Coupling distinct MOLP interactive approaches with a novel DEA hybrid model

Henriques, C. O.[1,2,3*], Luque, M.[4], Marcenaro-Gutierrez, O. D.[5]

[1]Polytechnic Institute of Coimbra, Coimbra Business School,

Quinta Agrícola, Bencanta, 3040-316 Coimbra, Portugal,

[2]INESC Coimbra – DEEC, University of Coimbra, Polo 2, 3030-290 Coimbra, Portugal

[3] University of Malaga PhD Programme in Economics and Business [Programa de Doctorado en Economía y Empresa de la Universidad de Malaga]

[4]Department of Applied Economics (Mathematics),

University of Malaga, C/ Ejido, 6. 29071 - Malaga, Spain

[5]Department of Applied Economics (Statistics and Econometrics)

University of Malaga, C/ Ejido, 6. 29071 - Malaga, Spain

Abstract

We propose a modelling framework which allows considering different priorities and individual expansion and contraction scales for distinct types of inputs and outputs, through the Weighted Russell Directional Distance Model (WRDDM). An equivalence model between the WRDDM and the super-ideal point model has also been established, which is then incorporated into several interactive Multiobjective Linear Programming (MOLP) approaches. The use of these diverse interactive methodologies allows obtaining the benchmark Decision Making Units (DMUs) which best suit the decision-maker's (DM's) preferences. This feature can be useful since traditional Data Envelopment Analysis (DEA) models tend to completely neglect the DM's preferences and value judgements in the computation of the DMUs used as a reference of best practices. Therefore, with this tool the DMs have the possibility of translating into the decision-making process management constraints (namely, budgetary) and aspiration levels regarding the inputs and outputs, providing much more realistic support for actual decision-making.

Keywords: Data Envelopment Analysis; Weighted Russell Directional Distance Model; Multiobjective Linear Programming; Interactive approaches.

[*] Corresponding author. Tel.: +34 95 213 1173; Fax: +34 95 213 2061.

E-mail adresses: chenriques@iscac.pt (C. O. Henriques); odmarcenaro@uma.es (O. D. Marcenaro-Gutierrez); mluque@uma.es (M. Luque).

1 INTRODUCTION

One of the main advantages of the application of DEA in efficiency assessment is the possibility of finding the benchmarks of inefficient DMUs, providing valuable information for managers about the best practices being followed to reach efficiency. The benchmarks of an inefficient DMU are usually computed through linear programming (LP) models and are obtained just by using the original inputs and outputs.

Still, one of the limitations of traditional DEA models is that they do not typically incorporate the preference structure or value judgments of DMs (Allen et al., 1997). This can be particularly relevant in management decisions since these benchmarks might not be reachable due to budgetary constraints, or they may not reflect the DMs' aspirations. Thus, the DMs' preferences should be specifically contemplated in the selection of the most preferred DMUs to be used as benchmarks for the DMU under evaluation.

Several methods have been suggested to account for the DMs' preference information in DEA models. In this regard, there are two types of models (Halme *et al.*, 1999): the efficiency score models which use the DMs' preference information to generate more valuable efficiency scores; and the target setting models which contemplate the DMs' preference information to obtain more reasonable targets. Out of these, we will focus on target setting models. In this context, two distinct frameworks for efficiency evaluation are generally employed for targeting (Fukuyama *et al.*, 2014): the greatest and the least distance frameworks. In the first case, the efficient targets are computed through the farthest projections to the DMU under evaluation via the maximization of the *p*-norm relative to either the strongly efficient frontier or the weakly efficient frontier. Non-radial measures such as the slacks based measure (SBM) proposed by Tone (2001), the enhanced Russell graph measure (ERGM) proposed in Pastor *et al.* (1999) and the range-adjusted measure developed by Cooper *et al.* (1999), belong to this sort of models. In the second approach, the closest targets are computed for a given DMU according to a previously specified criterion of similarity, i.e. considering the closeness between the values of the inputs and/or outputs of the DMU under assessment and the corresponding targets (see e.g. Aparicio *et al.*, 2007; Fukuyama *et al.*, 2014; Aparicio *et al.*, 2017; Vakili *et al.*, 2020; Ruiz and Sirvant, 2020). Although the least distance approach has the merit of potentially identifying the closest or easily acceptable efficient target for the DMs, we start by utilizing the greatest distance framework that has traditionally been employed because of its computational easiness (Fukuyama *et al.*, 2014), then progressing towards the most preferred efficient target according to the DM's preferences.

In this context, within the methodological approaches that explicitly consider the DM's preferences or value judgements in the attainment of the input and output targets and benchmarks[1] for inefficient DMUs, the MOLP approach is one of the most used. This type of methodology was first introduced by Golany (1988) and it was conducted by asking the DMs to assign a set of input levels as resources and to select the most preferred set of output levels from a set of feasible points on the efficient frontier, leading to the joint use of DEA with MOLP.

In this framework, Tavana *et al*. (2018) established an extended equivalence model between the combined-oriented DEA model and the super-ideal point model of the minimax reference point formulation such that: the increase in the total outputs and the decrease in the total inputs are simultaneously contemplated. Regarding other approaches, this DEA-MOLP method enables the DM to select the variables, as well as their relative intensities, in order to move the DMU closer to the efficient frontier. As a result, the DM can assess the strategic trade-offs between the different factors that are required to reduce the DMU's distance to the efficient frontier. This might be an important feature if there are budget constraints that prevent the DM from improving all the inefficient factors from a given DMU.

Despite the merits of the comprehensive hybrid DEA-MOLP model developed in Tavana *et al*. (2018), this model assumes that the expansion of all outputs and contraction of all inputs is considered to have the same rate. In fact, many real-world problems require the use of non-radial measures of technical efficiency (Gomes Junior *et al*., 2013; 2016; Aparicio *et al*., 2018). In this case, as noted by Chen *et al*. (2015), on the one hand, the inefficiency linked to the use of a specific input by a DMU is not necessarily related to the inefficiency regarding the use of another input by the same DMU; on the other hand, a DMU may produce distinct outputs at the same time, but with a different production capacity, and thus the production efficiency for different outputs may also be distinct. Therefore, besides being non-radial, the modelling framework herein considered is also non-oriented.

Hence, unlike other hybrid DEA-MOLP approaches proposed in the scientific literature, the modelling framework herein established allows accounting for different

[1] In this regard we would like to stress that targets are the coordinates of a projection point whereas benchmarks are the observed DMUs.

priorities and individual expansion and contraction scales regarding the outputs and inputs, making use of the non-radial and non-oriented WRDDM.

We have used this latter DEA model, because when contrasted with the SBM and the ERGM models, which account for input and output efficiency measures in a nonlinear fractional form, the WRDDM is evaluated in a linear form, thus yielding the advantage of a lower computational burden (Chen *et al*., 2015).

Therefore, the main novelties of our work are twofold: we establish an equivalence model between the WRDDM and the super-ideal point model based on the reference point approach; we combine this new model with several well-known interactive MOLP algorithms (in our particular case, the Wierzbicki, Tchebycheff, STEM and STOM methods) to allow for the explicit incorporation of the DM's preferences in finding the target DMUs to be viewed as benchmarks. These interactive methods are selected according to the preferences provided by the DM, who is either asked to specify the new target values to be achieved or to establish the target inputs and outputs that can be increased, maintained or reduced, seeking values within his/her grasp. Besides, since there is no MOLP interactive method (or procedure) that performs best in all circumstances, the use of several methods or procedures allows exploring different search strategies and computation techniques (Antunes *et al*. 2016). Furthermore, by using such diverse approaches, the DM gains further insights into the problem and can correct his/her preceding decisions if he/she wishes to do so. Finally, by employing more than one interactive method in the solution process, the DM is also able to acquire a broader spectrum of possible targets according to his/her preferences[2].

The remainder of this paper is given as follows. Section 2 provides a detailed explanation of the methodological approach followed. Section 3 presents an illustrative example of the application of the methodology. Finally, the main conclusions will be presented in Section 4.

2 The Methodological Approach

In the next sections, we first describe the generalized directional distance function approach (under the assumptions of strong availability of inputs/outputs), following a non-radial and non-oriented model. Then, we present the main underpinning assumptions

[2] Further details regarding the combination of several interactive methods in the same solution process can be found in Miettinen and Makela (1999), Kaliszewski (2004), Luque *et al*. (2011), Ruiz *et al*. (2012) and Antunes *et al*. (2016).

underlying the MOLP approach that will be used to establish the hybrid DEA-MOLP framework herein developed. Finally, we propose to use several interactive methods to obtain solutions to the hybrid DEA-MOLP model previously suggested.

2.1. The directional distance function approach

Fukuyama and Weber (2009, 2010) developed a measure of inefficiency, also known as the directional slacks-based inefficiency (SBI) measure, to obtain a generalized measure of technical inefficiency which considered all slacks in input and output constraints. This measure allows obtaining the same information provided by the SBM model suggested by Tone (2001) as long as the directional vectors for inputs and outputs are equal to the corresponding input and output vectors, being also regarded as a generalization of the Russell's[3] measure of efficiency. In this context, it is important to mention that the WRDDM is a closely related measure of ERGM, though the ERGM and SBM are special cases of the WRDDM (Chen *et al*., 2015). While the objective function of the ERGM is specified for calculating an efficiency measure, those corresponding variables in the WRDDM are inefficiency measures. Besides, the WRDDM has an additive form objective function, whereas the ERGM has a ratio form.

More recently, Färe and Grosskopf (2010) also proposed a generalization of the SBM based on the directional distance function, where the optimization problem is based on the sum of the directional distance function being able to express how much inputs have excessively been used and how much shortage of outputs have been produced regarding their efficiency level.

The directional distance function aiming to increase the outputs and decrease the inputs directionally can be defined as:

$$sup\{\beta: (\boldsymbol{x} - \beta \boldsymbol{g}_x, \boldsymbol{y} + \beta \boldsymbol{g}_y) \in T\} \quad (1)$$

where the non-zero vector $\boldsymbol{g} = (-\boldsymbol{g}_x, \boldsymbol{g}_y)$ establishes the "directions" in which inputs and outputs are scaled, and the technology reference set satisfies the assumptions $T = \{(\boldsymbol{x}, \boldsymbol{y}): \boldsymbol{x} \text{ can produce } \boldsymbol{y}\}$ of constant returns to scale, with strong disposability of inputs

[3] The Russell's measure was first suggested by Färe and Lovell (1978) following an input-orientation modelling approach. This measure only considers the input slacks, failing to account for the inefficiencies related to outputs. Färe et al. (2013) later extended this measure into a nonlinear form, also known as the "Russell graph measure", by combining the input and output Russell measures in an additive way and allowing for the incorporation of all the input and output slacks. Pastor et al. (1999) also revised this latter proposal with the suggestion of a new measure called the "Enhanced Russell graph measure" (ERGM), by combining input and output Russell measures in a ratio form.

and outputs (Chen *et al.*, 2015). A production process shows strong disposability of inputs/outputs if the inputs/outputs are freely disposable.

Given two vectors $\boldsymbol{x} = (x_1, \dots, x_n)^T$ and $\boldsymbol{y} = (y_1, \dots, y_n)^T$, the DEA piecewise reference technology can be obtained as follows:

$T = \{(\boldsymbol{x}, \boldsymbol{y}) \colon \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_r,\, r \,\epsilon\, O,$

$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_i,\, i \,\epsilon\, I,$

$\lambda_j \geq 0, (\forall j)\},$ (2)

where *O* and *I* are the index sets that designate the outputs and inputs, respectively.

In what regards the reference technology *T* given in (2), traditionally, for each DMU under assessment, DMU$_o$, the directional distance function can be obtained by solving the following LP problem[4]:

max β_o

s.t. $\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} + \beta_o g_{yr},\, r \,\epsilon\, O,$

$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io} - \beta_o g_{xi},\, i \,\epsilon\, I,$

$\lambda_j \geq 0$ $(\forall_j)$ (3)

where β_o measures simultaneously the maximum enlargement of outputs and reduction of inputs that remain technically feasible and can serve as a measure of technical inefficiency. If $\beta_o = 0$, then DMU$_o$ operates on the frontier of *T* with technical efficiency. If $\beta^o > 0$, then DMU$_o$ operates inside the frontier of *T* and it is inefficient. Finally, the parameter $\beta_o g_{xi}$ indicates the level by which DMU$_o$ has to reduce its *i*-th input to become efficient. Analogously, the parameter $\beta_o g_{yr}$ provides information on the level by which DMU$_o$ has to enlarge its *r*-th output in order to become efficient.

Besides being a generalization of the Shephard's distance functions, the directional distance function can be specified to embed different assumptions. If $\boldsymbol{g} = (-\boldsymbol{g}_x, \boldsymbol{g}_y)$= $(-\boldsymbol{x}^o, \boldsymbol{y}^o)$, i.e., the direction is set to account for the observed data, β^o corresponds to the potential proportional variation in outputs and inputs. If alternatively $\boldsymbol{g} = (-\boldsymbol{g}_x, \boldsymbol{g}_y)$= $(-1, 1)$, then the solution value can be viewed as the net improvement in performance in terms of feasible enlargement in outputs and feasible reduction in inputs (Färe and Grosskopf, 2004). Conversely, with $\boldsymbol{g} = (0, \boldsymbol{g}_y)$, the directional output

[4] According to Kuosmanen (2005) under the weakly disposable technology assumption the evaluation of efficiency in terms of Variable Returns to Scale technology, goes beyond the imposition of the additional constraint $\sum_{j=1}^{n} \lambda_j$= 1 in (3).

distance function is thus obtained. Nevertheless, this approach does not account for inefficiencies associated with non-zero slacks and it eventually has the problem of miss specifying some evaluated DMUs as efficient units (Chen *et al*., 2015).

2.1.1. A generalized directional distance function approach (under the assumption of strong availability of inputs/outputs)

The efficiency measurement obtained in (3) expands all outputs and inputs and contracts all inputs and outputs by the same rate, β. However, there is no guarantee that the proportional contraction rate for input factors and expansion rate output factors must be equal. Moreover, as noted by Chen *et al*. (2015), the inefficiency linked to the use of a specific input by a DMU is not necessarily related to the inefficiency regarding the use of another input by the same DMU. Additionally, a DMU may produce distinct outputs at the same time, but with a different production capacity, and hence the production efficiency for different outputs may also be distinct. Therefore, we consider the WRDDM developed by Chen et al. (2015), which, besides allowing for the technical inefficiency associated with inputs and outputs to be different, also allows for the technical inefficiency among each of the inputs and outputs to be distinct. The formulation of (3) can hence be generalized and adapted to individual expansion and contraction scales as follows:

$$\max \beta_o^R = max\ (w_y(\sum_{r\,\epsilon\,O} \varpi_y^r \alpha_o^r) + w_x(\sum_{i\,\epsilon\,I} \varpi_x^i \zeta_o^i))$$

$$\text{s.t. } \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} + \alpha_o^r g_{yr},\ r\,\epsilon\,O,$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io} - \zeta_o^i g_{xi},\ i\,\epsilon\,I,$$

$$\sum_{j=1}^{n} \lambda_j = 1,\ \lambda_j \geq 0\ (\forall_j), \quad (4)$$

where the vectors of inputs and outputs of DMU$_o$ are given as $\boldsymbol{x}_o$ and $\boldsymbol{y}_o$, correspondingly; the parameters α_o^r and ζ_o^i are the individual inefficiency measures for each output and input, respectively, and all variables are nonnegative except for β_o^R. The coefficients w_y and w_x may be regarded as the given priorities associated with the outputs and inputs, and their sum should be one. On the other hand, the inefficiencies of each corresponding output and input can also be assigned with different priorities and their sums are also assumed to be one, i.e.: $\sum_{r\,\epsilon\,O} \varpi_y^r = 1$, $\sum_{i\,\epsilon\,I} \varpi_x^i = 1$. In this case, it is necessary that the directional vectors are measured according to the same measurement units as the original vectors of inputs and outputs in order to add α_o^r and ζ_o^i. Finally, we assume the variable returns to scale technology, which implies the imposition of the additional constraint $\sum_{j=1}^{n} \lambda_j = 1$, $\lambda_j \geq 0$ $(\forall_j)$.

If the WRDDM inefficiency measure is zero ($\beta_o^R = 0$), then the DMU is fully efficient. Further developments regarding the WRDDM approach might be found in (Chen *et al*., 2015).

The reference set of the inefficient DMU$_o$ based on (4) is obtained by solving the following LP problem, assuming that α_o^{r*} and ζ_o^{i*} are computed in the optimal solution to (4):

max $\sum_{r \in O} s_r^+ + \sum_{i \in I} s_i^-$,

s.t. $\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} + \alpha_o^{r*} g_{yr}, r \in O,$

$\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io} - \zeta_o^{i*} g_{xi}, i \in I,$

$\sum_{j=1}^{n} \lambda_j = 1$, $\lambda_j \geq 0$ $(\forall_j)$,

$s_r^+ \geq 0$ $(\forall_{r \in O})$, $s_i^- \geq 0$ $(\forall_{i \in I})$, (5)

Let (s_r^{+*}, s_r^{-*}, λ_j^*) be the optimal solution to (5) and α_o^{r*}, ζ_o^{i*} are given by problem (4). Consider the reference set of the WRDDM-inefficient DMU$_o$ as follows:

E$_o$ = {j: $\lambda_j^* > 0$, j=1, …, n}.

The point of the efficient frontier which can be viewed as a target DMU for the WRDDM -inefficient DMU$_o$ is given by:

$(\hat{\boldsymbol{x}}_o, \hat{\boldsymbol{y}}_o) = (\sum_{j \in E_o} \lambda_j^* \boldsymbol{x}_j \ , \sum_{j \in E_o} \lambda_j^* \boldsymbol{y}_j \ )$.

If the direction vectors do not have the same units of measurement as the vectors of the observed data, we can alternatively see the weights ϖ_y^r and ϖ_x^i in problem (4) as values which can normalize the direction vectors, by using the sample standard deviations of inputs and outputs (see for example Chen *et al*. (2015)).

2.2 DEA models and MOLP models

In the next section, we describe the main underpinning assumptions regarding MOLP models in the framework of our proposal for a new hybrid DEA-MOLP model. In this context, we provide the overall steps used in the transformation of the general-combined non-oriented DEA model (4) into an equivalent super-ideal point model. Subsequently, this model is incorporated into an interactive framework that can encompass a sequence of computation and dialogue phases, after which, a solution is shown to the DM who reacts by providing his/her preferences to carry out a new computation phase and to attain a new solution. The DM learns about the possible solutions that can be obtained from the model, and he/she can progressively adjust his/her preferences until reaching the most preferred solution (MPS). Throughout this interactive framework, the DMs can explore different benchmarks and best practices that might be target DMUs for the non-efficient DMUs under evaluation, explicitly accounting for the management constraints and aspiration levels that DMs might be facing.

2.2.1. Basic MOLP concepts

Consider the following MOLP problem:

$$\max \left(f_1(\boldsymbol{\lambda}), \dots, f_{s+m}(\boldsymbol{\lambda})\right) = (\textstyle\sum_{j=1}^{n} \lambda_j^* y_{1j}, \quad \dots, \quad \sum_{j=1}^{n} \lambda_j^* y_{sj}, \quad -\sum_{j=1}^{n} \lambda_j^* x_{1j}, \quad \dots, \quad -\sum_{j=1}^{n} \lambda_j^* x_{mj})$$

$$\text{s.t. } \boldsymbol{\lambda} \in \Lambda \tag{6}$$

where the decision variables $\boldsymbol{\lambda} = (\lambda_1, \dots, \lambda_n)^T$ belong to the non-empty feasible space Λ.

Definition 1. A solution $\boldsymbol{\lambda}' \in \Lambda$ is weakly efficient to problem (6) if and only if there is no other $\boldsymbol{\lambda} \in \Lambda$ such that $f_k(\boldsymbol{\lambda}') < f_k(\boldsymbol{\lambda})$ for all $k = 1, \dots, s+m$.

Definition 2. A solution $\boldsymbol{\lambda}' \in \Lambda$ is efficient to problem (6) if and only if there is no other $\boldsymbol{\lambda} \in \Lambda$ such that $f_k(\boldsymbol{\lambda}') \leq f_k(\boldsymbol{\lambda})$ for all $k = 1, \dots, s+m$ with at least one strict inequality.

A way of expressing preferences about efficient solutions in multiobjective programming is through the use of a *reference point* $\mathbf{f}^{ref} = (f_1^{ref}, \dots, f_{s+m}^{ref})^T$, which consists of a desirable or reference value for each objective function. One of the most used achievement scalarizing function (ASF) to solve problem (6) was proposed by Wierzbicki (1980):

$$s(\mathbf{f}^{ref}, f(\boldsymbol{\lambda}), \boldsymbol{\tau}) \max_{k=1,\dots,s+m} \left\{\tau_k(f_k^{ref} - f_k(\boldsymbol{\lambda}))\right\}, \tag{7}$$

where $\boldsymbol{\tau} = (\tau_1, \dots, \tau_{s+m})^T$ is a vector of weights for reaching these values which must be positive. This function must be minimized in the feasible region:

$$\min \max_{k=1,\dots,s+m} \left\{\tau_k(f_k^{ref} - f_k(\boldsymbol{\lambda}))\right\},$$

$$s.t.: \boldsymbol{\lambda} \in \Lambda \tag{8}$$

where $\Lambda = \{\boldsymbol{\lambda} = (\lambda_1, \dots, \lambda_n)^T \mid \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0 \ \forall j = 1, \dots n\}$ is the weight vectors space (decision space in this case).

Every solution to (8) is weakly efficient to problem (6) and it is efficient if it is unique. One possible drawback of problem (8) is that it is generally non-differentiable even if the functions in the original problem (6) are all differentiable or even linear[5]. However, this drawback can be overcome if we use an equivalent differentiable formulation:

$$\begin{aligned} &\min \quad \beta_o \\ &s.t.: \ \tau_k\left(f_k^{ref} - f_k(\boldsymbol{\lambda})\right) \leq \beta_o, \ \ k = 1, \dots, s+m \\ &\qquad \boldsymbol{\lambda} \in \Lambda \end{aligned} \tag{9}$$

which implies in our case that problem (9) is linear.

In order to require the uniqueness of the solution, an augmentation term can be added to the objective function of problem (9) (Wierzbicki, 1980).

Problem (9) can be used to account for simultaneous modifications of the input and output values of DMUs. In fact, Ebrahimnejad and Lotfi (2012) established an equivalence model between the general combined oriented radial DEA model and model (9) with the use of the ideal point, i.e. the individual optimal solutions of each objective function, as a reference point, also known as the super-ideal point model.

2.2.2. The super-ideal equivalent model to the WRDDM model

Let the following MOLP problem be given as:

$$\begin{aligned} &\max \ (f_r(\boldsymbol{\lambda}), r \in O, f_i(\boldsymbol{\lambda}), i \in I) \\ &\text{s.t. } \boldsymbol{\lambda} \in \Lambda \end{aligned} \tag{10}$$

Considering the same reasoning of the previous Section, but bearing in mind that we are in the presence of a non-radial model and thus new conditions have to be employed, the next super-ideal point model can be used to generate any efficient solution to MOLP problem (10):

$$\begin{aligned} &\min \beta, \\ &\text{s.t. } \tau_{yr}(f_r^{ref} - f_r(\boldsymbol{\lambda})) \leq \alpha^r, \ r \in O, \\ &\tau_{xi}(f_i^{ref} - f_i(\boldsymbol{\lambda})) \leq \zeta^i, \ i \in I, \\ &(w_y(\textstyle\sum_{r \in O} \varpi_y^r(\alpha^r)) + w_x\left(\textstyle\sum_{i \in I} \varpi_x^i(\zeta^i)\right) \leq \beta, \end{aligned}$$

[5] For example, if the problem is linear (all the objective functions and the constraints are linear), an appropriate single objective optimization solver for LP can be used, which is usually more efficient and accurate than a solver for non-differentiable problems.

$\sum_{r \,\epsilon\, O} \varpi_y^r = 1,$

$\sum_{i \,\epsilon\, I} \varpi_x^i = 1,$

$w_y + w_x = 1,$

$\boldsymbol{\lambda} \in \Lambda$ (11)

where f_r^{ref}, $r \,\epsilon\, O$ and f_i^{ref}, $i \,\epsilon\, I$ are the reference values established for each objective function.

Finally, it can be demonstrated that problem (11) is equivalent to problem (4) if certain conditions are met.

In model (11), the r^th composite output (with $r \,\epsilon\, O$) can be given as follows:

$f_r(\boldsymbol{\lambda}) = \sum_{j=1}^{n} \lambda_j y_{rj} - y_{ro}, r \,\epsilon\, O,$ (12)

In a similar way the i^th composite input (with $i \,\epsilon\, I$) can be as follows:

$f_i(\boldsymbol{\lambda}) = x_{io} - \sum_{j=1}^{n} \lambda_j x_{ij}, i \,\epsilon\, I,$ (13)

The maximum feasible value of the r^th composite output for the observed DMU*o* can be given by $\widehat{f_{ro}} = f_r(\boldsymbol{\lambda}^*)$, for $r \,\epsilon\, O$, where $\boldsymbol{\lambda}^*$ can be computed by solving the following problem:

$\widehat{f_{ro}} = \max_{\boldsymbol{\lambda} \in \Lambda} f_{ro}(\boldsymbol{\lambda}), r \,\epsilon\, O,$ (14)

Analogously, the maximum feasible value of the i^th composite input for the observed DMU*o* can be given by $\widehat{f_{io}} = f_i(\boldsymbol{\lambda}^*)$, $i \,\epsilon\, I$, where $\boldsymbol{\lambda}^*$ can be computed by solving the following problem:

$\widehat{f_{io}} = \max_{\boldsymbol{\lambda} \in \Lambda} f_{io}(\boldsymbol{\lambda}), i \,\epsilon\, I,$ (15)

The equivalence relationship between the general DEA Model (4) and the minimax formulation (11) can be established by the following theorem.

Theorem 1. Assume g_{yr} >0 ($\forall r \epsilon O$) and g_{xi} >0 ($\forall i \epsilon I$). The general combined-oriented DEA Model (4) can be equivalently transformed into the super-ideal point Model (11) using Equations (12)-(15) and the following definitions:

$\tau_{yr} = \frac{1}{g_{yr}}, r \epsilon O,$ (16)

$\tau_{xi} = \frac{1}{g_{xi}}, i \epsilon I,$ (17)

$f_r^{ref} = \frac{F_r^{max}}{\tau_{yr}}, r \,\epsilon\, O,$ (18)

$f_i^{ref} = \frac{F_i^{max}}{\tau_{xi}}$, $i \epsilon I$, (19)

$F_r^{max} = \{ \tau_{yr} \widehat{f_{ro}} \} = \{\frac{\widehat{f_{ro}}}{g_{yr}}\}$, $r \epsilon O$ (20)

$F_i^{max} = \{ \tau_{xi} \widehat{f_{io}} \} = \{\frac{\widehat{f_{io}}}{g_{xi}}\}$, $i \epsilon I$ (21)

$\alpha^r = F_r^{max} - \alpha_o^r, r \epsilon O,$ (22)

$\zeta^i = F_i^{max} - \zeta_o^i, i \epsilon I,$ (23)

$\beta = ((w_y(\sum_{r \epsilon O} \varpi_y^r F_r^{max}) + w_x(\sum_{i \epsilon I} \varpi_x^i F_i^{max})) - \beta_o^R.$ (24)

Proof

Using Equations (12)-(17), the general non-radial non-oriented DEA Model (4) can be rewritten as:

max β_o^R = max $(w_y(\sum_{r \epsilon O} \varpi_y^r \alpha_o^r) + w_x(\sum_{i \epsilon I} \varpi_{xg}^i \zeta_o^i))$,

s.t. $\alpha_o^r \frac{1}{\tau_{yr}} - f_r(\boldsymbol{\lambda}) \leq 0$, $r \epsilon O$,

$\zeta_o^i \frac{1}{\tau_{xi}} - f_i(\boldsymbol{\lambda}) \leq 0$, $i \epsilon I$,

$\boldsymbol{\lambda} \in \Lambda$ (25)

From (20) the first set of constraints of problem (25) for $r \epsilon O$ can be equivalently transformed into

$\alpha_o^r \frac{1}{\tau_{yr}} - f_r(\boldsymbol{\lambda}) \leq 0 \Leftrightarrow \alpha_o^r \frac{1}{\tau_{yr}} \leq f_r(\boldsymbol{\lambda}) \Leftrightarrow - \tau_{yr} f_r(\boldsymbol{\lambda}) \leq -\alpha_o^r \Leftrightarrow$

$F_r^{max} - \tau_{yr} f_r(\boldsymbol{\lambda}) \leq F_r^{max} - \alpha_o^r \Leftrightarrow \tau_{yr} (\frac{F_r^{max}}{\tau_{yr}} - f_r(\boldsymbol{\lambda})) \leq F_r^{max} - \alpha_o^r \Leftrightarrow$

$\tau_{yr} (f_r^{ref} - f_r(\boldsymbol{\lambda})) \leq F_r^{max} - \alpha_o^r \Leftrightarrow \tau_{yr} (f_r^{ref} - f_r(\boldsymbol{\lambda})) \leq F_r^{max} - \alpha_o^r \Leftrightarrow$

$\tau_{yr} (f_r^{ref} - f_r(\boldsymbol{\lambda})) \leq \alpha^r$. (26)

Analogously from (21), the second set of constraints of problem (25) can be equivalently transformed into:

$\tau_{xi} (f_r^{ref} - f_i(\boldsymbol{\lambda})) \leq F_i^{max} - \zeta_o^i \Leftrightarrow \tau_{xi} (f_r^{ref} - f_i(\boldsymbol{\lambda})) \leq \zeta^i$, $i \epsilon I$, (27)

Additionally, the objective function of model (25) becomes:

max β_o^R = min $(- \beta_o^R)$ = min β. (28)

Since $\widehat{f_{ro}} = f_{ro}(\boldsymbol{\lambda}^*)$, $r \epsilon O$, expression (18) implies that for any $\boldsymbol{\lambda} \in \Lambda_o$

$f_r^{ref} = \frac{F_r^{max}}{\tau_{yr}} \geq \frac{\tau_{yr} \widehat{f_{ro}}}{\tau_{yr}} = \widehat{f_{ro}} = \max_{\boldsymbol{\lambda} \in \Lambda} f_{ro}(\boldsymbol{\lambda})$, $r \epsilon O$, (29)

Analogously from (19) it is obtained, respectively,

$f_i^{ref} = \frac{F_i^{max}}{\tau_{xi}} \geq \frac{\tau_{xi}\ \widehat{f_{io}}}{\tau_{xi}} = \widehat{f_{io}} = \max_{\boldsymbol{\lambda} \in \Lambda} f_{io}(\boldsymbol{\lambda})$, $i \in I$, (30)

Equations (29) and (30) imply for any $\boldsymbol{\lambda} \in \Lambda$

f_r^{ref} - $f_r(\boldsymbol{\lambda}) \geq 0$, $r \in O$, (31)

f_i^{ref} - $f_i(\boldsymbol{\lambda}) \geq 0$, $i \in I$, (32)

Hence, from (24) it is verified for any $\boldsymbol{\lambda} \in \Lambda_o$ that

$\beta = (w_y(\sum_{r \in O} \varpi_y^r F_r^{max}) + w_x(\sum_{i \in I} \varpi_x^i F_i^{max}) - \beta_o^R$.

Moreover, since

$\beta_o^R = w_y(\sum_{r \in O} \varpi_y^r(\alpha_o^r)) + w_x\left(\sum_{i \in I} \varpi_x^i(\zeta_o^i)\right)$

Thus, from (20) - (21), it is known that

$\beta = ((w_y(\sum_{r \in O} \varpi_y^r F_r^{max}) + w_x\left(\sum_{i \in I} \varpi_x^i F_i^{max}\right)) - \beta_o^R \Leftrightarrow \beta =$

$(w_y(\sum_{r \in O} \varpi_y^r(F_r^{max} - \alpha_o^r)) + w_x\left(\sum_{i \in I} \varpi_x^i(F_i^{max} - \zeta_o^i)\right) =$

$(w_y(\sum_{r \in O} \varpi_y^r(\alpha^r)) + w_x\left(\sum_{i \in I} \varpi_x^i(\zeta^i)\right)$

With the foregoing in mind, it is proven that the general combined-oriented DEA Model (4) can be equivalently transformed into the super-ideal point Model (11).

2.2.3. Interactive approaches to obtain solutions to the hybrid DEA-MOLP model

Interactive methods comprise a series of computation stages intertwined with dialogue phases conducted with the DM (Miettinen and Makela, 1999). After each computation phase, the solution attained is presented to the DM, who is asked to provide the required information to proceed with a new computation phase to obtain a new solution more consistent with his/her preferences, or by stopping the procedure when a satisfactory solution is found which is supposedly the MPS. Most interactive methods in MOLP generate one or more solutions that are efficient, except for a family of methods called NAUTILUS that starts from the nadir point (Miettinen et al., 2010, Ruiz et al., 2015). Each interactive procedure has specific dialogue and computation phases, as well as stopping conditions. In this section, we describe several interactive approaches that will be used to obtain solutions to the hybrid DEA-MOLP model previously developed.

The choice of the interactive method to be used is dependent on the preferences provided by the DM as follows: i) if the DM wishes to directly provide new target values to be achieved for the DMUs, then the Wierzbicki or the Tchebycheff methods should be used to generate new targets; ii) if the DM rather prefers to set the required adjustments to be made to the target input or output values, specifying those factors that should be improved, maintained or relaxed, then the STEM and/or STOM methods should be employed to generate new targets.

The switch of method, considering these two variants of preferences, does not imply restarting the interactive procedure, allowing to preserve the information gathered previously (Luque et al., 2011). Furthermore, the DM can correct his/her prior decisions if he/she wishes to do so.

Generically, the following steps are considered:

Step 1. Solve problem (4) to identify the efficient and inefficient DMUs using a selected combination of weights for each input and output and remain with this same combination for all the steps of the algorithm. Additionally, use the same weight profiles for all DMUs to guarantee the ordering of the DMUs based on the resulting efficiency scores (Ruiz and Sirvent, 2016). Solve the following problem to rank the efficient DMUs:

max $\beta_o^R = max\ (w_y(\sum_{r\,\epsilon\,O} \varpi_y^r \alpha_o^r) + w_x(\sum_{i\,\epsilon\,I} \varpi_x^i \zeta_o^i)))$

s.t. $\sum_{j \neq o} \lambda_j y_{rj} \geq y_{ro} + \alpha_o^r g_{yr}, r\,\epsilon\,O,$

$\sum_{j \neq o} \lambda_j x_{ij} \leq x_{io} - \zeta_o^i g_{xi}, i\,\epsilon\,I,$

$\sum_{j \neq o} \lambda_j = 1,\ \lambda_j \geq 0\ (\forall_j),$ (33)

Step 2. For each problem given in Step 1, solve problem (5) for each inefficient DMU and obtain the corresponding DMU target set. If the DM is satisfied with the previous target set, then the solution procedure ends. Otherwise, continue.

Step 3. Set $h = 1$. Obtain through expressions (18) and (19) the values of f_r^{ref}, $r\,\epsilon\,O$, f_i^{ref}, $i\,\epsilon\,I$.

Step 4. Compute $f_r(\boldsymbol{\lambda}^*)$, $r\,\epsilon\,O$, $f_i(\boldsymbol{\lambda}^*)$, $i\,\epsilon\,I$ through expressions (12) and (13), respectively, where $\boldsymbol{\lambda}^*$ is the optimal solution to each model obtained with formulation (5).

Step 5. The DM is either asked directly to provide new target values (case 1) or to set the required adjustments to be made to the target input or output values according to three categories (case 2): the ones that require further improvement (I^h), the ones that are to be maintained (M^h) and the ones that must be relaxed (R^h). In case 1, the DM should choose the Wierzbicki and/or Tchebycheff methods, whereas, in case 2, the DM should select the STEM and/or STOM methods.

Step 6. Consider the new reference points according to the method used.

Step 7. Consider the weights according to the method used.

Step 8. Solve the required model according to the method used.

Step 9. Obtain the optimal values α^{r*} and ζ^{i*}.

Step 10. Let:

$\alpha_o^{r*} = F_r^{max} - \alpha^{r*}, r \in O,$

$\zeta_o^{i*} = F_i^{max} - \zeta^{i*}, i \in I,$

Step 11. Solve problem (5) and obtain the new reference target units of the DMU under assessment. If the DM accepts the new reference target unit as the MPS, then stop. Otherwise, let $h := h+1$. Go to Step 4 or Step 1 if new weight profiles for the inputs and outputs should be explored (accounting for the same weight profiles for all DMUs).

Figure 1 provides a diagrammatical illustration of the overall algorithm, considering different interactive methods.

Choice of the interactive method to be used in each iteration

Wierzbicki method

The Wierzbicki (1980) method is based on the use of ASF (6). At each iteration, several efficient solutions are computed through the minimization of an ASF by using a reference point selected by the DM, which is representative of his/her aspiration levels. A specific feature of this method refers to the fact that the weights used in the ASF are fixed, and they are kept unchanged during the entire solution process.

Step 6. Ask the DM to stipulate the new reference points given as:

q_r^h for each $r \in O$,

q_i^h for each $i \in I$.

Step 7. Consider the weights τ_{yr}, $r \in O$, and τ_{xi}, $i \in I$, computed according to expressions (16) and (17).

Step 8. Solve the following problem:

min β,

s.t. $\tau_{yr}(q_r^h - f_r(\boldsymbol{\lambda})) \leq \alpha^r, r \in O,$

$\tau_{xi}(q_i^h - f_i(\boldsymbol{\lambda})) \leq \zeta^i, i \in I,$

$(w_y(\sum_{r \in O} \varpi_y^r(\alpha^r)) + w_x(\sum_{i \in I} \varpi_x^i(\zeta^i)) \leq \beta,$

$\sum_{r \in O} \varpi_y^r = 1,$

$\sum_{i \in I} \varpi_x^i = 1,$

$w_y + w_x = 1,$

$\boldsymbol{\lambda} \in \Lambda$ (34)

Let:

$F_r^{max} := \tau_{yr}(q_r^h - y_{ro}),\ r \epsilon O$

$F_i^{max} := \tau_{xi}(x_{io} - q_i^h),\ i \epsilon I$

Go to Step 9.

Tchebycheff method

The *Tchebycheff* method was proposed by Steuer and Choo (1983). This method begins with a set of dispersed weight vectors which are used to create a set of radially dispersed probing directions emanating from the ideal point and solves problem (6) once for each of them (Steuer and Choo, 1983). The efficient solutions thus generated are then shown to the DM, who must choose his/her MPS. Subsequently, a group of dispersed vectors is obtained from a neighborhood of the weight vector that generated the MPS. Problem (6) is, once more, solved for each of them, and a new, but more concentrated set of efficient solution candidates is computed. After the DM selects the MPS from this new set of efficient solutions, a new group of dispersed weight vectors is obtained departing from a neighborhood around the weight vector that produced it, and so forth. Provided that the neighborhoods keep narrowing, the algorithm will converge to a final solution.

Step 6. Consider the ideal reference point given as (18) and (19).

Step 7. Obtain the new weights for the current solution under scrutiny as follows. Let $\tau_k = \tau_{yk},\ k \in O$ and $\tau_k = \tau_{xk},\ k \in I$, where $\tau_k \in [0,1]$, $k \in \{O\} \cup \{I\}$ and $\sum_{k=1}^{s+m} \tau_k = 1$ can be selected according to the DM's preferences.

Step 8. Solve super-ideal model (11) with the weights considered in Step 7.

Go to Step 9.

STOM method

This method was published by Nakayama and Sawaragi (1984) and proposes the use of an achievement function similar to the ASF proposed by Wierzbicki (1980). A distinctive feature of this method refers to the fact that in its original version, the reference point considered is kept constant throughout the entire procedure and it is the ideal point and the weights are updated considering the DM's preferences. Based on this method the following additional steps should be considered:

Step 6. For each $r, i \in I^h$, ask the DM how much he/she wants to improve the corresponding output (Δf_r^h) or input (Δf_i^h) and, for each $r, i \in R^h$, specify how much the

DM allows to worsen the corresponding output Δf_r^h or input Δf_i^h. Update the new reference values given as:

$q_r^h = f_r(\boldsymbol{\lambda}^*) + \Delta f_r^h$ for each $r \epsilon I^h \cap O$

$q_i^h = f_i(\boldsymbol{\lambda}^*) - \Delta f_i^h$ for each $i \epsilon I^h \cap \ I$

$q_r^h = f_r(\boldsymbol{\lambda}^*) - \Delta f_r^h$ for each $r \epsilon R^h \cap O$

$q_i^h = f_i(\boldsymbol{\lambda}^*) + \Delta f_i^h$ for each $i \epsilon R^h \cap I$

$q_r^h = f_r(\boldsymbol{\lambda}^*)$ for each $r \epsilon M^h \cap O$

$q_i^h = f_i(\boldsymbol{\lambda}^*)$ for each $i \epsilon M^h \cap O$

Step 7. According to the new reference values given in Step 6, compute:

$$\tau_{yr} = \frac{1}{f_r^{ref} - q_r^h},\ r \ \epsilon \ O, \qquad \tau_{xi} = \frac{1}{f_i^{ref} - q_i^h},\ i \ \epsilon \ I.$$

With these weights, it is possible to obtain a normalized degree of non-attainability of the objective function regarding the reference point (which corresponds to the ideal point).

Step 8. Solve super-ideal model (11) with the weights computed in Step 7.

Go to Step 9.

STEM method

STEM was the first interactive method, proposed by Benayoun et al. (1971). In each interaction, the DM is asked to stipulate how much he/she wishes to sacrifice the objective functions whose value he/she views as acceptable, to enhance those whose values are still not satisfactory. With this information, a weighted Tchebycheff distance from the ideal point to the criterion space, which is reduced, is minimized. Thus, the following additional steps should be incorporated into the algorithm:

Step 6. Taking into account the three categories of outputs and inputs, the ones that require further improvement (I^h), the ones that are to be maintained (M^h) and the ones that must be relaxed (R^h), for each $r, i \in R^h$, specify how much the DM allows to worsen the corresponding output Δf_r^h, $r \ \epsilon \ R^h \cap O$ or input Δf_i^h, $i \ \epsilon \ R^h \ \cap \ I$.

Step 7. Compute the new weights as follows:

$$\tau_k = \frac{f_k^{ref} - \min\limits_{\boldsymbol{\lambda} \in \Lambda} f_{ko}(\boldsymbol{\lambda})}{f_k^{ref}} \left(\frac{1}{\sqrt{\sum_{k=1}^{s+m} (f_k)^2}} \right),\ k \in \{O\} \cup \{I\},$$

where f_k corresponds to the gradients of each objective function (*i.e.* to the original inputs and outputs). The first term of this expression is aimed at giving higher importance to those factors subject to higher relative variations, whereas the second term is a normalization factor. Let $\tau_k = \tau_{yk}$, $k \in \{O\}$ and $\tau_k = \tau_{xk}$, $k \in \{I\}$.

Step 8. Solve the following problem with the weights computed in Step 7:

min β,

s.t. $\tau_{yr}(f_r^{ref} - f_r(\boldsymbol{\lambda})) \leq \alpha^r$, $r \in I^h \cap O$,

$\tau_{xi}(f_i^{ref} - f_i(\boldsymbol{\lambda})) \leq \zeta^i$, $i \in I^h \cap I$,

$f_r(\boldsymbol{\lambda})) \geq f_r(\boldsymbol{\lambda}^*) - \Delta f_r^h$, $r \in R^h \cap O$,

$f_i(\boldsymbol{\lambda})) \leq f_i(\boldsymbol{\lambda}^*) + \Delta f_i^h$, $i \in R^h \cap I$,

$f_r(\boldsymbol{\lambda})) \geq f_r(\boldsymbol{\lambda}^*)$, $r \in M^h \cap O$,

$f_i(\boldsymbol{\lambda})) \leq f_i(\boldsymbol{\lambda}^*)$, $i \in M^h \cap I$,

$(w_y(\sum_{r \in O} \varpi_y^r(\alpha^r)) + w_x\left(\sum_{i \in I} \varpi_x^i(\zeta^i)\right) \leq \beta$,

$\sum_{r \in O} \varpi_y^r$=1,

$\sum_{i \in I} \varpi_x^i = 1$,

$w_y + w_x = 1$,

$\boldsymbol{\lambda} \in \Lambda$ (35)

Go to Step 9.

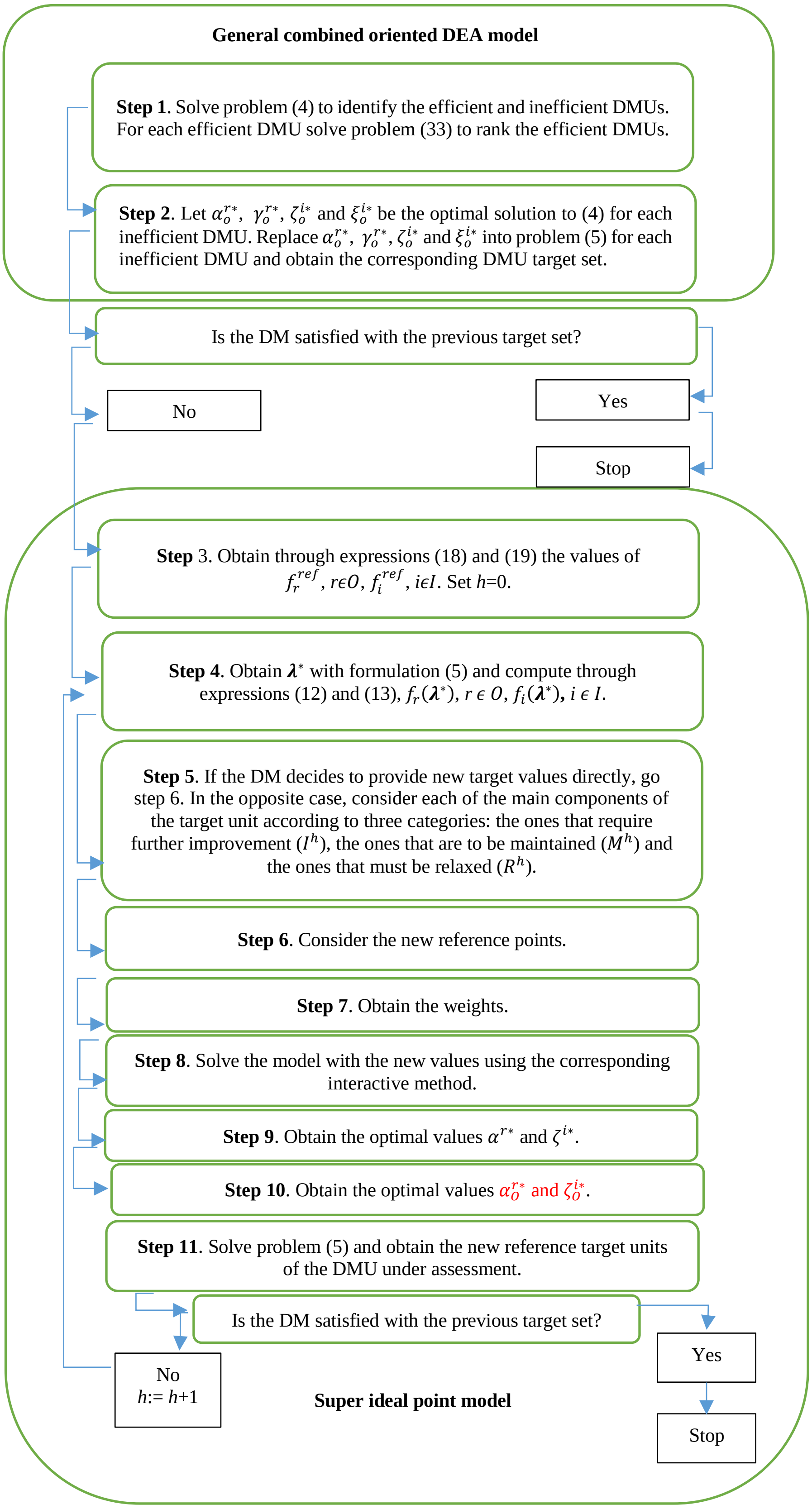


Figure 1. Diagrammatical illustration of the interactive algorithm.

3 AN ILLUSTRATIVE EXAMPLE

We have considered the example provided in Wong et al. (2009) that addresses the performance measurement of seven UK retail banks, i.e. seven DMUs. The outputs used are total revenue, corporate image and customer satisfaction, whereas the inputs considered are the number of branches, ATMs and staff (see Table 1). After solving problems (4) and (33) it is possible to conclude that the bank with the lowest efficiency score corresponds to DMU 6. The solution of problem (5) provides information regarding the benchmarks in terms of best practices for each inefficient bank (DMUs 5 and 6) and the corresponding projections – see Table 2. In this context, we will focus on the required improvements that need to be operated regarding the inputs and outputs of the DMU with the lowest performance in terms of aggregate efficiency to become efficient. According to Figure 1, to become efficient, DMU 6 should increase its total revenue from £12.04 m to £15.05 m, its corporate image evaluation from 2.53 to 4.83 and the customer satisfaction evaluation from 4.86 to 7.68, while reducing the number of branches from 1.73 to 1.22 thousand and the number of ATMs from 3.30 to 2.95 thousand, keeping the original staff number.

Table 1. Data regarding the inputs and outputs and efficiency scores

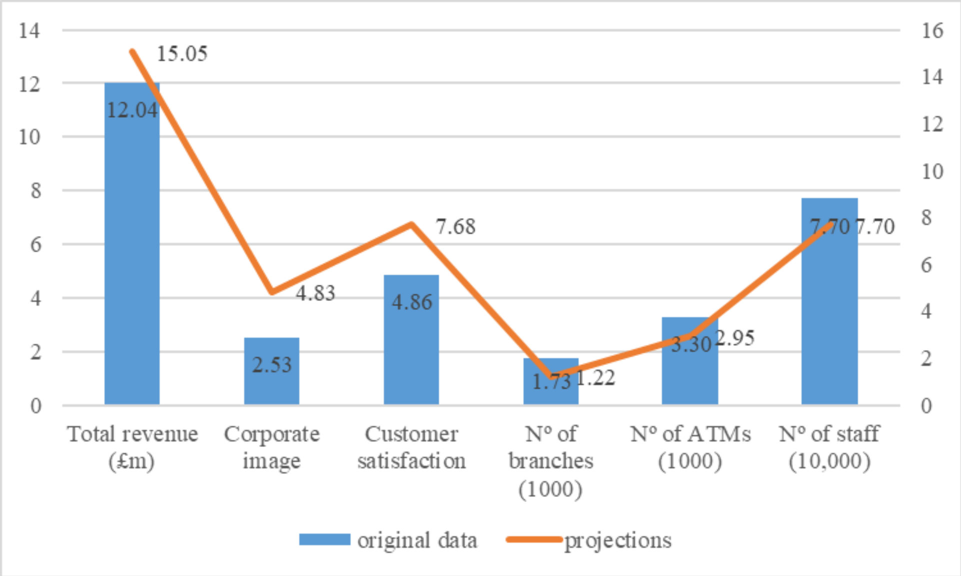
DMU	Total revenue (£m)	Corporate image	Customer satisfaction	Nº of branches (1000)	Nº of ATMs (1000)	Nº of staff (10,000)	Efficiency Score
1	10.57	3.40	6.79	2.00	2.18	2.35	1.17
2	13.35	6.66	2.55	1.95	3.19	8.43	1.01
3	8.14	1.92	9.17	0.80	2.10	3.21	1.05
4	23.67	8.47	5.82	1.75	4.00	13.30	1.17
5	14.01	3.44	6.57	2.50	4.30	9.27	0.71
6	12.04	2.53	4.86	1.73	3.30	7.70	0.64
7	7.36	1.26	7.28	0.65	1.53	2.67	1.13
Minimum	**7.36**	**1.26**	**2.55**	**0.65**	**1.53**	**2.35**	**0.64**
Maximum	**23.67**	**8.47**	**9.17**	**2.50**	**4.30**	**13.30**	**1.17**
Average	**12.73**	**3.95**	**6.15**	**1.63**	**2.94**	**6.70**	**0.98**

Source: Authors' own calculations

Table 2. Initial DEA efficiency results

DMU	Nº Ref.	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
1	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00
2	1	0.00	1.00	0.00	0.00	0.00	0.00	0.00
3	3	0.00	0.00	1.00	0.00	0.00	0.00	0.00
4	3	0.00	0.00	0.00	1.00	0.00	0.00	0.00
5	0	0.00	0.00	0.40	0.60	0.00	0.00	0.00
6	0	0.00	0.00	0.56	0.44	0.00	0.00	0.00
7	1	0.00	0.00	0.00	0.00	0.00	0.00	1.00

Note: Nº ref. is the number of times that the DMU is selected as a benchmark in the WRDDM non-oriented model.
Source: Authors' own calculations



Source: Authors' own calculations

Figure 1. Required adjustments in DMU 6 according to the original projections

Nonetheless, the projections obtained in Figure 1 do not encompass the DM's preferences. Therefore, we will next describe other possible solutions obtainable with different interactive MOLP algorithms to compute other solutions more consistent with the DM's preferences and/or aspiration levels. To proceed with our analysis, we will consider a hypothetical DM which establishes the following targets, i.e. the revenue will be targeted at £16m, the level of customer satisfaction should be improved to 7.5 and the level of corporate image should be maintained, but to increase its outputs he/she is willing to increase the Nº of staff to 90 thousand people, while increasing the number of branches to 2.5 thousand and the number of ATMs to 5 thousand.

Table 3 provides a comparison of the solutions obtained with the different methods considered. Out of the methods used, the Tchebycheff method was the only one that allowed attaining exactly the target aimed for total revenue of £16m, ensuring at the same time the highest level of customer satisfaction of 7.00 (a little bit away from the target of 7.50). To obtain this solution, we have assigned higher weights to the total revenue and customer satisfaction, i.e. 0.25 to each factor, while the remaining factors were assumed a weight of 0.13 each. The benchmarks in terms of the best practices also change regarding the initial solution, i.e. besides DMU 3 and 4 we now have DMU 1. Furthermore, there is an improvement of corporate image, but at the expense of increasing the number of branches by 330 and the number of ATMs by 60. The Wierzbicki approach allows reaching the highest level of total revenue, but the lowest level of customer satisfaction, concerning the sample of solutions under scrutiny. But, to obtain this outcome, it must opt for the highest increase of both the number of branches and ATMs when contrasted with the remaining solutions. To achieve the final solution, the STOM method requires an additional iteration when compared with all the other methods. The required number of iterations is higher because in the computation of the weights needed to solve problem (11) the method considered the initial target, which led to a higher weight of 0.60 regarding customer satisfaction and 0.54 concerning the number of branches. It is worth mentioning that these latter two factors are the ones that have a target closer to the corresponding ideal point of 9.17 and 0.65, respectively. Finally, the STEM method allows computing results quite near the ones reached with the STOM method but allocating different weights regarding the benchmarks considered.

Overall, it might be concluded that the values generated by the different methods are not very discrepant. However, the use of such diverse approaches allows the DM to select the method according to the easiness of the information required from him/her, and the possibility of switching methods enables him/her to pick the method that is making more significant progress towards his/her MPS.

Table 3. Comparison of final solutions

								Benchmarks		
Interactive Method	Nº of iterations	Total revenue (£m)	Corporate image	Customer satisfaction	Nº of branches (1,000)	Nº of ATMs (1,000)	Nº of staff (10,000)	λ_1	λ_3	λ_4
Target	-	16.00	2.53	7.50	2.50	5.00	9.00			
Starting solution	0	15.05	4.83	7.68	1.22	2.95	7.70	0.00	0.56	0.44
Wierzbicki	1	16.97	5.88	6.32	1.88	3.07	7.70	0.51	0.00	0.49
Tchebycheff	1	16.00	5.35	7.00	1.55	3.01	7.70	0.25	0.28	0.47
STOM	2	16.54	5.64	6.62	1.73	3.04	7.70	0.06	0.72	0.22
STEM	1	16.42	5.58	6.71	1.69	3.03	7.70	0.36	0.16	0.48

4 CONCLUSIONS

One of the most important features of DEA is that it provides information on how inefficient DMUs are performing against their peers, offering relevant information to DMs regarding the best practices that should be followed to reach efficiency. Nevertheless, in traditional DEA models, the selection of the benchmark DMUs is not influenced by the DMs' preferences. In fact, the reference DMUs more often nominated generally vary according to the models used. Moreover, multiple reference sets may occur for a DMU in a non-radial DEA perspective. Therefore, in real-world situations, the DMs might want to reflect their preferences and aspirations in the choice of the efficient DMUs to be used as benchmarks.

There are several methods that allow including the DMs' preferences. Out of these, the target setting models might be particularly appropriate, being usually combined with MOLP models.

In this context, unlike other hybrid DEA-MOLP approaches previously suggested, we propose a modelling framework which allows considering different priorities and individual expansion and contraction scales for the inputs and outputs. Additionally, an equivalence model between the WRDDM and the super-ideal point model has been established which enables incorporating a broad category of factors.

Several interactive methods that allow selecting the target DMUs to be viewed as benchmarks (i.e. best practices) of the non-efficient DMUs under evaluation have also been proposed based on different ways to incorporate the DM's preferences. This feature can be useful since traditional DEA models tend to completely neglect the DM's preferences in the computation of the DMUs to be used as benchmarks. Therefore, with this tool the DMs are enabled with the possibility of translating into the decision-making process management constraints (namely, budgetary) and aspiration levels regarding the inputs and outputs, providing much more realistic support for actual decision-making.

With these interactive methods to search for the MPS (i.e. the most preferred benchmarks), DMs are capable of handling input and output factors according to their preferences, i.e. inputs and outputs can be increased, maintained or reduced, depending on what they believe it is reasonable and realistic for them.

Finally, it is worth mentioning that the procedure for articulating the computation and dialogue phases through the use of these distinct search methods rather than being concerned with obtaining a minimum distance projection has an essential purpose of building progressively the DM's preferences in order to obtain the most preferred efficient target.

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Conflict of Interest: The authors declare that they have no conflict of interest.

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