 98% upper and lower confidence intervals for all the coefficients in each of the regressions in three objective functions, i.e. earnings, job security and working times, respectively. Concretely, this is the form that these functions adopt:

$$Max\ Z_1 = +[0.105,0.126]x_1 + [-0.237,-0.077]x_2 + \cdots + [0.054,0.278]x_{33} + [2.067,2.673]$$

[9] Regional unemployment rates are provided by the European Statistic Database REGIO, using NUTS-2 level region as a definition.

$$Max\, Z_2 = +[0.011{,}0.032]x_1 + [-0.305, -0.138]x_2 + \cdots + [-0.027{,}0.205]x_{33} + [2.552{,}3.183]$$

$$Max\, Z_3 = +[0.024{,}0.046]x_1 + [-0.268, -0.097]x_2 + \cdots + [-0.352, -0.114]x_{33} + [3.348{,}3.994]$$

In order to ensure that the problem being solved is realistic, a set of technical constraints has been defined. Each constraint of (C1) to (C7) represents a group of binary categorical variables, which allows ensuring that they do not simultaneously take the value 1 in the multiobjective optimization problem[10]:

1) Education level

$$x_2 + x_3 \leq 1 \qquad \text{(C1)}$$

2) Job seniority

$$x_6 + x_7 + x_8 + x_9 \leq 1 \qquad \text{(C2)}$$

3) Occupational status

$$x_{12} + x_{13} \leq 1 \qquad \text{(C3)}$$

4) Worker's health

$$x_{16} + x_{17} \leq 1 \qquad \text{(C4)}$$

5) Industry of current job

$$x_{19} + x_{20} + x_{21} + x_{22} + x_{23} \leq 1 \qquad \text{(C5)}$$

6) Firm size

$$x_{24} + x_{25} + x_{26} + x_{27} \leq 1 \qquad \text{(C6)}$$

7) Year dummies

$$x_{28} + x_{29} + x_{30} + x_{31} + x_{32} + x_{33} \leq 1 \qquad \text{(C7)}$$

Other constraints have also been set in order to ensure that the profile of the male workers we are looking for is realistic. These are the following:

-Relationship between age (x_5), unemployment duration (x_{15}) and seniority ($x_6 - x_9$). The sum of unemployment duration plus job seniority cannot be higher than worker's age

[10] In other words, this set of technical constraints are implemented to ensure that we do not fall into the dummy variable trap.

minus 16, as it is the minimum legal working age (x_{15} is measured in months):

$$x_5 - (3x_6 + 5x_7 + 10x_8 + 15x_9) - (1/12)x_{15} \geq 16 \qquad \text{(C8)}$$

-Relationship between gross real hourly wage (x_1) and education level (x_2 and x_3). It is not realistic to consider that workers with only primary education can get a higher salary than those with. e.g., higher education. Based on the data, 98% of the male workers with a higher education level (x_2) do not have a gross real hourly wage (x_1) higher than 22.16 Euros, while 98% of the male workers with secondary education level (x_3) do not have a gross real hourly wage (x_1) higher than 17.64 Euros, and so on. Given these relationships, we have stablished the following upper bounds on the gross real hourly wage for different education levels:

(a) If $x_3 = 0$ then $x_1 \leq 22.16$

(b) If $x_3 = 1$ then $x_1 \leq 17.64$

(c) If $x_2 + x_3 = 0$ then $x_1 \leq 12.38$

(d) If $x_2 + x_3 = 1$ then $x_1 \leq 22.16$

All these bounds are summarized in the two following constraints:

$$x_1 + (22.16 - 17.64)x_3 \leq 22.16 \qquad \text{(C9)}$$

$$x_1 - (22.16 - 12.38)(x_2 + x_3) \leq 12.38 \qquad \text{(C10)}$$

-Relationship between gross real hourly wage (x_1) and occupational status (x_{12} and x_{13}). Following the same logic as in the previous set of constraints, we have established upper bounds on the gross real hourly wage for male workers' occupational statuses:

(a) If $x_{13} = 0$ then $x_1 \leq 26.09$

(b) If $x_{13} = 1$ then $x_1 \leq 18.35$

(c) If $x_{12} + x_{13} = 0$ then $x_1 \leq 12.99$

(d) If $x_{12} + x_{13} = 1$ then $x_1 \leq 26.09$

The previous bounds can be reflected in the next two constraints:

$$x_1 + (26.09 - 18.35)x_{13} \leq 26.09 \qquad \text{(C11)}$$

$$x_1 - (26.09 - 12.99)(x_{12} + x_{13}) \leq 12.99 \qquad \text{(C12)}$$

In order to illustrate the building of these constraints, an example using the "Gross real hourly wage" (x_1) and "Higher education" (x_2) variables is provided in the following:

The dependency between x_1 and x_2 is given by the following linear regression:

$$x_1 = a \cdot x_2 + b$$

where the confidence intervals of the coefficients are (at 98%):

$$a \in [a^l, a^u] = [2.787, 3.1514] \text{ and } b \in [b^l, b^u] = [6.285, 6.4564]$$

which implies:

$$a^l \cdot x_2 + b^l \leq x_1 \leq a^u \cdot x_2 + b^u$$

These inequalities provided us with two new constraints:

$$x_1 - (3.1514x_2 + 6.4564) \leq 0 \qquad \text{(C13)}$$

$$x_1 - (2.787x_2 + 6.285) \geq 0 \qquad \text{(C14)}$$

Following this procedure, we can get constraints (C15) to (C20):

$$x_4 + 5.779x_{14} - 9.975 \leq 0 \qquad \text{(C15)}$$

$$x_4 + 6.404x_{14} - 9.503 \geq 0 \qquad \text{(C16)}$$

$$x_{14} - 0.0279x_5 + 0.3076 \leq 0 \qquad \text{(C17)}$$

$$x_{14} - 0.02603x_5 + 0.3733 \geq 0 \qquad \text{(C18)}$$

$$x_{18} + 0.3741x_5 - 27.9013 \leq 0 \qquad \text{(C19)}$$

$$x_{18} + 0.4082x_5 - 26.7037 \geq 0 \qquad \text{(C20)}$$

The limits of the variables considered in the model for the sample under analysis are:

$$2.340 \leq x_1 \leq 21.573,\ 0 \leq x_4 \leq 32.868,\ 25 \leq x_5 \leq 64,\ 0 \leq x_{15} \leq 244,\ 4.2 \leq x_{18} \leq 41.6$$

$$x_2, x_3, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{16}, x_{17}, x_{19}, x_{20} \in \{0,1\}$$

$$x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}, x_{27}, x_{28}, x_{29}, x_{30}, x_{31}, x_{32}, x_{33} \in \{0,1\}$$

$$x_1, x_4, x_5, x_{18} \in \mathbb{R},\ x_{15} \in \mathbb{Z}$$

4. An algorithm to obtain "possibly" efficient solutions

Multiobjective programming models for decision support have been found to be really useful to solve problems of different nature (Pérez-Galarce et al., 2017; Medaglia et al., 2008). In order to solve them, this methodology should take explicitly into account the treatment of the inherent uncertainty associated with the model's coefficients. Interval programming is one of the approaches used to handle uncertainty in mathematical programming models, which entails some interesting characteristics. In contrast to stochastic programming or to fuzzy programming which start with the specification or the assumption of probabilistic distributions and possibilistic distributions, respectively, and to robustness optimization techniques which inherently consider a max-min formulation (i.e., worst-case), interval programming only requires information about the range of variation of some (or all) of the parameters (Oliveira and Antunes, 2007).

In general, in interval multiobjective linear programming (MOLP), there are several approaches which allow for the enumeration of all "possibly" efficient solutions (see an illustrative review of some of these methods in Oliveira and Antunes, 2007); however, the computational burden required might be considerable if the number of interval coefficients on the objective function is high.

Another issue refers to the fact that even if checking possible efficiency can be done effectively (in polynomial time), the set of "possibly" efficient solutions may contain an infinite number of elements, in many cases with just slight differences among the objective function values (Rivaz et al., 2016). Moreover, necessary efficiency is also hard to check and to calculate them is a NP-hard problem (Oliveira and Antunes, 2007; Hladík, 2012). On the other hand, although the "necessarily efficient" solutions are the most robust ones, in most situations they may not exist (Oliveira and Antunes, 2007).

In order to overcome the main problems raised by these approaches Inuiguchi and Sakawa (1995) have proposed a methodology to solve Linear Programming (LP) problems with interval coefficients on the objective function which considers a "min-max regret" objective function and uses relaxed LP problems. More recently, Rivaz and Yaghoobi (2013) and Rivaz et al. (2016) also propose a "min-max regret" solution to MOLP problems. The "min-max regret" solution takes into account all possibilities of objective functions' coefficients. Additionally, a "min-max regret" solution is "possibly" efficient and coincides with the necessarily efficient solution when it exists. However, if no necessarily efficient solutions are available the computational burden required just to obtain a "possibly" efficient solution can be significant.

Therefore, this section of the paper presents an extension of the idea proposed in Oliveira and Antunes (2009) combined with a reference point-based approach, through the use of surrogate scalarizing problems to solve Multiobjective Linear Programming problems with interval objective function coefficients. The proposed surrogate scalarizing problems allow generating the "possibly" efficient solutions which are closest as possible to the interval ideal solution without requiring a considerable computational effort and using a simple mathematical formulation. Some underpinning assumptions used in the framework of interval MOLP are also provided and the properties of the obtained solutions are also analysed.

4.1. Background concepts and notation

Consider, without loss of generality, the following multiobjective programming model with interval coefficients:

$$\begin{aligned} \max Z_k(\mathbf{x}) &= \sum_{j=1}^{n} [c_{kj}^L, c_{kj}^U]\, x_j, & k &= 1,\dots,p, \\ s.t.: \sum_{j=1}^{n} a_{ij}x_j &\leq b_i, & i &= 1,\dots,m, \\ x_j &\geq 0, & j &= 1,\dots,n. \end{aligned} \tag{1}$$

where $\mathbf{x} = (x_1,\dots,x_n)^T$ is the vector of variables and $X = \left\{\mathbf{x} = (x_1,\dots,x_n)^T \in \mathbb{R}^n \mid \sum_{j=1}^{n} a_{ij}x_j \leq b_i \;\; i = 1,\dots,m,\;\; x_j \geq 0,\;\; j = 1,\dots,n \right\}$ is the feasible region.

It is evident that if each of the coefficients $[c_{kj}^L, c_{kj}^U]$ is a real value, then problem (1) is a MOLP problem:

$$\begin{aligned} \max \overline{Z}_k(\mathbf{x}) &= \sum_{j=1}^{n} \bar{c}_{kj}\, x_j, & k &= 1,\dots,p, \\ s.t.: \sum_{j=1}^{n} a_{ij}x_j &\leq b_i, & i &= 1,\dots,m, \\ x_j &\geq 0, & j &= 1,\dots,n. \end{aligned} \tag{2}$$

where $\bar{c}_{kj} \in [c_{kj}^L, c_{kj}^U]$ for all $k = 1,\dots,p,\ j = 1,\dots,n$.

Analogously to the concept of ideal solution values in MOLP programming, the optimal values with the upper bounds of each objective function (best case scenario in a maximization problem) and with the lower bounds of each objective function (worst case scenario in a maximization problem) are computed.

For each objective function $Z_k(\mathbf{x})$, $k = 1,\dots,p$, the following linear programming problems are solved (Chinneck and Ramadan, 2000):

$$\max Z_k^U(\mathbf{x}) = \sum_{j=1}^{n} c_{kj}^U x_j$$
$$s.t.: \sum_{j=1}^{n} a_{ij} x_j \le b_i, \qquad i = 1, \dots, m, \qquad (3)$$
$$x_j \ge 0, \qquad j = 1, \dots, n.$$

and

$$\max Z_k^L(\mathbf{x}) = \sum_{j=1}^{n} c_{kj}^L x_j$$
$$s.t.: \sum_{j=1}^{n} a_{ij} x_j \le b_i, \qquad i = 1, \dots, m, \qquad (4)$$
$$x_j \ge 0, \qquad j = 1, \dots, n.$$

Let model (3) and model (4) be identified, respectively, with $\beta = 0$ and $\beta = 1$ and the optimal solution to each model be identified with $\mathbf{x}_k^{\beta}$, $k = 1, \dots, p$. The optimal values are denoted by:

$$Z_k^{U^*} = Z_k^U(\mathbf{x}_k^0), \qquad k = 1, \dots, p, \qquad (5)$$
$$Z_k^{L^*} = Z_k^L(\mathbf{x}_k^1), \qquad k = 1, \dots, p. \qquad (6)$$

The intervals $\left[Z_k^{L^*},\ Z_k^{U^*}\right]$ with $k = 1, \dots, p$ are considered to be the bounds of the interval ideal solution (Oliveira and Antunes, 2009).

The concept of efficiency in multiobjective programming (Miettinen, 1999) is also adapted to interval MOLP (Bitran, 1980).

Definition 1

A solution $\mathbf{x}' \in X$ is efficient to problem (2) if and only if there is no other $\mathbf{x} \in X$ such that $\overline{Z}_k(\mathbf{x}') \le \overline{Z}_k(\mathbf{x})$ for all $k = 1, \dots, p$ with at least one strict inequality.

Definition 2

A solution $\mathbf{x}' \in X$ is necessarily efficient to problem (1) if and only if it is efficient to problem (2) for any $\bar{c}_{kj} \in [c_{kj}^L, c_{kj}^U]$ for all $k = 1, \dots, p$, $j = 1, \dots, n$. The necessarily efficient solution set is given by N_E.

Definition 3

A solution $\mathbf{x}' \in X$ is “possibly” efficient to problem (1) if it is efficient to problem (2) for at least one $\bar{c}_{kj} \in [c_{kj}^L, c_{kj}^U]$ for all $k = 1, \dots, p$, $j = 1, \dots, n$. The “possibly” efficient solution set is given by P_E.

A suitable solution to an interval MOLP should be selected among the elements of N_E or P_E.

In this context, a way of expressing preferences about efficient solutions in multiobjective programming is through a *reference point* $\mathbf{q} = (q_1, \dots, q_p)^T$, which consists of a desirable or a reference value for the objective functions. One of the most common achievement scalarizing function (ASF) to problem (2) was proposed by Wierzbicki (1980):

$$s(\mathbf{q}, \bar{Z}(\mathbf{x}), \boldsymbol{\mu}) = \max_{k=1,\dots,p} \{\mu_k (q_k - \sum_{j=1}^{n} \bar{c}_{kj}\, x_j)\} \quad (7)$$

where $\boldsymbol{\mu} = (\mu_1, \dots, \mu_p)^T$ is a vector of weights for reaching these values. This function must be minimized in the feasible region:

$$\begin{aligned} &\min \max_{k=1,\dots,p} \{\mu_k (q_k - \sum_{j=1}^{n} \bar{c}_{kj}\, x_j)\} \\ &s.t.: \mathbf{x} \in X \end{aligned} \quad (8)$$

Every solution of (8) is weakly Pareto optimal of (2) and it is Pareto optimal if it is unique.

One possible drawback of the problem (8) is that it is generally non-differentiable even if the functions in the original problem (2) are all differentiable or even linear[11]. However, this drawback can be overcome if we use an equivalent differentiable formulation:

$$\begin{aligned} \min \quad & \alpha \\ s.t.: \quad & \mu_k \big(q_k - \sum_{j=1}^{n} \bar{c}_{kj} x_j\big) \leq \alpha \\ & \mathbf{x} \in X \end{aligned} \quad (9)$$

which implies in our case that the problem (9) is linear.

4.2. Scalarizing problems for interval MOLP problems

In this section, we propose an approach based on a simple mathematical surrogate formulation (considering reference point-based techniques) to obtain “possibly” efficient solutions to problem (1) which are closest (according to the concept of “necessary”

[11] For example, if the problem is linear (all the objective functions and the constraints are linear), an appropriate single objective optimization solver for linear programming can be used, which is usually more efficient and accurate than a solver for non-differentiable problems.

subtraction) to the interval ideal solution (further developments on "necessary" subtraction are obtainable from Inuiguchi and Kume, 1991).

If the DM wishes to minimize the maximum "necessary" regret, $E(\mathbf{x})$, of each interval objective function from its corresponding interval target problem (1), (s)he has the following surrogate problem:

$$\begin{aligned}
&\min \max E(\mathbf{x}) \\
&s.t.: \textstyle\sum_{j=1}^{n} c_{kj}^{L} x_j + e_k^{L-} = Z_k^{L*}, && k = 1, \dots, p, \\
&\textstyle\sum_{j=1}^{n} c_{kj}^{U} x_j + e_k^{U-} = Z_k^{U*}, && k = 1, \dots, p, \\
&\textstyle\sum_{j=1}^{n} a_{ij} x_j \leq b_i, && i = 1, \dots, m, \\
&x_j \geq 0, && j = 1, \dots, n. \qquad (10)
\end{aligned}$$

where the deviational variables, e_k^{L-} and e_k^{U-} are defined in such a way that:

$$\textstyle\sum_{j=1}^{n} c_{kj}^{L} x_j + e_k^{L-} = Z_k^{L*}, \sum_{j=1}^{n} c_{kj}^{U} x_j + e_k^{U-} = Z_k^{U*} \qquad (11)$$

Problem (10) can either be solved by considering an optimistic (the lower bound of the "necessary" deviation) or a pessimistic procedure (the upper bound of the "necessary" deviation). The second procedure allows obtaining the smallest distance from the target considered (Oliveira and Antunes, 2007; Inuiguchi and Kume, 1991). Therefore, we consider the following surrogate problem, which guarantees the smallest deviation possible form the interval ideal solution:

$$\begin{aligned}
&\min v, \\
&s.t.: \textstyle\sum_{j=1}^{n} c_{kj}^{L} x_j + e_k^{L-} = Z_k^{L*}, && k = 1, \dots, p, \\
&\textstyle\sum_{j=1}^{n} c_{kj}^{U} x_j + e_k^{U-} = Z_k^{U*}, && k = 1, \dots, p, \\
&\textstyle\sum_{j=1}^{n} a_{ij} x_j \leq b_i, && i = 1, \dots, m, \\
&e_k^{L-} \leq u_k, && k = 1, \dots, p, \\
&e_k^{U-} \leq u_k, && k = 1, \dots, p, \\
&u_k \leq u^U, && k = 1, \dots, p, \\
&\lambda \textstyle\sum_{k=1}^{p} \gamma_k u_k + (1-\lambda) u^U \leq v, \\
&0 \leq \lambda \leq 1, \\
&\gamma_k \geq 0, && k = 1, \dots, p, \\
&\textstyle\sum_{k=1}^{p} \gamma_k = 1, \\
&x_j \geq 0, && j = 1, \dots, n. \qquad (12)
\end{aligned}$$

Since that e_k^{L-} and e_k^{U-} for all $k = 1, \dots, p$ are always positive values (Z_k^{L*} and Z_k^{U*} are ideal optimal values for $\sum_{i=1}^{n} c_{ki}^{L} x_i$ and $\sum_{i=1}^{n} c_{ki}^{U} x_i$, respectively), the first two sets of constraints of problem (12) can be simplified and e_k^{L-} and e_k^{U-} for all $k = 1, \dots, p$ are not necessary. Thus, we propose a scalarizing problem equivalent to (12) whose formulation is simplified:

$$
\begin{aligned}
&\min\ v \\
&\text{s.t. } Z_k^{L*} - \textstyle\sum_{j=1}^{n} c_{kj}^{L} x_j \le u_k, && k = 1, \dots p, \\
&\quad Z_k^{U*} - \textstyle\sum_{j=1}^{n} c_{kj}^{U} x_j \le u_k, && k = 1, \dots p, \\
&\quad \textstyle\sum_{j=1}^{n} a_{ij} x_j \le b_i, && i = 1, \dots m, \\
&\quad u_k \le u^U, && k = 1, \dots p, \\
&\quad \lambda \textstyle\sum_{k=1}^{p} \gamma_k u_k + (1-\lambda)\, u^U \le v, \\
&\quad 0 \le \lambda \le 1, \\
&\quad \gamma_k \ge 0, && k = 1, \dots p, \\
&\quad \textstyle\sum_{k=1}^{p} \gamma_k = 1, \\
&\quad x_j \ge 0, && j = 1, \dots n. \qquad (13)
\end{aligned}
$$

Considering problem (13) and the reference point-based approach, we propose the following scalarizing problem:

$$
\begin{aligned}
&\min\ v \\
&\text{s.t. } \mu_k^L \left(Z_k^{L*} - \textstyle\sum_{j=1}^{n} c_{kj}^{L} x_j\right) + \mu_k^U \left(Z_k^{U*} - \textstyle\sum_{j=1}^{n} c_{kj}^{U} x_j\right) \le v, \; k = 1, \dots, p, \\
&\quad \textstyle\sum_{j=1}^{n} a_{ij} x_j \le b_i, && i = 1, \dots, m, \\
&\quad x_j \ge 0, && j = 1, \dots, n. \qquad (14)
\end{aligned}
$$

which is even more simplified than the previous one. This approach takes into account the Tchebychev distance to the interval ideal values Z_k^{L*} and Z_k^{U*} and the importance for reaching these values is made by means of the weights $\mu_k^L, \mu_k^U > 0$ with $\mu_k^L + \mu_k^U = 1$ for all $k = 1, \dots, p$.

Now let us see that if the optimal solution to (14) is unique, then it is a “possibly” efficient solution to (1).

Theorem 1

Let $\mathbf{x}^* = (x_1^*, \dots, x_n^*)^T$ and v^* be an optimal solution to the problem (14) where μ_k^L, $\mu_k^U > 0$ and $\mu_k^L + \mu_k^U = 1$ for all $k = 1, \dots, p$, with μ_k^L and μ_k^U given as fixed values. If $\mathbf{x}^*$ is unique, then it is a "possibly" efficient solution to problem (1).

Proof

Let us prove the Theorem by *reductio ad absurdum.* Assuming that $\mathbf{x}^*$ is not a "possibly" efficient solution to problem (1), for any $\bar{c}_{kj} \in \left[c_{kj}^L, c_{kj}^U\right]$ for all $k = 1, \dots, p$, $j = 1, \dots, n$, there is a feasible solution $\mathbf{x}^1$ such that dominates $\mathbf{x}^*$ for these coefficients. Let us consider the following coefficients:

$$\bar{c}_{kj} = \mu_k^L\, c_{kj}^L + \mu_k^U\, c_{kj}^U \text{ for all } k = 1, \dots, p, j = 1, \dots, n,$$

where $\bar{c}_{kj} \in \left[c_{kj}^L, c_{kj}^U\right]$ for all $k = 1, \dots, p$, $j = 1, \dots, n$ since that μ_k^L, $\mu_k^U > 0$ and $\mu_k^L + \mu_k^U = 1$.

There is a feasible solution $\mathbf{x}^1$ such that:

$\sum_{j=1}^n \left(\mu_k^L\, c_{kj}^L + \mu_k^U\, c_{kj}^U\right) x_j^* \leq \sum_{j=1}^n \left(\mu_k^L\, c_{kj}^L + \mu_k^U\, c_{kj}^U\right) x_j^1$ for all $k = 1, \dots, p$ with at least one strict inequality. This implies:

$\mu_k^L \sum_{j=1}^n c_{kj}^L\, x_j^* + \mu_k^U \sum_{j=1}^n c_{kj}^U\, x_j^* \leq \mu_k^L \sum_{j=1}^n c_{kj}^L\, x_j^1 + \mu_k^U \sum_{j=1}^n c_{kj}^U\, x_j^1$ for all $k = 1, \dots, p$.

If the sign of the previous inequality is changed and we add $\mu_k^L\, Z_k^{L*} + \mu_k^U\, Z_k^{U*}$ to both terms:

$$\mu_k^L \left(Z_k^{L*} - \sum_{j=1}^n c_{kj}^L\, x_j^1\right) + \mu_k^U \left(Z_k^{U*} - \sum_{j=1}^n c_{kj}^U\, x_j^1\right) \leq$$
$$\leq \mu_k^L \left(Z_k^{L*} - \sum_{j=1}^n c_{kj}^L\, x_j^*\right) + \mu_k^U \left(Z_k^{U*} - \sum_{j=1}^n c_{kj}^U\, x_j^*\right) \leq v^* \text{ for all } k = 1, \dots, p.$$

which implies that $\mathbf{x}^1$ is also optimal solution to (17) and it is a contradiction of $\mathbf{x}^*$ being unique.

□

In order to require the uniqueness of the solution, an augmentation term can be added to the objective function being minimized in a similar way to Wierzbicki (1980):

$$\begin{aligned} &\min\ v + \rho \sum_{k=1}^p \left(\mu_k^L \left(Z_k^{L*} - \sum_{j=1}^n c_{kj}^L\, x_j\right) + \mu_k^U \left(Z_k^{U*} - \sum_{j=1}^n c_{kj}^U\, x_j\right)\right) \\ &\text{s.t. } \mu_k^L \left(Z_k^{L*} - \sum_{j=1}^n c_{kj}^L\, x_j\right) + \mu_k^U \left(Z_k^{U*} - \sum_{j=1}^n c_{kj}^U\, x_j\right) \leq v, \quad k = 1, \dots p, \\ &\quad \sum_{j=1}^n a_{ij} x_j \leq b_i, \qquad i = 1, \dots, m, \\ &\quad x_j \geq 0, \qquad j = 1, \dots n. \qquad (15) \end{aligned}$$

where $\rho > 0$ is a so-called augmentation coefficient. In this case, every optimal solution to (15) is "possibly" efficient to (1).

Theorem 2

Let $\mathbf{x}^* = (x_1^*, \dots, x_n^*)^T$ and v^* be an optimal solution to the problem (15) where μ_k^L, $\mu_k^U > 0$ and $\mu_k^L + \mu_k^U = 1$ for all $k = 1, \dots, p$. Then, $\mathbf{x}^*$ is a "possibly" efficient solution to problem (1).

Proof

Analogous to the proof of Theorem 1.

□

Finally, for each "possibly" efficient solution obtained, the following information is presented to the DM, which allows him/her to express his/her preferences.

1) The acceptability of the solution obtained being inferior to the interval ideal solution, i.e.

$A \prec (Z_k(\mathbf{x}), Z_k^*) = \frac{(m[Z_k^*] - m\,[Z_k(\mathbf{x})])}{(w[Z_k^*] + w\,[Z_k(\mathbf{x})])}$, where $m[Z_k^*]$ and $m\,[Z_k\,(\mathbf{x})]$ are the central values of the intervals and $w[Z_k^*]$ and $w\,[Z_k(\mathbf{x})]$ are the width of the intervals as defined in Sengupta et al. (2001).

2) The distance from $Z_k(\mathbf{x})$ to Z_k^*, i.e.

$d\left(Z_k^*, Z_k(\mathbf{x})\right) = \text{Max}\left(|Z_k^{L*} - Z_k^L(\mathbf{x})|, \left|Z_k^{U*} - Z_k^U(\mathbf{x})\right|\right).$

When both the distance and the acceptability index are close to zero, the interval objective functions are closer to their corresponding interval ideal solutions.

3) The achievement rate of the solution obtained regarding the lower target and upper target, respectively i.e.

$tc_k^L = 1 - \frac{(Z_k^{L*} - Z_k^L(\mathbf{x}))}{(Z_k^{L*} - m_k^L)}$ and $tc_k^U = 1 - \frac{\left(Z_k^{U*} - Z_k^U(\mathbf{x})\right)}{\left(Z_k^{U*} - m_k^U\right)}$,

where m_k^L and m_k^U are the worst values attained in the expanded pay-off table (considering solutions $\mathbf{x}_1^0, \mathbf{x}_2^0, \mathbf{x}_3^0, \mathbf{x}_1^1, \mathbf{x}_2^1, \mathbf{x}_3^1$, which are the solutions to models (3) and (4), as previously specified).

The closer the values of tc_k^L and tc_k^U are to 1, the closer the DM is to meet his/her aspiration level Z_k^*.

5. Illustrative results

In order to obtain the "possibly" efficient solutions to our interval multiobjective problem, the first step is to obtain the interval ideal solutions using the upper and lower coefficients of the objective functions, respectively. From these problems we obtain:

$$Z_1^{U^*} = Z_1^U(\mathbf{x}_1^0) = 2.908,\ Z_2^{U^*} = Z_2^U(\mathbf{x}_2^0) = 3.793,\ Z_3^{U^*} = Z_3^U(\mathbf{x}_3^0) = 2.758$$

$$Z_1^{L^*} = Z_1^L(\mathbf{x}_1^1) = 0.414,\ Z_2^{L^*} = Z_2^L(\mathbf{x}_2^1) = 1.292,\ Z_3^{L^*} = Z_3^L(\mathbf{x}_3^1) = 0.334$$

The analysis of these solutions provides a global overview of the trade-offs between the different aspects of job satisfaction under different scenarios, i.e. both in the best and worst case coefficient scenarios (by considering the upper bound and the lower bound of the objective function coefficients, respectively), i.e. considering an optimistic and a pessimistic scenario.

Under an optimistic scenario, the profile of the most ambitious workers (i.e. workers more focused on earnings) is consistent with a married employee of about 47 years of age, has higher education completed and is willing to work more than 40 hours per week. He has reached a seniority position between 10 to 14 years, holds a supervisory status and considers the relevance of a permanent contract. This type of worker wants to be hired by companies within the activity sector of "other industries" with 5 to 19 workers, aiming for a net family income of $4.196*10^3$ Euros, which corresponds to a gross real hourly wage of 9.61 Euros. In this context, under a more pessimistic scenario these workers are satisfied with a lower net family income of about $3.099*10^3$ Euros (but with the same real hourly wage) and are willing to work in a firm with less than 5 workers.

If we assume the existence of an optimistic scenario, the workers less prone to risk, i.e. focused on job security, correspond to married employees of about 47 years of age, who have a lower education level than the one obtained previously for the ambitious type of worker (both in a best and worst case scenario), corresponding to the first level of secondary education or lower and are also willing to work more than 40 hours per week. These less prone to risk workers hold a permanent contract and reach a supervisory status with a high seniority level of 10 to 14 years. Regarding the more ambitious workers, a reduction of the gross real hourly wage of 6.46 Euros is acceptable, but he expects to obtain the same previous net family income of $4.196*10^3$ Euros. The firms that this worker looks for are big sized with 20 to 99 workers and belong to the public education sector. If a more pessimistic scenario is accounted for, these workers become satisfied with a lower net family income of about $3.761 *10^3$ Euros, which is curiously higher than the net family income that satisfies the most ambitious workers in a worst case scenario (but with a substantially lower real hourly wage). The activity sector that less prone to risk workers look for under a pessimistic scenario also changes to “other industries” and a reduced firm size with less than 5 workers becomes acceptable.

Finally, considering an optimistic scenario, workers more concerned with their quality of life i.e. with their working times are married employees with 53 years’ old who are not willing to work more than 40 hours per week. These type of workers have reached a supervisory status, aim for a gross hourly wage of 9.61 Euros (the same level attained by the most ambitious workers), hold higher education, but become less eager regarding the net family income, accepting the lowest level of income under an optimistic scenario (i.e. $3.099*10^3$ Euros). This worker’s profile also suggests that a seniority of 15 or more years is reached, with a permanent contract within the public education sector in a big sized firm of 20 to 99 workers. In a worst case scenario these workers differ from the previous ones in that they have a gross hourly wage of 6.46 Euros, but the level of education becomes reduced to first level of secondary education or lower and the level of seniority is also reduced to 10 to 14 years. On the other hand, less importance is given to the type of contract, since the workers are satisfied without a permanent contract. The targeted activity sectors are utilities and construction and small sized firms with less than 5 workers.

The results obtained suggest the existence of a trade-off between earnings and quality of life, particularly under a pessimistic scenario, with the maximization of earnings leading to the lowest value of the working times considering a worst case scenario. On the other hand, the lowest value obtainable for earnings is reached in both scenarios when the maximization of the satisfaction of the quality of life seekers is obtained. Finally, the trade-off between less prone to risk workers and quality of life seekers is also attainable, with the lowest job security levels obtained in the solution that maximizes working times.

Considering the values $Z_1^{U*}, Z_1^{L*}, Z_2^{U*}, Z_2^{L*}, Z_3^{U*}, Z_3^{L*}$ in problem (18), which is identical to consider these values as $q_1^U, q_1^L, q_2^U, q_2^L, q_3^U, q_3^L$, respectively, in problem (22), and given the same importance for reaching these values (equal weights), in order to obtain a more balanced solution, we solve problem (18) – see Table 2:

Table 2. Optimal solution to the interval multiobjective model - Spanish male workers

Decision Variables			
Names	Solution	Names	Solution
Gross real hourly wage	6.4564	Regional unemployment rate	10.3682
Higher education	0	Utilities and construction	0
Secondary education	0	Hotel sales	0
Net family income (10^3€)	4.196	Transport	0
Age	46.8674	Public education social	0
Seniority 3-4	0	Other industry	1
Seniority 5-9	0	Firm size 5-19	0
Seniority 10-14	0	Firm size 20-99	0
Seniority 15+	1	Firm size 100-499	0
Working + 40 hours/week	0	Firm size 500+	0
Permanent contract	1	Year 1996	0
Supervisory	1	Year 1997	0
Intermediate	0	Year 1998	0
Married	1	Year 1999	0
Unemployment duration	0	Year 2000	0
Good health	1	Year 2001	1
Fair health	0		

Objective functions[12]			
Satisfaction	Lower value	Central value	Upper value
Earnings	2.254	3.603	4.951
Job security	3.788	5.269	6.751
Working times	3.229	4.823	6.417

Source: Authors' own calculations.

If the DM considers the same weight for all the axes of evaluation (suggesting the workers assign the same importance to all aspects of job satisfaction under different scenarios), the solution obtained seems to indicate the profile of a man that is aiming for a gross real hourly wage of 6.46 Euros (consistent with a less prone to risk profile both in pessimistic and optimistic scenarios or with a quality of life seeker under a pessimistic scenario). This low ambition regarding the earnings obtained is reflected on the level of education achieved for this type of worker, corresponding to first level of secondary

[12] In order to calculate the objective function values in each case (lower, central and upper values), we have added as constant term the lower, central and upper values, respectively, for each interval of $\hat{\alpha}^1$, $\hat{\alpha}^2$ and $\hat{\alpha}^3$.

education or lower. Nevertheless, the satisficing net family income is around 4.196*10^3 Euros (which corresponds to the same level obtained for the most ambitious and less prone to risk workers under an optimistic scenario).

It is also relevant to highlight that middle-aged male workers around 47 years are more satisfied than younger or older ones, additionally to having higher experience in the job, i.e., having 15 or more years of seniority (as required by the quality of life seeking worker in a best case scenario), and have not been unemployed previously to the present job. However, working only 40 hours or less per week seems to increase satisfaction (as obtained for workers that want to maximize their working times both with optimistic and pessimistic scenarios), as so does possessing a permanent contract and a supervisory occupational status (both characteristics are present under an optimistic scenario for all the type of workers considered). Working in other industries different from utilities and construction, hotel sales, transport, public education (and those in the reference category, like mining and quarrying, manufacturing and finance property) and with a firm size of less than 5 people may be positive for increasing male workers' satisfaction (some of these factors are present for all the type of workers considered under a pessimistic scenario).

In terms of contextual variables, the best year to get this optimal satisfaction seems to be 2001, when the unemployment rate in Spain reached the level of 10.37%. This basically corresponds to the year with greater economic expansion, within the period, and lower unemployment rate.

In this context, it is also important to see that more balanced profiles are more demanding, regarding the features that satisfy the type of worker, with 47-year-old workers asking for characteristics requested by quality of life seekers, i.e. working less than 40 hours per week, at the expense of a lower gross real hourly wage of 6.46 Euros. Nevertheless, the level of net income reached is the highest attainable for more ambitious and less prone to risk workers under an optimistic scenario.

An overview of the main characteristics of the solution obtained is provided in Table 3.

Table 3. Information regarding the solution obtained in Table 2

$A \prec (Z_k(\mathbf{x}), Z_k^*)$	$d(Z_k^*, Z_k(\mathbf{x}))$	tc_k^L	tc_k^U

Z_1	0.1630	0.5769	0.8305	0.5843
Z_2	0.0565	0.2220	0.9648	0.9015
Z_3	0.1552	0.4436	0.7482	0.7012

Source: Authors' own calculations.

After providing this information to the DM, (s)he is asked to express his/her preferences regarding the solution under analysis. Let us consider that the DM wants to improve the achievement rate regarding the upper bound of the first objective function in order to obtain a more balanced solution regarding the worker's satisfaction on Earnings, considering $\mu_1^U = 0.8$.

The main characteristics of the solution thus obtained are provided in Table 4.

Table 4. Information regarding the new solution obtained with $\mu_1^U = 0.8$

	$A \prec (Z_k(\mathbf{x}), Z_k^*)$	$d(Z_k^*, Z_k(\mathbf{x}))$	tc_k^L	tc_k^U
Z_1	0.1565	0.4682	0.7171	0.6559
Z_2	0.1730	0.4875	0.7753	0.8207
Z_3	0.1608	0.6174	0.6496	0.7941

Source: Authors' own calculations.

In this problem, it is possible to observe that all the analysed solutions with different μ_k^L, μ_k^U have a non-dominance relation regarding the achievement rates of the upper and lower bounds of the objective functions, meaning that it is not possible to improve the achievement rate of one objective function (regarding its upper or lower bounds) without worsening at least the achievement rate of another objective function (regarding its upper or lower bounds). In particular, in Table 4 it is possible to see that the achievement rate of the upper bound of the first objective function (i.e. earnings) has improved at the cost of reducing both achievement rates of job security and lower achievement rate of working times (highlighting their trade-offs). With this new more balanced solution, this type of worker is satisfied with a gross hourly wage of 9.6078 Euros (near the value obtained for all type of workers under a best case scenario), has higher education levels completed (being consistent with more ambitious and quality life seeking workers under an optimistic scenario), does not work more than 40 hours per week (like the workers aiming for better working times in both scenarios), and has a supervisory status with a permanent contract (a characteristic sought by all type of workers under an optimistic scenario). These workers are about 52 years of age, reach a job seniority of 15 years or more (aspects consistent with quality life seekers in both scenarios), obtain a net family income of about $4.196*10^3$ Euros (an income pursued by the most ambitious and less prone to risk workers in a best case scenario) and work in "Other industries" for firms of less than 5 workers (a common characteristic for all type of workers under a pessimistic scenario).

In this framework, it is also worth noting that balanced profiles more demanding towards earnings lead to an increase of the worker's age to 53-year-old searching for features consistent with quality of life seekers, i.e. working less than 40 hours per week, but without reducing gross real hourly wage (i.e. keeping the values near to the highest values per hour). Finally, the level of net income reached is once more the highest attainable for more ambitious and less prone to risk workers under an optimistic scenario. It is also interesting to realize that the size of firms becomes less important with more balanced profiles.

The assessment of this particular solution regarding the achievement rates of the interval objective functions allows us to conclude that these attain high values (either in the upper or in the lower bounds) and are significantly close to one another (as it can be seen by the near zero acceptability indexes obtained and by the small distances between intervals provided in Table 4). If the DM wants to improve a particular achievement rate regarding one of the objective functions or wants to better explore the trade-offs under different scenarios (s)he might change the values of μ_k^L, μ_k^U and the algorithm stops when the DM considers satisfactory the achievement rates regarding the targets employed.

6. Conclusions

Conventional multiobjective programming models usually address practical problems in which all coefficients and parameters are *a priori* given. However, the inexactness and uncertainty aspects of these problems should be explicitly taken into account. Uncertainty handling can be dealt with in various ways, namely by means of stochastic, fuzzy and interval programming techniques. In the stochastic approach the coefficients are treated as random variables with known probability distributions (Gabriel et al., 2006). In the fuzzy approach, the constraints and objective functions may be regarded as fuzzy sets with known membership functions. However, it is not always easy for the DM to specify these probability distributions and membership functions. In the interval approach it is considered that the uncertain values are perturbed simultaneously and independently within known fixed bounds, being therefore intuitively preferred by the DM in practice.

In this context, a multiobjective interval model is suggested to be applied to a socio-economic problem which is aimed at exploring the trade-offs of distinct axes of evaluation of satisfaction levels of the Spanish workers under distinct coefficient settings. In a first step the estimation of the coefficients and the choice of the decision variables used to build the model are made by using econometric techniques (ordinary least squared estimations). These coefficients are then allowed to vary within a feasible range of variation.

In the second step, an extension of the idea proposed in Oliveira and Antunes (2009), combined with a reference point-based approach, is proposed through the use of scalarizing problems to solve the multiobjective interval model obtained based on the "necessary" subtraction concept. The surrogate scalarizing problems allow generating possibly efficient solutions without requiring a considerable computational effort and using a simple mathematical formulation.

Finally, this study highlights the advantages of considering uncertainty handling techniques based on the interval multiobjective linear programming approach for this kind of real-world models and can be used to carry out some policy recommendations to manage workers' satisfaction levels, possibly leading to productivity improvements. Particularly, the analysis allows to conclude that the DMs must be aware of the trade-offs involved in their decisions, particularly when they are related to future hiring of human resources. For example, increasing working hours is consistent with satisfaction in terms of job security and earnings at the expense of reducing satisfaction with working times, with important implications in terms of the debate about the rationality of working times in Spain. Additionally, the implications for DMs (managers and human resources managers) and workers of the 'flexicurity' could be evaluated, to the extent that, e.g., permanent contracts seem to positively contribute to achieve higher levels of satisfaction for almost all type of workers, except for those seeking for higher quality of life levels under a pessimistic scenario, although some relevant DMs participating in the policy debate in Spain call into question the benefits of this type of contract.

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Appendix

Table A1 Decision variables

Name	*Notation*	*Variable*	*Type*	*Values*	*Description*
ghw	x_1	**Gross real hourly wage**	Continuous	[0, ∞)	Gross real hourly wage (€)
		Education level:			Highest education level completed *(reference group: first level of secondary education or lower)*
edh	x_2	Higher education	Binary	0 or 1	Higher education
eds	x_3	Secondary education	Binary	0 or 1	Secondary (2nd level) education completed
nfi	x_4	**Net Family Income**	Continuous	[0, ∞)	Net Equivalent family income (10^3 €)
age	x_5	**Age**	Continuous	[26, 64]	Age (years)
		Job seniority:			Seniority in the company *(reference group:0-2 years)*
$j_{3\text{-}4}$	x_6	Seniority 3-4	Binary	0 or 1	Seniority in the company (3-4 years)
$j_{5\text{-}9}$	x_7	Seniority 5-9	Binary	0 or 1	Seniority in the company (5-9 years)
$j_{10\text{-}14}$	x_8	Seniority 10-14	Binary	0 or 1	Seniority in the company (10-14 years)
j_{15+}	x_9	Seniority 15+	Binary	0 or 1	Seniority in the company (15- years)
mh40	x_{10}	**Working + 40 hours/week**	Binary	0 or 1	Working more than 40 hours per week (*reference group: working 40 or less hours per week*)
per	x_{11}	**Permanent contract**	Binary	0 or 1	Type of contract signed: permanent *(reference group: non-permanent; i.e. fixed term, etc.)*
		Occupational status:			Job status *(reference group: non-supervisory/non-intermediate)*
sup	x_{12}	Supervisory	Binary	0 or 1	Supervisory status
int	x_{13}	Intermediate	Binary	0 or 1	Intermediate status
mar	x_{14}	**Married**	Binary	0 or 1	Civil State
und	x_{15}	**Unemployment duration**	Integer	[0, 288]	Worker's unemployment duration previous to current job (months)
		Worker's Health:			General health status *(reference group: poor or very poor)*
ghe	x_{16}	Good health	Binary	0 or 1	Health status (Good)
fhe	x_{17}	Fair health	Binary	0 or 1	Health status (Fair)
rur	x_{18}	**Regional unemployment rate**	Continuous	[0, 100]	Regional unemployment rate
		Industry of current job:			Main activity in current job *(reference group: Mining and quarrying, Manufacturing and Finance Property)*
in3	x_{19}	Utilities and construction	Binary	0 or 1	Industry (Utilities and construction)
in4	x_{20}	Hotel sales	Binary	0 or 1	Industry (Sales, hotels and restaurants)
in5	x_{21}	Transport	Binary	0 or 1	Industry (Transport)
in6	x_{22}	Public education social	Binary	0 or 1	Industry (Public education social)
in7	x_{23}	Other industry	Binary	0 or 1	Industry (Other industry)
		Firm size:			Number of employees in current job *(reference group: fewer than 5)*
$fs_{5\text{-}19}$	x_{24}	Firm size 5-19	Binary	0 or 1	Firm size (5-19 workers)
$fs_{20\text{-}99}$	x_{25}	Firm size 20-99	Binary	0 or 1	Firm size (20-99 workers)
$fs_{100\text{-}499}$	x_{26}	Firm size 100-499	Binary	0 or 1	Firm size (100-499 workers)
fs_{500+}	x_{27}	Firm size 500+	Binary	0 or 1	Firm size (>=500 workers)
		Year dummies:			Year of the data (*reference group: 1995*)

y_2	x_{28}	Year 1996	Binary	0 or 1	Year 1996
y_3	x_{29}	Year 1997	Binary	0 or 1	Year 1997
y_4	x_{30}	Year 1998	Binary	0 or 1	Year 1998
y_5	x_{31}	Year 1999	Binary	0 or 1	Year 1999
y_6	x_{32}	Year 2000	Binary	0 or 1	Year 2000
y_7	x_{33}	Year 2001	Binary	0 or 1	Year 2001

Note: The duration of unemployment for those who have been out of the labour force is the sum of the duration of the first unemployment spell and the duration of the spell out of the labour force (excluding time spent in formal education).

Source: Authors' own calculations.

Table A2 Descriptive statistics by gender (Spanish workers)

	Both		Male		Female	
Variable	**Mean**	**Std. Dev.**	**Mean**	**Std. Dev.**	**Mean**	**Std. Dev.**
Satisfaction in terms of:						
Earnings	3.23	1.30	3.24	1.28	3.21	1.33
Job security	3.92	1.49	3.92	1.47	3.94	1.54
Working times	4.03	1.36	4.03	1.32	4.04	1.42
Gender (female = 1)	0.36	0.48				
Gross real hourly wage	6.67	3.49	7.08	3.56	5.96	3.24
Education level:						
Higher education	0.26	0.44	0.22	0.42	0.31	0.46
Secondary education	0.23	0.42	0.21	0.41	0.25	0.44
Net Family Income (10^3 €)	7.27	7.87	6.24	7.47	9.07	8.24
Age	33.19	9.74	33.83	10.00	32.05	9.15
Job seniority:						
Seniority 3-4	0.17	0.37	0.17	0.37	0.16	0.37
Seniority 5-9	0.20	0.40	0.20	0.40	0.20	0.40
Seniority 10-14	0.11	0.32	0.12	0.33	0.10	0.31
Seniority 15+	0.04	0.20	0.05	0.21	0.03	0.17
Working + 40 hours/week	0.32	0.46	0.37	0.48	0.22	0.41
Permanent contract	0.53	0.50	0.54	0.50	0.51	0.50
Occupational status:						
Supervisory	0.07	0.25	0.08	0.27	0.05	0.21
Intermediate	0.14	0.35	0.16	0.37	0.12	0.32
Married	0.54	0.50	0.58	0.49	0.47	0.50
Unemployment duration	65.85	69.04	61.04	59.51	74.34	82.55
Worker's Health:						
Good health	0.87	0.34	0.87	0.34	0.86	0.34
Fair health	0.11	0.32	0.11	0.32	0.11	0.32
Regional unemployment rate	17.95	10.75	13.94	8.35	25.02	10.88
Industry of current job:						
Utilities and construction	0.16	0.37	0.24	0.43	0.02	0.14
Hotel Sales	0.26	0.44	0.22	0.42	0.34	0.47
Transport	0.06	0.23	0.07	0.26	0.03	0.17
Public education social	0.06	0.23	0.02	0.15	0.12	0.32
Other industry	0.07	0.26	0.04	0.19	0.13	0.34
Firm size:						
Firm size 5-19	0.31	0.46	0.34	0.47	0.27	0.45
Firm size 20-99	0.27	0.44	0.28	0.45	0.24	0.43
Firm size 100-499	0.12	0.33	0.12	0.33	0.12	0.33
Firm size 500+	0.07	0.26	0.08	0.26	0.07	0.25
Year dummies:						
Year 1996	0.13	0.34	0.13	0.34	0.13	0.34
Year 1997	0.14	0.34	0.14	0.35	0.13	0.34
Year 1998	0.14	0.35	0.14	0.35	0.15	0.35
Year 1999	0.15	0.36	0.15	0.36	0.15	0.35
Year 2000	0.15	0.36	0.15	0.36	0.16	0.36
Year 2001	0.15	0.36	0.15	0.36	0.16	0.37
Observations	16,165		10,318		5,847	

Source: Authors' own calculations.