Endogenous Home Bias in Portfolio Diversification and Firms Entry

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September 3, 2014

Abstract

The home bias in portfolios is considered a main puzzle in international macroeconomics. This paper provides a new benchmark for its analysis in a tractable new open economy macroeconomic model, where the home-biased position is an optimal allocation. I specify an equilibrium model of perfect risk-sharing, with endogenous portfolios and firm entry.

Unlike in the previous work, the international portfolio diversification is driven by home bias in capital goods -independently of home bias in consumption when countries are of equal size-. The model explains considerably well the recent patterns of portfolio allocations in developed economies. Most important, optimal portfolio shares are independent of market dynamics.

KEYWORDS: Home bias, equity puzzle, New open economy macroeconomics, NOEM, extensive margin.

JEL CLASSIFICATION CODES: F41, G11

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Acknowledgement 1 *I thank G. Corsetti for his invaluable guidance and S. Krautheim, J. Caballé, F. Gouret and the referees for their useful comments and suggestions during the completion of this paper. I also thank the participants in SMYE 2007 and RES 2008 for dynamic discussions on earlier drafts of the paper. Any remaining errors are mine. The financial support from the Spanish Ministry of Science and Innovation through grant ECO2012-34046 is gratefully acknowledged.*

1 Introduction

Home bias in international investment portfolio is one of the main puzzles in international finance. Empirical evidence show that agents invest mostly in domestic assets, apparently without taking advantage of the possibilities of international risk diversification. Lying on the border between international macroeconomics and finance, the home bias in portfolio selection has important implications for economic analysis and policy-making. The main question is whether this evidence exists due to market imperfections and distortionary policies or it is the result from private economic agents' optimal decisions.

This paper explores the demand for diversification due to investment fluctuations in a Cole and Obstfeld (1991) economy, where terms of trade (TOT) adjustments are a *natural* mechanism to hedge consumption risk after a turbulence. Any variation in the relative value of home output is compensated by a change in relative prices, keeping the nominal intercountry difference of consumption equal to zero. Hence, the effect of country-specific productivity shocks can be perfectly offset through its international transmission via changes in the prices of imports and exports. However, the mechanism is not enough to reach perfect risk sharing when there is some intertemporal transmission of consumption.[1] In other words, the mere existence of some investment destroys the capacity of TOT to offset productivity shocks and, hence, TOT provides insurance only in a "static sense". If households take into account future expectations they need another strategy to ensure perfect risk sharing: some portfolio diversification. One interesting paper addressing the latter idea is Heathcote and Perri (2013, henceforth H&P). They build upon the two-symmetric-country, two-good extension of the stochastic growth model developed by Backus, Kehoe and Kydland (1994 and 1995), and conclude that the preferences in consumption, drive not only the openness to trade of a country, but also the structure of its portfolio. This portfolio is more biased the more households prefer domestic products.

I construct a tractable new open economy macroeconomics (NOEM) model a là H&P. However, my model is novel in a number of ways. First, I differentiate between consumption goods and capital goods market. Whereas the former maybe mainly driven by preferences, including habits and feelings, the latter is specially tight to technological requirements. This distinction is crucial to ensure one is not overlooking the actual role preferences play. Indeed, I show that the parameter tied to investment demand determines the optimal bias in portfolio.[2] Nevertheless, preferences in consumption become relevant for countries of different sizes. The result shows that the optimal proportion of diversification predicted by the model is close to those found in the data, which are home biased and driven by the share of imports in capital goods. Since the intratemporal risk is hedged by TOT (in other words, via changes in the prices of imports and exports), in a static model (where intertemporality does not exist), the portfolio choice becomes totally irrelevant

[1] Except under certain assumptions discussed in Cole and Obstfeld (1991).

[2] Apart from H&P, Coeurdacier (2009) also proposes a theoretical model with frictions in markets and finds a similar relationship. However, both papers place consumption goods and physical capital goods in a single market.

and, hence, the home bias in the consumption goods does not matter. Second, I explore the interrelation between investment allocation and firms allocation by adding the extensive margin in monopolistic markets.[3] The introduction of a new variety requires an initial sunk cost and some time to build up the plant before beginning production. I can conclude that, at least in the case of flexible prices and symmetric countries, the allocation of the firms, and thus of the number of varieties supplied in the market, is independent of the ownership of their shares. Consequently, the constant allocation of investment is optimal even with market dynamics.[4] Finally, although I focus on a two-symmetric-country world, I also show some differences that arise when population sizes differ between countries. Only shares of firms and an international riskless bond are traded in the world. It turns out that the available stocks are enough to replicate the complete market allocation.

Certainly, there are not two identical countries in the world and, of course, differences in preferences must affect international portfolios. These asymmetries between countries would represent an explanation by themselves for the asymmetries between biases of country-portfolios. This paper shows how the biases are present even between fully symmetric countries. I left for the appendix the departure from this assumption, which has no analytical solution.

Empirically, the relevant gains for international asset diversification have been reported by authors like Van Wincoop (1999) and Lewis (2000). If the potential gains are so significant, why have financial markets not achieved greater risk sharing? Literature does not have a clear consensus on the issue. See the extensive discussion provided in a recent literature review by Coeurdacier and Rey (2013).

Since Lucas' (1982) seminal theoretical paper -where households keep identical equity shares of each country output- and Baxter and Jermann (1997) -who conclude that households should go short in home assets to hedge the risk generated by the undiversifiable labour income-, economic research has moved in several directions to explain the home-biased equities puzzle.

This paper abstracts, for instance, from explanations like informational asymmetries (see Gehrig (1993), Brennan and Cao (1997), Van Nieuwerburgh and Veldkamp (2009) and Grinblatt and Keloharju (2000) for progresses based on this explanation and Kang and Stulz (1997) and Mankiw and Zeldes (1991) on equity holdings concentration); financial frictions and transaction costs (see Obstfeld and Rogoff (2001), French and Poterba (1991) and Tesar and Werner (1995)), potentially combined with low gains from international risk-sharing as in a Cole and Obstfeld (1991) economy. The paper belongs to the strand of literature

[3] See Bergin and Corsetti (2008) and Bilbiie et al. (2012) for theoretical assessments of the relevance of the extensive margin in different contexts.

[4] Although this experiment is beyond the scope of the paper, if one assumes nominal rigidities and a monetary policy that replicates the flexible price equilibrium allocation, the previous result holds independently of the price regime of the economy. I tried to introduce nominal rigidities in prices, assuming first producer currency pricing (PCP) and then local currency pricing (LCP). In both cases, the optimal level of diversification coincides with that of the flexible price regime. However, when the authorities apply a general monetary policy, the endogenous portfolio arising from the benchmark setup is no longer constant.

focusing on hedging demands in frictionless financial markets. This literature has put forward the hedging of real exchange rate due to non-tradable goods (see Tesar (1993), Serrat (2001), Kollmann (2006), Pesenti and Van Wincoop (2002), Hnatkovska (2010) and Collard et al. (2009)), trade frictions in tradable goods (Coeurdacier (2009)), non-diversifiable labour income (Engel and Matsumoto (2009)) or redistributive and relative demand shocks (Coeurdacier et al. (2007)).

The aforementioned Cole and Obstfeld (1991) was one of the first papers that, like mine, departs from the widespread view that home bias is the result of market frictions or agents' unoptimal behaviour. Following their view, H&P and a small collection of quite recent papers argue that the home bias corresponds to the strategies of optimal rational agents for portfolio diversification. They also rely on relative international price adjustment after shocks as the main mechanism to ensure the diversification of risk, but allow for capital investment dynamics and imperfect substitutability among traded goods.

Concerning the bias on capital goods, a large part of the literature agrees that physical capital is mostly bought or built domestically. First, construction (of the plants and some equipment installation) is almost entirely local, and it represents a large proportion of the total set-up costs; second, equipment trade is tied to costs arising from overseas marketing, the negotiations for foreign purchases, transportation, tariffs and non-tariff barriers, the distribution in foreign markets, adaptations to foreign conditions and standards, installation in foreign production facilities, the need to train foreign workers to use the equipment and the provision of parts, maintenance and customer service from abroad. All of these features make capital home bias even greater than that of consumption goods. I explore this empirical evidence for 24 OECD countries in Section 4 and find large home biases in the capital market.[5]

The remainder of the paper is organized as follows: Section 2 presents the setup of the model. Section 3 gives us the equilibrium results. Section 4 develops the empirical analysis. Conclusions appear in Section 5. The appendix with an extension of the model which analyses the case of asymmetric preferences and asymmetric capital intensity in the start-up costs is available at the end.

2 The Model

The world consists of two countries, denoted by H (Home) and F (Foreign), *that may differ in size*, and an endogenously determined number of varieties, all of them perfectly tradable. The Home (Foreign) country is inhabited by a constant number of households L (L^*) that elastically supply their labour to domestic firms. There is no capital accumulation but only a cost of entry into the market with a lag of one period. For

[5] See Eaton and Kortum (2001) for an exhaustive empirical study on equipment trade. Notice, however, that the analysis refers only to equipment and disregards construction. In the model I present, one must consider "construction goods" to be aggregated in the composites for consumption and capital. Thus, the correct proportion of capital produced domestically must necessarily be higher than the levels indicated by Eaton and Kortum.

simplicity, this fixed cost of entry is assumed to be repaid each period, as in Bergin and Corsetti (2008), Kim (2004), and Devereux et al. (1996).

Bergin and Corsetti (2008) consider the entry cost as the unique physical capital of the economy, as I do here. However, they offer an extension of their model in which standard capital is added to the production function. They find that aggregate variables respond in the same way as in their basic set-up with small-scale effects.

Firms and agents are homogeneous within countries. However, preferences are symmetrically biased towards domestically-produced goods. The monopolistic firms set prices flexibly by maximizing profits. As regards notation, where applicable and unless noted otherwise, Home and Foreign variables *are expressed in per capita terms* of the Home and Foreign population, respectively. I detail the equations of the Home country and limit the report of the equations of the Foreign country to those essential to understand the model. The rest are symmetric. Foreign-country variables are indexed by an asterisk.

2.1 Households

Each country is populated by homogeneous households whose preferences are defined over the consumption of $n_t + n_t^*$ goods, which is a composite of Home plus Foreign final produced varieties. The preferences of Home households are represented by

$$U_t = E_t \Sigma_{t=0}^{\infty} \beta^t \left[\ln C_t - \kappa \ell_t (j) \right], \tag{1}$$

where $0 < \beta < 1$ is the discount factor and $U(.)$ is a utility function defined over the consumption of a basket C_t and a linear disutility of labour effort represented by parameter κ. Finally, $\ell_t (j)$ is the elastic labour supply of household j. The consumption basket is given by a Cobb-Douglas aggregator over the bundles of tradeable goods produced in the Home (C_H) and Foreign (C_F) countries (i.e., a constant elasticity of substitution (CES) basket with unitary elasticity),[6]

$$C_t = C_{H,t}^{\gamma} C_{F,t}^{1-\gamma}, \tag{2}$$

where $\gamma < 1$. C_H and C_F are CES aggregators over the $n(n^*)$ varieties produced in the Home(Foreign) country. For simplicity, I assume identical elasticities of substitution, σ, in both countries:

$$C_{H,t} = \left(\int_0^{n_t} c_t (h)^{\frac{\sigma-1}{\sigma}} dh \right)^{\frac{\sigma}{\sigma-1}}, \; C_{F,t} = \left(\int_0^{n_t^*} c_t (f)^{\frac{\sigma-1}{\sigma}} df \right)^{\frac{\sigma}{\sigma-1}}. \tag{3}$$

Here, h and f denote a specific variety of the corresponding country. Households all over the world finance the creation of firms in both countries. The Home household constructs her portfolio of investment by

[6] To keep the algebra as clear as possible and the main results in closed form, I assume that households at home and abroad have symmetric preferences. The appendix offers a reflection on the relevant differences caused by the cases in which $\gamma \neq \gamma^*$, $\delta \neq \delta^*$ and for asymmetries in σ.

purchasing a fraction $\lambda_{F,t+1}(j)$ of the equities issued by foreign-country firms and $\lambda_{H,t+1}(j)$ of the shares issued by domestic firms, which will start producing in the next period. As it is detailed in the following subsection, productive firms are alive only for one period. Therefore, by purchasing equities, households pay a share of the cost of creation of firms. Due to full amortization, households cannot sell these equities in the future.

Due to homogeneity, every household purchases the same fraction, being $\lambda_{F,t+1}(j) = \frac{\lambda_{F,t+1}}{L}$ and $\lambda_{H,t+1}(j) = \frac{\lambda_{H,t+1}}{L}$ The household affords her consumption expenditure and investment using the dividends received from currently active firms at home and abroad, in proportion to her current portfolio allocation: $\lambda_{H,t}(j)$, $\lambda_{F,t}(j)$, and her labour income. The budget constraint is

$$\begin{gathered} B_{t+1} + \lambda_{H,t+1}(j) \int^{n_{t+1}} q_t(h)\, dh + e_t \lambda_{F,t+1}(j) \int^{n^*_{t+1}} q^*_t(f)\, df + \\ + \int^{n_t} p_t(h)\, c_t(h)\, dh + \int^{n^*_t} p_t(f)\, c_t(f)\, df = \\ = \lambda_{H,t}(j) \int^{n_t} \pi_t(h) dh + e_t \lambda_{F,t}(j) \int^{n^*_t} \pi^*_t(f) df + w_t \ell_t(j) + (1+i_t)\, B_t, \end{gathered} \tag{4}$$

where $\pi_t(h)$ $(\pi^*_t(f))$ are the profits of a single Home (Foreign) firm in Home (Foreign) currency; e_t is the nominal exchange rate $(p_t(h) = e_t p^*_t(h))$; $c_t(h)$ the domestic demand for good h; n_t is the number of firms allocated at home; and w_t is the wage. B_t is the international riskless bond. Finally, γ indicates the home bias on consumption preferences. An initial investment is needed for a new firm to start producing. The cost to conduct this investment at home (abroad) is $q_t(h)$ $(q^*_t(f))$.

2.2 Firms

A continuum of $n(n^*)$ tradable-good firms in the Home (Foreign) country act in a monopolistically competitive economy. All of them sell their products in both Home and Foreign markets. A sunk cost is paid at time t to develop a new variety, which will enter the market at $t+1$ and disappear at the end of that period (full amortization). This cost is financed by issuing equities in the international stock market (in other words, both Home and Foreign agents have access to shares of any firm created all over the world).

In order to produce a new Home variety at time $t+1$, entrepreneurs must incur a startup cost of $q_t(h) = P_{k,t} K_t$ at time t. Firms are fully depreciated after one year of production. K_t is a composite good containing both Home and Foreign varieties and following a Cobb-Douglas aggregator the size of which is randomly determined every period,

$$K_t = K^{\delta}_{H,t} K^{(1-\delta)}_{F,t}, \tag{5}$$

where $K_{H,t}$ and $K_{F,t}$ are the baskets of Home and Foreign final goods used in capital. Technically speaking, this basket is determined due to capital intensity requirements and not due to preferences. The lower the

$K_t\,(K_t^*)$ the more efficient the Home (Foreign) country is in the creation of new firms or varieties. $P_{k,t}$ is the CPI for the basket K_t.[7] Finally, δ indicates the bias in the preferences of capital goods. Furthermore,

$$K_{H,t} = \left(\int_{h=0}^{n_t} k_t\,(h)^{1-\frac{1}{\sigma}}\, dh \right)^{\frac{\sigma}{\sigma-1}} \; ; \; K_{F,t} = \left(\int_{f=0}^{n_t^*} k_t\,(f)^{1-\frac{1}{\sigma}}\, df \right)^{\frac{\sigma}{\sigma-1}}, \tag{6}$$

Hence, total investment at home is

$$I_{H,t} = n_{t+1} q_t\,(h) = n_{t+1} P_{K,t} K_t. \tag{7}$$

Once created, firms produce a differentiated variety with a homogeneous technology that requires only labour:

$$Y_t(h) = A_{H,t} \ell_t(h)^{\theta}. \tag{8}$$

The state of the economy is then summarised by

$$\{A_{H,t}, A_{F,t}\}\,.$$

θ is the share of output going to labour. $(1-\theta)$, which belongs to capital, is distributed among investors via dividends. $Y_t(h)$ is the production of one firm, and $k_t\,(h)$ is the demand for the final good h by new entrants to build up their plants. $p_t(h)$ is the price of variety h, which is flexibly set by the monopolistic firm, and $\ell_t\,(h)$ is labour demand for good h.

3 Equilibrium

3.1 The Household's Problem

Households maximize utility subject to the budget constraint. The first-order conditions are the following:

$$\frac{\kappa}{w_t} = \xi_t = \frac{1}{P_t C_t} \tag{9}$$

$$C_{H,t} = \gamma \frac{P_t C_t}{P_{H,t}},\; C_{F,t} = (1-\gamma)\frac{P_t C_t}{P_{F,t}}, \tag{10}$$

$$c_t\,(h) = C_{H,t} \left(\frac{p_t\,(h)}{P_{H,t}} \right)^{-\sigma},\; c_t\,(f) = C_{F,t} \left(\frac{p_t\,(f)}{P_{F,t}} \right)^{-\sigma}, \tag{11}$$

[7] One may easily have different CES aggregators and/or an extra parameter of productivity for K in the model (e.g., of the type $K_{i,t} = A_{Ki,t} \left(\int_{h=0}^{n_t} k_t\,(h)^{\frac{\varrho-1}{\varrho}}\, dh \right)^{\frac{\varrho}{\varrho-1}}$, where $i = H, F$ and ϱ stands for the elasticity of substitution between capital goods, which may differ from σ, the elasticity between consumption goods). However, the setup presented in this paper disregards this alternative to concentrate only on the scope explained in the introduction. In this case, the closed-economy version will have $P = P_K$, as the unique differentiation between C and K is, by assumption, the Cobb-Douglas parameter ($\delta \neq \gamma$).

$$\frac{1}{P_t C_t} = \beta (1 + i_t) E_t \frac{1}{P_{t+1} C_{t+1}}, \tag{12}$$

$$q_{H,t} = E_t Q_{t,t+1} \Pi_{H,t+1}, \tag{13}$$

$$e_t q^*_{F,t} = E_t Q^*_{t,t+1} e_{t+1} \Pi^*_{F,t+1}, \tag{14}$$

where $Q_{t,t+1}$ is the discount factor of future dividends,[8] and $q_{H,t}$ ($q^*_{F,t}$) is the country aggregate of $q_t (h)$ ($q^*_t (f)$). Equation (9) is the endogenous supply of hours of labour; (10) shows the allocation of the consumption expenditure among home- and foreign-produced goods, which is constant due to the Cobb-Douglas assumption; (13) and (14) provide us with the free entry conditions for new firms. Firms will enter the market as long as the initial fixed cost is lower than or equal to the expected profits. $\Pi_{H,t}$ are the aggregate profits of all domestic firms. Finally, (12) is the usual Euler equation, the intertemporal rate of substitution between the consumption in periods t and $t+1$. The welfare-based price index is

$$P_t = \frac{P^{\gamma}_{H,t} P^{1-\gamma}_{F,t}}{\Gamma}, \tag{15}$$

where $\Gamma = \gamma^{\gamma} (1-\gamma)^{1-\gamma}$. Foreign households solve an analogous problem with symmetric preferences, i.e., they prefer the foreign-produced goods, f, as much as Home households prefer home-produced ones, h.

3.2 The Firm's Problem

During the creation of the variety, Home firms choose the demand of capital goods, $k_t (h)$ and $k_t (f)$. They minimize, separately, the cost of purchasing each of them, subject to the basket of h or f capital goods in equation (6).The first-order conditions are

$$k_t (h) = \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\sigma} K_{H,t}, \tag{16}$$

$$k_t (f) = \left(\frac{p_t(f)}{P_{F,t}} \right)^{-\sigma} K_{F,t}, \tag{17}$$

where the Lagrange multiplier for the Home firm problem is $\zeta_t = P_{H,t} = n_t^{\frac{1}{1-\sigma}} p_t (h)$ and $\zeta^*_t = P_{F,t}$ for the Foreign firm problem. The optimal aggregate baskets of Home and Foreign capital for domestic firms are

$$K_{H,t} = \delta \frac{P_{k,t} K_t}{P_{H,t}} \ , \ K_{F,t} = (1-\delta) \frac{P_{k,t} K_t}{P_{F,t}}. \tag{18}$$

Firm h today has a demand for variety h, to be used in building firms, of $n_{t+1} k_t (h)$. $\sigma > 1$ is the intratemporal elasticity of substitution among goods.[9]

[8] See the appendix for the full expression of $Q_{t,t+1}$

[9] The condition for stability requires that $1 > \theta \frac{\sigma-1}{\sigma}$. See the appendix for details.

Moreover, firms choose the amount of labour that minimizes costs, $w_t \ell_t(h)$,subject to the technology constraint. The first order condition is

$$\phi_t = \frac{w_t}{\theta A_{H,t}} \ell_t (h)^{1-\theta} = \text{mg cost}, \tag{19}$$

where ϕ_t is the Lagrange multiplier. Once operative, firms maximize profits, subject to the technology restriction and demand. Thus, the optimal price is

$$p_t(h) = \frac{\sigma}{\sigma - 1} \frac{1}{\theta} \frac{w_t}{A_{H,t}^{\frac{1}{\theta}}} Y_t(h)^{\frac{1}{\theta}-1}. \tag{20}$$

Prices consist of a constant mark-up over the expression of marginal costs which depends crucially on the level of production, due to the non-linear technology.

3.3 Market Clearing

The clearing conditions for the Domestic and Foreign goods markets are the following:

$$c_t(h) L + c_t^*(h) L^* + n_{t+1} k_t(h) + n_{t+1}^* k_t^*(h) = Y_t(h), \tag{21}$$

$$c_t(f) L + c_t^*(f) L^* + n_{t+1} k_t(f) + n_{t+1}^* k_t^*(f) = Y_t(f). \tag{22}$$

A firm satisfies four sources of demand: those of the Home and Foreign households and those of the firms that will produce next year in the Home and Foreign country.

The labour markets are emptied when the labour demand generated by all firms in a country equals the labour supply generated by all households in that country:

$$n_t \ell_t(h) = \ell_t(j) L, \tag{23}$$

$$n_t^* \ell_t^*(f) = \ell_t^*(j^*) L^*. \tag{24}$$

Finally, the financial markets in equilibrium must fulfill

$$B_t L = -B_t^* L^*, \tag{25}$$

$$\lambda_{H,t} = 1 - \lambda_{H,t}^*, \tag{26}$$

$$\lambda_{F,t} = 1 - \lambda_{F,t}^*. \tag{27}$$

Under this non-linear technology, one can write Home aggregate profits as a constant fraction of total revenue.[10] This fraction depends both on the elasticity of substitution and the technological parameter. Hence,

$$\Pi_{H,t} = P_{H,t} Y_{H,t} \left(1 - \frac{\sigma - 1}{\sigma}\theta\right) = \left(\frac{\sigma(1-\theta)+\theta}{\sigma}\right) P_{H,t} Y_{H,t}. \tag{28}$$

[10] Although this fraction is different from that found under constant returns to scale (with a linear technology $\Pi_H^{CRS} = \frac{1}{\sigma} P_H Y_H < \Pi_H^{DRS}$).

Notice that $\frac{\sigma(1-\theta)+\theta}{\sigma}-\frac{1}{\sigma}>0$. The amount of profits over total income is higher due to the diminishing returns to scale in the technology. One can also write the labour cost (which, in aggregate terms becomes the labour income) as a fraction of the output of the firms:

$$w_t \ell_t(h) = \frac{\sigma-1}{\sigma}\theta p_t(h) Y_t(h). \tag{29}$$

3.4 Terms of Trade

TOT are defined as the price of a country's exports in terms of their imports, i.e., $TOT=\frac{P^*_{H,t}}{P_{F,t}}$. One can derive TOT from the resource constraints,

$$C_{H,t}L + C^*_{H,t}L^* + n_{t+1}K_{H,t} + n^*_{t+1}K^*_{H,t} = Y_{H,t}, \tag{30}$$

$$C_{F,t}L + C^*_{F,t}L^* + n_{t+1}K_{F,t} + n^*_{t+1}K^*_{F,t} = Y_{F,t}. \tag{31}$$

Taking the ratio of the two equations on $\frac{P_tC_t}{P_{F,t}}$ and $\frac{P^*_tC^*_t}{P^*_{H,t}}$ yields an expression for the TOT,

$$TOT_{H,t} = \frac{1}{e_t}\frac{\left[\gamma L + (1-\gamma^*)L^*\right]\left[Y_{F,t} - n^*_{t+1}K^*_{F,t} - n_{t+1}K_{F,t}\right]}{\left[(1-\gamma)L + \gamma^* L^*\right]\left[Y_{H,t} - n_{t+1}K_{H,t} - n^*_{t+1}K^*_{H,t}\right]}. \tag{32}$$

The terms of trade depend on the relative supply of the output net of investment, and on the international relative preference for home goods, weighted by population sizes. Given investment, the international transmission is positive: an increase in net Home output benefits Foreign households by lowering the Home output prices. At the same time, a positive productivity shock at Home raises investment.

For $\gamma \neq 1/2$ or $\gamma^* \neq 1/2$, home bias in consumption implies that the real exchange rate (RER) is not constant, but changes with the terms of trade:[11]

$$RER = \frac{P_t}{e_t P^*_t} = \frac{\Gamma^*}{\Gamma}\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\gamma-\gamma^*}. \tag{33}$$

With perfect risk sharing, it follows that the ratio between consumption levels is also equal to RER.

$$\frac{C^*_t}{C_t} = \left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\gamma-\gamma^*}. \tag{34}$$

[11] For $\gamma=\gamma^*=1/2$ and $L=L^*$, households' preferences are identical, and the real exchange rate P/P^* equals 1. In other words, purchasing power parity holds. This is the case in Cole and Obstfeld (1991).

3.5 Optimal Diversification Level λ

Let us assume that per capita risk-sharing is efficient with a constant portfolio structure, $\lambda^*_{H,t} = \lambda_{F,t} = \lambda$, for the symmetric-country scenario,[12] and $B = 0$ such that

$$P_t C_t = e_t P^*_t C^*_t. \tag{35}$$

So that $Q_t = e_t Q^*_t$, stochastic discount rates are the same across countries. Hereafter, it is shown that this allocation is indeed an equilibrium allocation by characterizing the associated vector of equilibrium prices and verifying that prices and quantities satisfy households' first-order conditions, market clearing conditions and the resource constraints.

First, one needs to define the following relative variables in nominal terms:

$$\begin{aligned} \triangle\mathbb{C} &= P_t C_t L - e_t P^*_t C^*_t L^*, \\ \triangle\mathbb{k} &= n_{t+1} P_{k,t} K_t - e_t n^*_{t+1} P^*_{k,t} K^*_t = I_{H,t} - e_t I^*_{F,t}, \\ \triangle\mathbb{Y} &= P_{H,t} Y_{H,t} - e_t P^*_{F,t} Y_{F,t}. \end{aligned} \tag{36}$$

These equations are the intercountry differences in total consumption, investment and output in nominal terms. Moreover, from the goods market clearing conditions,

Home Output

$$\begin{aligned} P_{H,t} Y_{H,t} = P_{H,t} \frac{\gamma P_t C_t}{P_{H,t}} L + e_t P^*_{H,t} \left(1 - \gamma^*\right) \frac{P^*_t C^*_t}{P^*_{H,t}} L^* + n_{t+1} P_{H,t} \frac{\delta P_{k,t} K_t}{P_{H,t}} + \\ + n^*_{t+1} e_t P^*_{H,t} \left(1 - \delta^*\right) \frac{P^*_{k,t} K^*_t}{P^*_{H,t}}. \end{aligned} \tag{37}$$

Foreign Output

$$\begin{aligned} P^*_{F,t} Y_{F,t} = \gamma^* P^*_t C^*_t L^* + \frac{P_t C_t}{e_t} \left(1 - \gamma\right) L + n_{t+1} \frac{P_{H,t}}{e_t} \left(1 - \delta\right) \frac{P_{k,t} K_t}{P_{H,t}} + \\ + n^*_{t+1} P^*_{H,t} \frac{\delta^* P^*_{k,t} K^*_t}{P^*_{H,t}}. \end{aligned} \tag{38}$$

By taking the differences, and assuming $\gamma^* = \gamma$ and $\delta^* = \delta$,[13] I have an expression for the output absorption in the economy -or, in other words, the allocation of output into different uses-:

$$\triangle\mathbb{Y} = \left(2\gamma - 1\right) \triangle\mathbb{C} + \left(2\delta - 1\right) \triangle\mathbb{k}. \tag{39}$$

[12] It would be interesting to relax this assumption and allow for $\lambda_H \neq \lambda_F$. This would permit the analysis of the construction of portfolios in a fully asymmetric world. However, this scenario has no analytical solution and, their results would answer a question different from the one addressed here. The appendix provides expressions for λ_H and λ_F that give an idea of relevant terms driving portfolios in this asymmetric world.

[13] See the appendix for a discussion on the consequences of relaxing this assumption.

The difference in nominal output is due to the differences in consumption and investment. The size of each of them in $\triangle\mathbb{Y}$ depends on the corresponding parameter of the Cobb-Douglas aggregator in C or K, δ or γ.

Our conjectured equilibrium, implies that consumption per capita equalizes across countries. However, differences in aggregate consumptions can still fluctuate with world output and these fluctuations, if any, are equal to $\triangle\mathbb{C} = P_t C_t L - e_t P_t^* C_t^* L^* = (L - L^*)\, P_t C_t$ when perfect risk sharing is achieved. In order to understand the forces that may cause changes in country-differences in consumption, I derive an expression for $\triangle\mathbb{C}$ which depends on parameters, the portfolio and $\triangle\mathbb{k}$.[14] Let us start with the case in which $L = L^*$. Then, $\triangle\mathbb{C} = 0$ and

$$\triangle\mathbb{Y}|_{\triangle\mathbb{C}=0} = (2\delta - 1)\, \triangle\mathbb{k}. \tag{40}$$

The expression for $\triangle\mathbb{C}$ becomes

$$\triangle\mathbb{C} = \left(1 + 2\lambda\left(\frac{\sigma - 1}{\sigma}\theta - 1\right)\right)(2\delta - 1)\, \triangle\mathbb{k} - (1 - 2\lambda)\, \triangle\mathbb{k}. \tag{41}$$

3.5.1 Transmission Mechanism

Equation (41), which I restate below, is the key equation yielding explanations for investors' behaviour:[15]

$$\triangle\mathbb{C} = \underbrace{\overbrace{(2\delta - 1)}^{\text{via Y}}\overbrace{\left(1 - 2\lambda\left(1 - \frac{\sigma - 1}{\sigma}\theta\right)\right)}^{\text{via prices}}}_{\text{indirect effect}}\triangle\mathbb{k} \underbrace{- (1 - 2\lambda)\, \triangle\mathbb{k}}_{\text{direct effect}}. \tag{42}$$

Let us analyse the mechanism: Assume that a fully anticipated shock consistent with a rise in relative investment occurs (i.e., $\triangle I_{H,t+1}$ whereas $I^*_{F,t+1}$ remains constant).[16] First, consider an environment where the basket of capital goods is biased towards domestic varieties ($\delta > \frac{1}{2}$). Thus, $\lambda < \frac{1}{2}$.

In brief, one can state that the $\triangle I_{H,t+1}$ causes a quantity and a valuation effect, and these disturb perfect risk sharing. The shock generates an increment in Home households' wealth whereas Foreign wealth decreases. This difference is reflected in the valuation of Home output via the increase in prices.

I can split the overall impact into two simultaneous effects that move in opposite directions. On the one hand, $\triangle\mathbb{k}$ has a negative direct effect on $\triangle\mathbb{C}$. The relative demand for Home goods increases because they are used to satisfy the extra investment. Although part of this cost is financed by foreigners through ownership ($\frac{1}{2} > \lambda > 0$), the Home household is forced to reduce her relative consumption. This impact on $\triangle\mathbb{C}$ helps to regulate the financial flows and thus avoids disturbing perfect risk sharing.

[14] See the appendix for algebraic details. Notice that I concentrate in the case where $\triangle\mathbb{k} \neq 0$. Otherwise, λ cannot be derived from this condition.

[15] To keep the explanation as clear and intuitive as possible, I assume here $L = L^*$. The end of the section considers the differences for a model where L, L^* are constant but different.

[16] A typical example of ΔI_H is the expectation of a future increase in home productivity, so that agents want to create more firms to take advantage of such improvement.

Notice that when $\lambda = 0$ (no diversification), the term $(-(1-2\lambda))$ equals -1. Thus, an increment of one euro in domestic investment directly implies a one-euro reduction in domestic consumption because it generates a one-unit decline in dividends received by Home households. By contrast, if $\lambda = 1$ (Home households own only Foreign assets), the direct term equals 1. Thus, an extra euro of Home investment generates a reduction of one euro in Foreign consumption because the Foreign households finance the whole cost of this investment.

In contrast, the indirect effect, the impact of $\triangle\mathbb{Y}$ on $\triangle\mathbb{C}$, is positive. This effect can also be separated into two parts. The first, $(2\delta - 1)$, captures the extent to which an increase in domestic absorption (in this case, investment) increases the relative value of Home output. The second, $\left(1 - 2\lambda\left(1 - \frac{\sigma-1}{\sigma}\theta\right)\right)$, reflects the impact of a change in relative output on relative consumption. It shows that an increment in relative demand for Home goods has a positive effect on the TOT for the domestic economy. This effect is negatively related to λ and positively to $\frac{\sigma-1}{\sigma}\theta$ because the larger the non-diversifiable labour's share, the larger the impact of an improvement in the domestic economy's TOT on relative consumption, given λ.[17] Similarly, the smaller the diversification level λ, the larger the impact of a variation in relative prices on $\triangle\mathbb{C}$.

To sum up, when the shock is anticipated, Home output has a higher relative value due to the increment of the demand. In consequence, the distributed dividend, which belongs partly to Foreign households, is larger. The increase in the output demand pushes the quantity of labour up, and therefore the total labour income increases, causing households to become richer.

Indeed, the magnitude of this general equilibrium effect is greater than the magnitude of the direct effect when λ (the proportion of Foreign assets) is inefficiently high and *vice versa*.

The household willing to compensate for a situation like this and to re-establish perfect risk sharing, must increase λ. Therefore, a larger proportion of dividends is redistributed to the Foreign households to produce a smooth consumption. In this way, Home households pass part of their wealth to the other country and simultaneously reduce the demand effect. The latter occurs because, although they are importing more, these imports are partly financed by giving extra ownership to the foreigners.

In the case in which capital goods are mostly composed of foreign varieties ($\delta < \frac{1}{2} \Rightarrow \lambda > \frac{1}{2}$), all the effects act in the opposite direction. The direct effect is positive, while the indirect effect becomes negative. This change is reasonable because the demand generated by the extra investment must now be mostly covered by Foreign goods, and therefore Foreign output increases in value with respect to Home production.

This result yields a basic conclusion: diversification is such that redistribution of income across countries makes sure that consumption expenditures are equalized.[18] And it is the existence of investment that makes

[17] This feature is because most of the revenue goes directly to labour, via wages. Real wages are affected by changes in relative prices. In contrast, when a large part of the household's income comes from dividends, TOT lose their capacity to offset the impact of the shocks.

[18] Notice that δ and λ appear multiplied by 2. When investment at Home goes up, the Home country increases the demand

diversification necessary, because TOT are not able to neutralise the consequences of the shocks.

Different country sizes When country sizes are different, $\triangle\mathbb{C}$ is always different from zero and $\triangle\mathbb{C} = (L - L^*)\, P_t C_t$. Equation (41) becomes

$$\triangle\mathbb{C} = \frac{(2\delta - 1)\left(1 + 2\lambda\left(\frac{\sigma-1}{\sigma}\theta - 1\right)\right)\triangle\mathbb{k} - (1 - 2\lambda)\triangle\mathbb{k}}{1 - (2\gamma - 1)\left(1 + 2\lambda\left(\frac{\sigma-1}{\sigma}\theta - 1\right)\right)}. \tag{43}$$

The main novelty of this expression is that γ appears. The denominator of the latter equation is positive and smaller than one for any $\gamma > \frac{1}{2}$ and positive but above one for $\gamma < \frac{1}{2}$. Therefore, its presence enhances the magnitude of the direct and the indirect effect for all countries with home bias in consumption.

3.5.2 The portfolio

By setting $\triangle\mathbb{C} = 0$ I solve for λ,

$$\lambda = \frac{1 - \delta}{1 + (2\delta - 1)\left(\frac{\sigma-1}{\sigma}\theta - 1\right)}. \tag{44}$$

With such a portfolio one can easily check that risk-sharing is efficient, this is then the equilibrium portfolio; i.e., the diversification level for which the direct and indirect effects of a shock disturbing per capita relative consumption (for instance, a shock in investment) are exactly offset.

The number of firms in the market do not play any role in λ. Households allocate a positive part of their portfolios to Foreign assets, $0 \leq \lambda \leq 1$. Notice that it is not the parameter from the preferences on consumption (γ) that plays a role in the diversification, but the parameter of the preferences on capital goods (δ). This happens because TOT, which do incorporate γ, has already hedged the risk associated to consumption, as Cole and Obstfeld (1991) showed. It is only when intertemporal decisions and savings appear that we really require an optimal allocation of our savings. Hence, it is important to disentangle these two, allowing them to be different.[19]

The larger the home bias in the preferences for capital goods, the less they diversify. The value of λ decreases with δ and is kept above $\frac{1}{2}$ for $\delta < \frac{1}{2}$ and below $\frac{1}{2}$ for $\delta > \frac{1}{2}$. Thus, I find a portfolio biased towards Home assets to be the optimal allocation for households to reach perfect risk sharing. A larger

both for domestic goods (by δ) and for Foreign goods (by $1-\delta$) and $e_t P^*_{F,t} K^*_{F,t} = (1-\delta)\, P_{K,t} K_t$,$P_{H,t} K_{H,t} = \delta P_{K,t} K_t$. Thus, $\Delta\mathbb{k}$ includes the term $(2\delta - 1)$. By the same token, $P_t C_t - e_t P^*_t C^*_t = ... - (1-\lambda)\, P_{K,t} K_t - (-\lambda)\, P_{K,t} K_t = (2\lambda - 1)\, P_{K,t} K_t$.

[19] Under the present specification with bond trading, the optimal portfolio of a log-investor and the one for CRRA utility are the same. As Coeurdacier and Gourinchas (2011 and their unpublished version (2013)) show, the optimal equity portfolio is independent of preference parameters such as the degree of risk aversion or the 'tradability' of goods in consumption. The complex and non-linear dependence of equilibrium equity portfolios on preference parameters in the equity-only model disappears once one introduces trade in bonds. If the economy was limited to equity and goods trade, the irrelevancy of home bias in consumption for the portfolio would be partially driven by the assumption of log-utility, since our agents do not care to hedge real exchange rate fluctuations (see Coeurdacier and Rey (2013)).

trade share (smaller δ) in capital goods implies a weaker terms-of-trade response to changes in relative final demand. Thus, for any given diversification level, the indirect effect of demand changes on relative consumption that works through prices is going to be smaller. Moreover, whilst $\delta \geq \frac{1}{2}$, λ decreases with the labour income share because when $\frac{\sigma-1}{\sigma}\theta$ is high, TOT do most of the work in equalizing consumptions. A smaller diversification is needed to produce perfect risk sharing. Coeurdacier and Gourinchas (2011) find empirical evidence in favour of this result. Canada, US and Japan show negative correlations between equity and labour income. This makes Home equities a good hedge for labour income.

In the extreme case of $\delta \to 1$, i.e., the country uses only domestically produced goods as capital in the creation of new firms, households do not diversify at all, $\lambda \to 0$. This result shows, again, that the home bias in consumption preferences is not relevant for diversification, but it also shows that the size of the labour income share alone, without the presence of some bias in demand (here in capital goods), is not important either. The reason is easy to understand: when Home agents use only their own goods to create firms, the first term of the indirect effect, the one explaining the impact of relative output on relative consumption, is zero. There is no valuation of Home output because flexible prices react one-to-one to the excess of demand, compensating for the shock and ensuring perfect risk sharing. Thus, this result agrees with Cole and Obstfeld's result. Finally, when the bundle of capital goods is equally divided between Home and Foreign varieties, households need perfect pooling (i.e., they divide their portfolios perfectly between Home and Foreign equities) to achieve perfect risk sharing.

The reason is that when the demand on capital goods is equally allocated between Home and Foreign goods, any increase in either Home or Foreign investment pushes the demand for Domestic and Foreign varieties in the same proportion, keeping the TOT invariable (the indirect effect is zero). Thus, if agents rely on perfect pooling, they share the weight of the financing whichever country is affected by the shock.

The model provides an example of complementarity between terms-of-trade movements and income transfers via asset holdings in insuring against consumption risk from productivity fluctuations.

Relative price movements already provide some consumption risk insurance, but it is not perfect because trade flows among countries move TOT in response, not only to consumption but also to investment needs. These needs are possibly driven by expectations of future returns to capital. Portfolio diversification provides a way to insulate terms of trade from the components of demand due to investment. Hence, income flows from assets cover the demand for local inputs by Foreign firms: the higher the proportion of investment that is local, the lower the need to diversify.

Different country sizes The portfolio providing perfect risk sharing to households has no longer a constant share of Foreign and Domestic assets when $\triangle\mathbb{C} = (L - L^*)\, P_t C_t$. It is now constructed in the following way:

$$\lambda_{L\neq L^*} = \frac{(1-\gamma)\,\triangle\mathbb{C} + (1-\delta)\,\triangle\mathbb{k}}{(2\gamma-1)\left(\frac{\sigma-1}{\sigma}\theta-1\right)\triangle\mathbb{C} + \left(1+(2\delta-1)\left(\frac{\sigma-1}{\sigma}\theta-1\right)\right)\triangle\mathbb{k}}. \tag{45}$$

λ depends on country-differences in investment and on the size of the country compared to the rest of the trade partners. As in the symmetric case, portfolio diversification is not necessary when there is no trade in goods ($\delta = \gamma = 1$). Notice that, from equation (43), when there are no differences in investment ($\triangle\mathbb{k} = 0$), the portfolio is indeterminate. $\lambda_{L\neq L^*}$ gives us an abstract picture of what the portfolio would be in a more complex scenario. To get an accurate result and analyse it in detail, one must develop a numerical exercise for the model and consider asymmetric portfolios λ_H and λ_F. However, this is beyond the scope of the paper, which aims at showing how portfolios can be biased regardless of country symmetry.

4 Empirical Analysis

Equation (44) provides a prediction of the share of Foreign assets in aggregate portfolios and links the imports share of physical capital $(1-\delta)$ with international diversification. In this section, I compare these predictions with the data to assess the extent to which the model can explain the behaviour of international diversification in developed countries during recent decades.

My model focuses on a two-symmetric-country model, whereas different international portfolios belong to highly heterogeneous countries. Although this restriction may seem to limit our comprehension of international diversification patters, it does not for the understanding of portfolio allocation in frictionless markets. I address this characteristic of the model by limiting the empirical analysis to 24 relatively homogeneous countries with open financial markets: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Israel, Italy, Japan, Korea, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom and the United States. These countries are high income economies under World Bank classification.

As is discussed in the following subsections, I use exactly the same measure and databases for aggregate portfolio diversification and for national wealth that are used by H&P. My data differs from their data in two ways: (i) due to my theoretical model, I consider the level of international openness in the capital goods market instead of the level of openness in total trade; and (ii) I use actual values of labour income share instead of a standard accepted value.

My analysis focuses on the relationship between openness in the capital goods market and in the financial markets and controls for differences in GDP per capita, population size and openness in the consumption goods market. A test for collinearity has been run before using capital trade openness and consumption trade openness simultaneously, with satisfactory results.[20]

[20]H&P also enrich their empirical study by assessing whether factors omitted in the model, such as size or level of develop-

4.1 Trade in Capital

Based on the theoretical interpretation of the transmission mechanism, I need the presence of home bias in physical capital to lead the biased aggregate portfolios. Capital in the model is built period by period to create new varieties and it is not accumulated. The parameter δ that appears in equation (44) belongs to the technology of creation of this capital. Hence, the most suitable measure of $(1-\delta)$ in the data should be the share of imports in gross fixed capital formation. The trade share in capital goods is defined as

$$(1-\delta)_{it} = \frac{M_{k,it}}{GFCF_{it}}, \tag{46}$$

where $M_{k,it}$ is the cost of imports in capital goods of country i in period t, and $GFCF_{it}$ is the gross fixed capital formation. Both the imports content of GFCF and GFCF itself come from the OECD database.

Since 1995, OECD has collected data on the imports content of gross fixed capital formation every five years. The series is not available for all OECD countries but only for 24 of them. All these countries show a share of imports below fifty percent in their capital formation.[21] Table 1 provides some examples of these shares.

	$(1-\delta)$		
Country	**1995**	**2000**	**2005**
Australia	0.13	0.14	0.12
France	0.12	0.13	0.11
Ireland	0.43	0.25	0.17
Japan	0.03	0.05	0.06
United States	0.10	0.08	0.08

Table 1: M over GFCF. Source: OECD, own calculations.

ment, are important empirical factors in explaining diversification patterns. They conclude that the control variables are not statistically related to diversification, as long as their openness variable (which drives the main result) is retained.

H&P repeat the regressions for a larger group of countries that includes developing economies and find that, although the strong link between trade and diversification remains, income per capita becomes an important determinant of diversification, with richer countries being more diversified. Hence, it is necessary to restrict the data to relatively homogeneous countries due to the structure of the present model.

[21] Ireland shows the highest level of imports over gross fixed capital formation in 1995, at 43%, although the share decreases to 17% in 2005. The lowest level is for Japan, which does not exceed 6%.

4.2 Trade in consumption

The theoretical model concludes that home bias in consumption, γ, does not drive portfolio allocation (once you control for population sizes). I include this variable in the empirical model to see whether this result is supported in actual data. OECD database offers data on *total imports in final consumption*, M_c, in mid-90s, beginning of 2000s and mid-2000s. These are, indeed, the aggregation of imports contents of Households Final Consumption; Non-Profit Institutions Serving Households; Government Final Consumption; and of Total Intermediate Consumption minus Import Contents of Exports. The latter is necessary to capture the presence of foreign components, different from capital, embodied in final consumption and to create the adequate empirical counterpart to the theoretical model, which does not consider intermediate-goods producers. Moreover, OECD offers annual data on government and private final consumption. Since I have abstracted from public expenditure in the theoretical model, I must consider the share of imports in total final consumption (the sum of public and private consumption, TC). All data is in constant prices. Finally, I consider an alternative measure of γ (called γ_2) that ignores the imports present in final goods that come from intermediate goods. Results do not change significantly regardless of the measure used. The trade share in consumption goods is defined as

$$(1-\gamma)_{it} = \frac{M_{c,it}}{TC_{it}}.$$

4.3 Diversification

The reciprocal of equation (44) provides us with a linear relationship between the reciprocal of diversification, $\frac{1}{\lambda}$, and the reciprocal of the trade share of capital goods, $\frac{1}{1-\delta}$:

$$\frac{1}{\lambda} = 2\left(1-\Pi\right) + \Pi\frac{1}{1-\delta}, \tag{47}$$

where $\Pi = \frac{\sigma-1}{\sigma}\theta$ and, hence, relates the reciprocal of international diversification with the labour income share. There are two aims of this section: to determinate whether the predicted link in equation (47) is present in the data and to observe the explanatory power of the theoretical model.

The level of international diversification in the general equilibrium macroeconomic model derived here, λ, is a broad measure of diversification. In the data, I must choose an aggregated measure of both the ratio of gross foreign assets and the ratio of gross foreign liabilities to country wealth. I use the Lane and Milesi-Ferretti (2007)[22] dataset to obtain total foreign assets and liabilities. These measures include portfolio equity investment, foreign direct investment, debt (including loans or trade credit), financial derivatives and reserve assets (excluding gold). Total country wealth is defined as the value of the entire domestic capital stock plus

[22] The original database the authors used in that paper ends in 2004. I based my analysis on their updated webpage version of the dataset, published online in 2009.

gross foreign assets, less gross foreign liabilities: $W = K + FA - FL$. H&P develop an accurate discussion of the method to construct the most suitable measure of capital. I rely on their series, which borrow the initial value for capital stock from Dhareshwar and Nehru (1993) and build time series by accumulating net investment from the Penn World Tables 6.2.[23] Hence, international diversification for country i in period t is[24]

$$\lambda_{it} = \frac{FA_{it} + FL_{it}}{2\left(K_{it} + FA_{it} - FL_{it}\right)}. \tag{48}$$

Theoretical macroeconomic literature tends to assume a labour income share around 0.66. Because diversification in my model depends crucially on this share (equal to Π here), I preferred to use the actual values provided by OECD. Although they are not drastically different among developed countries, they show some heterogeneity and change over time. See some examples in table 2.

	Π		
Country	**1995**	**2000**	**2005**
Australia	0.66	0.64	0.61
France	0.69	0.67	0.67
Ireland	0.66	0.57	0.58
Japan	0.63	0.62	0.58
United States	0.67	0.68	0.65

Table 2: Labour income share. Source: OECD.

A usual concern of the use of labour income share is the weight and the precise consideration made about self-employment. The share of self-employed people in the labour force vary a lot from country to country. Experts have discussed for long about the suitability of its inclusion in labour income. Gollin (2002) analyses the differences caused by alternative systems to allocate self-employed income between returns to labour and to capital. He finally advocates for splitting self-employment income between labour income and

[23] I also tried two other options. First, I based an analysis on Kamps (2006), who publishes his dataset on the Kiel Institute for World Economics webpage. This workbook contains net capital stock estimates for the period 1960 - 2001 for 22 OECD countries. These estimates are constructed by applying the Perpetual Inventory Method, following the practice of the U.S. Department of Commerce, Bureau of Economic Analysis. However, I would have been forced to drop two countries from my already reduced dataset. Second, I defined $\lambda = \frac{FA+FL}{Total\ Market\ capitalization}$. However, I omitted it because it does not cover such a large measure of assets and liabilities as total wealth in H&P does.

[24] Based on Collard et al. (2009) analysis, our theoretical λ is brought to data only imperfectly. This is true because I abstract from housing, the most important form of equity for households; and from multinational firms, which may imply an indirect foreign investment. However, as these authors admit, only the availability of data for US allows correcting these problems.

capital income: a labour income equal to the national average labour income of employees is assigned to every self-employed person and, the rest of his/her income, if any, is assigned to capital income. OECD has taken the same solution for its labour income series and, therefore, the data I use follows Gollin (2002).

4.4 Comparing the Predictions to the Data

In this subsection I use average diversification per country for the three years for which I have data (1995, 2000 and 2005) and explore the linear relationship of its reciprocal with the reciprocal of the average of imports content of capital goods depicted by equation (47). I estimate by Ordinary Least Squares (OLS) and Least Absolute Deviation (LAD) the following equation:

$$\frac{1}{\lambda} = \beta_0 + \beta_1 \frac{1}{1-\delta} + \varepsilon, \tag{49}$$

where ε is an error term. The theoretical relationship (47) states that $\beta_0 = 2(1 - \Pi\theta)$ and $\beta_1 = \Pi\theta$. Thus, to test if equation (47) is true, I will test the null hypothesis

$$H_0 : \beta_0 = 2\left(1 - \beta_1\right). \tag{50}$$

Table 3 presents the results. Column [1] reports the OLS regression of equation (49). As one can see, $\hat{\beta}_0 = 0.18$ and does not differ significantly from zero. The estimated slope coefficient $\hat{\beta}_1 = 0.34$ and differs significantly from zero at the one-percent significance level. At the bottom of the table, I present the F test corresponding to the null hypothesis in (50). The calculated Fisher statistic is $F(1;22) = 7.87$. The 1% critical value from an F distribution with $(1;22)$ degrees of freedom is 7.95, and therefore the null hypothesis (50) cannot be rejected. In other words, the theoretical relationship depicted by equation (47) is supported by the empirical evidence. To be sure that this result is not driven by specific countries, I have also excluded some specific countries in Column [1b] and performed a LAD regression (which is more robust to influential observation than OLS). In Column [1b], Ireland and Luxembourg are omitted from the regression. These countries are particularly unusual. Luxembourg is a small country but a huge financial center and the weight of foreign assets and liabilities over wealth is far from the second most diversified country. Ireland experienced a dramatic jump in its portfolio allocation towards foreign assets and liabilities between one observation in time and the next, making the mean of the three year points quite uninformative for this country. Excluding these two countries, I obtain $F(1;20) = 4.37$. The 1% critical value from an F distribution with $(1;20)$ degrees of freedom is 8.10, and again, I cannot reject the theoretical relationship (47). Column [2] presents the LAD regression. The F test is 5.03 and much lower than the 1% critical value from the F distribution with $(1;22)$ degrees of freedom (7.95, as already stated). Hence, I do not reject the relationship using a LAD either.[25]

[25] H&P's model finds exactly the same link between the constant and the coefficient tied to the reciprocal of trade openness,

The R^2 values in the table indicate that differences in openness to capital trade can alone explain around 50% of cross-country variation in portfolio diversification.

Column [1c] regresses the inverse of portfolio diversification against the inverse of openness in capital and in consumption markets. Both variables are significant, although the latter takes a negative sign and loses its statistical significance once you add population and/or GDP per capita as control variables (Columns [1d] and [1e]).[26] Indeed, the theoretical result for the case $L \neq L^*$ tights the role of openness in consumption markets to population size, L, and to output per capita, PC, similar to the conclusions extracted from the change in the significance of openness in consumption markets. Altogether, it provides robustness to the main theoretical result: trade openness in capital goods, and not in consumption goods, drives diversification. Collinearity is not a problem in this case. The variance inflation factors (VIF) and the tolerance are 1.14 and 0.88 respectively for the pair $\frac{1}{1-\delta}, \frac{1}{1-\gamma_1}$ and 1.89 and 0.53 for the pair $\frac{1}{1-\delta}, \frac{1}{1-\gamma_2}$.[27]

The latter outcomes may quieten the reasonable concerns regarding the relationships between population sizes and portfolio diversification and between population sizes and import shares. The symmetric-country theoretical model abstracts from these two potential negative relationships. The empirical outcomes seem to state that, for the group of countries analysed, these two simplifications do not affect the portfolio diversification.[28]

The results show that diversification and trade in capital are significantly related, with a coefficient slightly above $\frac{1}{3}$. Thus, they indicate that 1 percentage point more imports relative to total investment translates into more than $\frac{1}{3}$ percentage point increase in foreign assets relative to wealth.

Column [3] presents a calibrated version of the relationship. The $\hat{\beta}_1$ of Columns [1c] to [1e] do not differ significantly from $\Pi = 0.58$ an lay within the 99% confidence interval; concerning $\hat{\beta}_0$, it does not differ significantly from $2(1-\Pi) = 1.06$ in Columns [1] to [2b] (i.e., in Column [1], for instance, the estimated standard error is 0.518, and the estimated coefficient is 0.18; knowing that the student t is 2.81 for 22 degrees of freedom, the 99% confidence interval is $[-1.27; 1.63]$). Nevertheless, $\hat{\beta}_0$ is not statistically significant.

Although the model replicates the observed correlation between trade and portfolio diversification, it is still far from perfect. Indeed, it overpredicts the influence of capital trade suggesting that a 1% increase in the latter would increase diversification by 0.58%. This overprediction -obtained in a frictionless world-

although in their case $\beta_0 = 2(1-\theta)$ and $\beta_1 = \theta$. Moreover, due to the structure of their model, they use data on total international trade instead of the capital goods market. By replicating their empirical analysis with the data provided in Perri's webpage, one can see that their coefficients do not pass this test.

[26] Both control variables are in logs. Population is in millions.

[27] A commonly given rule of thumb is that VIFs of 10 or higher (or equivalently, tolerances of 0.10 or less) may be reason for concern. Other more conservative authors mistrust a regression if explanatory variables have VIFs over 2.5 and tolerance under 0.40.

[28] Indeed, dispersion graphs for δ and γ over population and over GDP per capita show that there is not a clear sign in any of the relationships. Neither is it when one uses GDP per capita instead of population as the size measure.

leaves some room for the compatibility of our explanation with others already mentioned. For instance, the importance of investment in multinationals as a form of indirect international diversification or the existence of asymmetric information that favours local investment, would reduce our coefficient.[29]

Dependent variable: inverse of diversification

	OLS					**LAD**		**Model**
	[1]	[1b]	[1c]	[1d]	[1e]	[2]	[2b]	[3]
$\frac{1}{1-\delta}$	0.34^{***}	0.32^{***}	0.39^{***}	0.35^{***}	0.35^{***}	0.36^{***}	0.37^{***}	0.58
	(0.067)	(0.069)	(0.067)	(0.091)	(0.080)	(0.040)	(0.060)	
$\frac{1}{1-\gamma_1}$			-0.28^{**}		-0.19			
			(0.129)		(0.16)			
$\frac{1}{1-\gamma_2}$				-0.21			-0.05	
				(0.24)			(0.210)	
$ln(gdpc)$				-2.34^{**}	-1.50		-2.21^{*}	
				(1.06)	(1.17)		(1.25)	
pop				0.004	0.002		-0.08	
				(0.007)	(0.005)		(0.350)	
$cons$	0.18	0.46	1.68	25.0	16.9	-0.15	23.2	1.06
	(0.518)	(0.548)	(0.852)	(11.2)	(11.9)	(0.448)	(13.3)	
$individuals$	24	22	24	24		24		24
R^2	0.53	0.51	0.62	0.64				
$\underset{H_0:\beta_0-2(1-\beta_1)}{F(1;n)}$	7.87	4.37				5.03		
$p-value$	0.0103	0.0495				0.0353		

Table 3: Cross-sectional regressions.

4.5 Changes in diversification

So far, the empirical analysis has not taken advantage of the time dimension of the dataset. The reader must be aware of the small size of the database, with an unbalanced panel of only 3 time-points for 24 countries. Therefore, the results must be interpreted with cautious. Notice, moreover, that the time elapse between one data point and the next is 5 years.

[29] There are potentially a range of alternative models that could also generate such a relationship. See, for example, Collard et al. (2007).

Dependent variable: 5-year change in diversification

	Data			Model
	[1]	[2]	[3]	[4]
$\Delta \frac{1}{1-\delta}$	-0.24^{***}	-0.23^{***}	-0.19^{**}	-0.026
	(0.072)	(0.073)	(0.080)	(0.010)
$\Delta \frac{1}{1-\gamma_1}$	0.11			
	(0.076)			
$\Delta \frac{1}{1-\gamma_2}$		0.11		
		(0.202)		
$\Delta \ln\_gdpc$	-6.19^{**}	-5.49^{*}		
	(2.624)	(2.751)		
$\Delta \ln\_pop$	4.47	4.65		
	(15.3)	(15.5)		
observations	22	22	22	22
R^2-within	0.56	0.50	0.23	

Table 4: Changes in diversification

The equilibrium found in the theoretical model is not changing over time. However, if for whatever reason the imports share in capital changes between t and $t+1$, also diversification does. This section develops a panel data to study how changes in δ affect changes in λ. First of all, I performed a Hausman test to decide on the introduction of random or fixed effects in the model. Differences in the results are significant and the test supports the suitability of country fixed effects (FE). Table (4) shows the results for the alternative models.

The first relevant conclusion we extract from table (4) is that, to the previous finding that countries trading more in capital goods are more diversified, we can add that countries with a faster growth in this kind of trade experience a slower growth in diversification. The coefficient produced by the theoretical model is far from those in the data, although it fits within the 95% confidence interval of model [3] and within the 99% confidence interval of models [1] and [2]. Finally, as in the cross-sectional analysis, consumption home bias and country-size are statistically non-significant.

5 Conclusions

This model is able to predict patterns for international portfolio diversification that are broadly consistent with those observed empirically in recent years. Therefore, in the context of the standard macroeconomic theory, in which this model is based, one concludes that the international home bias in investment is not a puzzle.

I develop a two-period, two-country model with ex-post perfect risk sharing. In other words, regardless of the fact that the only available assets are an international riskless bond and the shares of domestic and foreign firms, households reach perfect consumption risk sharing by combining them optimally in a portfolio. I follow Heathcote and Perri (2013)'s idea of the compatibility of the home bias in portfolio found in actual data with perfect risk sharing. I enrich the standard models in this literature with dynamic markets, making the dynamic number of firms (and varieties) endogenously determined.

The model presented here confirms that an equilibrium exists where a home-biased and constant portfolio allocation is able to provide households with perfect risk sharing, i.e., the equity puzzle is not such a puzzle. It shows that the terms of trade do play an important role in neutralizing the effects of country-specific shocks on relative consumption, as Cole and Obstfeld (1991)'s labour economy found. However, they are not able to offset the disturbances to investment. One needs to diversify assets to control for such disturbances.

There are two main contributions of this analysis. First, it highlights the fact that technological requirements (equivalent to the share of imports) in the production of physical capital goods determine the level of diversification. Preferences in consumption or consumption market openness becomes relevant only for asymmetric-size countries and its relevancy is tight to differences in population sizes and GDP per capita. Therefore, it is necessary to distinguish between the preferences of demand on capital -driven by the technology- and those on consumption goods. Previous literature does not identify it. Second, I investigated the role of the endogenous number of firms or varieties in the determination of the portfolio allocation. I find that these two endogenous variables are completely independent when the economy has flexible determination of prices, i.e., in the long run, and countries are symmetric.

Finally, the ability of the model to explain the patterns of international diversification is tested. An empirical analysis of a cross-section of 24 OECD countries shows a clear positive relationship between the home bias in capital goods and the home bias in aggregate portfolios. The model is able to match the correct sign, although it overestimates the effect of imports share in capital on diversification. Furthermore, it correctly predicts the link between the constant and the slope present in the actual data in the tested regression. The panel data analysis suggests that countries with a faster growth in capital market openness experience a slower growth in portfolio diversification. The theoretical model predicts the same negative relationship.

Therefore, although market imperfections that limit or distort capital flows explain the structure of international investment partially, this paper shows that even in a frictionless decentralized symmetric-country world with perfect financial integration, households find optimal to hold portfolios with large shares of domestic equities.

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A Appendix

A.1 Discount factor of future dividends

Under our notation, this discount factor, which is the intertemporal rate of substitution between the consumption in periods t and $t+1$ is

$$Q_{t,t+1} = \frac{1}{1+i_t} = \beta E_t \left(\frac{P_t C_t}{P_{t+1} C_{t+1}} \right) = \beta E_t \left(\frac{\mu_t}{\mu_{t+1}} \right). \tag{51}$$

A.2 Labour Demand

The f.o.c. for home firms was

$$\ell_t (h) = \left[\frac{w_t}{A_{H,t}} \frac{1}{\theta} \frac{1}{\phi_t} \right]^{\frac{1}{\theta-1}}. \tag{52}$$

I use the technology restriction to get the Lagrangian multiplier $\phi_t = p_t (h) \frac{\sigma-1}{\sigma}$ so that

$$\ell_t (h) = \left[\frac{\sigma-1}{\sigma} \frac{A_{H,t}}{w_t} p_t (h) \right]^{\frac{1}{1-\theta}}. \tag{53}$$

Households supply an elastic amount of labour. It increases with the increment of the returns to scale of their effort, i.e. the higher θ is, the more productive labour is, and the higher the willingness to work. labour supply goes up for higher levels of $A_{H,t}$, the productivity of technology, and for higher prices, because, in this case, they need more income to be able to consume the same amount of goods. Finally, given prices, they supply less labour when wages are high.

A.3 Price indexes

The price indexes can be rewritten as

$$P_t = \frac{P_{H,t}}{\Gamma}\left(\frac{\left[\gamma L + (1-\gamma^*) L^*\right]\left[Y_{H,t} - n_{t+1}K_{H,t} - n^*_{t+1}K^*_{H,t}\right]}{\left[(1-\gamma) L + \gamma^* L^*\right]\left[Y_{F,t} - n^*_{t+1}K^*_{F,t} - n_{t+1}K_{F,t}\right]}\right)^{1-\gamma}; \quad (54)$$

$$P^*_t = \frac{P_{H,t}}{e_t\Gamma^*}\left(\frac{\left[\gamma L + (1-\gamma^*) L^*\right]\left[Y_{H,t} - n_{t+1}K_{H,t} - n^*_{t+1}K^*_{H,t}\right]}{\left[(1-\gamma) L + \gamma^* L^*\right]\left[Y_{F,t} - n^*_{t+1}K^*_{F,t} - n_{t+1}K_{F,t}\right]}\right)^{\gamma^*}. \quad (55)$$

As a consequences, also the real exchange rate depends on population sizes,

$$RER = \frac{\Gamma^*}{\Gamma}\left(\frac{\left[\gamma L + (1-\gamma^*) L^*\right]\left[Y_{H,t} - n_{t+1}K_{H,t} - n^*_{t+1}K^*_{H,t}\right]}{\left[(1-\gamma) L + \gamma^* L^*\right]\left[Y_{F,t} - n^*_{t+1}K^*_{F,t} - n_{t+1}K_{F,t}\right]}\right)^{1-\gamma-\gamma^*}.$$

Instead, if countries are symmetric, the price indexes are

$$P_t = \frac{P_{H,t}}{\Gamma}\left(\frac{Y_{H,t} - n_{t+1}K_{H,t} - n^*_{t+1}K^*_{H,t}}{Y_{F,t} - n^*_{t+1}K^*_{F,t} - n_{t+1}K_{F,t}}\right)^{1-\gamma}; \quad (56)$$

$$P^*_t = \frac{P_{H,t}}{e_t\Gamma^*}\left(\frac{Y_{H,t} - n_{t+1}K_{H,t} - n^*_{t+1}K^*_{H,t}}{Y_{F,t} - n^*_{t+1}K^*_{F,t} - n_{t+1}K_{F,t}}\right)^{\gamma^*}. \quad (57)$$

For $\gamma = 1/2$, households' preferences are identical, and the real exchange rate P/P^* equals 1. In other words, purchasing power parity holds.[30] For $\gamma \neq 1/2$, instead, home bias in consumption implies that the real exchange rate (RER) is not constant, but changes with the terms of trade:

$$RER_t = \frac{P_t}{e_t P^*_t} = \left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-2\gamma}. \quad (58)$$

With perfect risk sharing, it follows that the ratio between consumption levels is also equal to RER.

$$\frac{C^*_t}{C_t} = \left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-2\gamma}. \quad (59)$$

Moreover, the specific price indexes for capital goods can be written as follows:

$$P_{K,t} = \frac{(P_{H,t})^{\delta}(P_{F,t})^{1-\delta}}{\Gamma_{\delta}}, \; P^*_{K,t} = \frac{\left(P^*_{H,t}\right)^{1-\delta^*}\left(P^*_{F,t}\right)^{\delta^*}}{\Gamma_{\delta^*}}, \quad (60)$$

where $\Gamma_{\delta} = \delta^{\delta}(1-\delta)^{1-\delta}$.

[30] This is the case in Cole and Obstfeld (1991).

A.4 Allocation of firms

Firms are allocated in the home or the foreign country based on the free entry conditions (FECs). These conditions provide us with a system of two difference equations to solve for n and n^{*}. At Home, the FEC is

$$P_{K,t}K_{t}=P_{H,t}K_{H,t}+P_{F,t}K_{F,t}=E_{t}Q_{t,t+1}\pi_{t+1}\left(h\right). \tag{61}$$

After some algebra, one finds a system of two non-linear differentiated equations on n and n^{*},

$$\begin{array}{c} n_{t+1}\left[K_{H,t}\dfrac{Y_{t}(h)^{\frac{1}{\theta}-1}}{A_{H,t}^{\frac{1}{\theta}}}\left(1-\beta\left(1-\dfrac{\sigma-1}{\sigma}\theta\right)\right)+K_{F,t}\dfrac{Y_{t}(f)^{\frac{1}{\theta}-1}}{e_{t}A_{F,t}^{\frac{1}{\theta}}}\right]=\beta\left(1-\dfrac{\sigma-1}{\sigma}\theta\right)\theta\dfrac{\sigma-1}{\sigma\kappa} \\ E_{t}\left[\begin{array}{c} L_{t+1}\gamma+L_{t+1}^{*}\left(1-\gamma\right)\left[\frac{A_{F,t+1}}{A_{H,t+1}}\right]^{\frac{1}{\theta}}\left[\frac{Y_{t+1}(h)}{Y_{t+1}(f)}\right]^{\frac{1}{\theta}-1}\frac{n_{t+1}}{n_{t+1}^{*}}+ \\ +\frac{1}{\theta}\frac{\sigma\kappa}{\sigma-1}\frac{Y_{t+1}(h)^{\frac{1}{\theta}-1}}{A_{H,t+1}^{\frac{1}{\theta}}}n_{t+2}^{*}K_{H,t+1}^{*} \end{array}\right] \end{array} \tag{62}$$

and, symmetrically,

$$\begin{array}{c} n_{t+1}^{*}\left[K_{F,t}^{*}\dfrac{Y_{t}(f)^{\frac{1}{\theta}-1}}{A_{F,t}^{\frac{1}{\theta}}}\left(1-\beta\left(1-\dfrac{\sigma-1}{\sigma}\theta\right)\right)+K_{H,t}^{*}\dfrac{Y_{t}(h)^{\frac{1}{\theta}-1}}{A_{H,t}^{\frac{1}{\theta}}}e_{t}\right]=\beta\left(1-\dfrac{\sigma-1}{\sigma}\theta\right)\theta\dfrac{\sigma-1}{\sigma\kappa} \\ E_{t}\left[\begin{array}{c} L_{t+1}^{*}\gamma+L_{t+1}\left(1-\gamma\right)\left[\frac{A_{H,t+1}}{A_{F,t+1}}\right]^{\frac{1}{\theta}}\left[\frac{Y_{t+1}(f)}{Y_{t+1}(h)}\right]^{\frac{1}{\theta}-1}\frac{n_{t+1}^{*}}{n_{t+1}}+ \\ +\frac{1}{\theta}\frac{\sigma\kappa}{\sigma-1}\frac{Y_{t+1}(f)^{\frac{1}{\theta}-1}}{A_{F,t+1}^{\frac{1}{\theta}}}n_{t+2}K_{F,t+1} \end{array}\right] \end{array} \tag{63}$$

Although an analytical solution for n and n^{*} cannot be provided, it is worth noticing that the expressions above do not depend on λ at all. Hence, the decision on the allocation of plants of production is completely disconnected from the decision on the ownership of firms made by agents in the home and foreign countries. Therefore, it follows that the dynamics of markets do not invalidate the main result, i.e., constant biased λ, found in a perfectly competitive world.[31]

A.5 Stability condition

In aggregate,

$$Y=\left[\int_{0}^{n}n^{-\theta\rho}dn\right]^{\frac{1}{\rho}},$$

[31] Coeurdacier (2009) shows that the logarithmic preferences are not a crucial assumption for the home biased portfolio result. However, the logarithmic functional form makes the need of exchange rate risk diversification disappear (see Adler and Dumas (1983)). Both papers discuss this result for the case of a partial equilibrium with nominal rigidities and local currency pricing. In a general equilibrium context, however, a recent paper by Coeurdacier and Gourinchas (2011) show that optimal equity positions coincide with the equity positions of a log-investor who doesn't care about hedging the real exchange rate risk. Although this paper limits its analysis to the flexible price case and, hence, this discussion is beyond its target, one should be aware of it.

where $\rho = \frac{\sigma}{\sigma-1}$. Thus,

$$Y = n^{\frac{1-\theta\rho}{\rho}}.$$

To ensure stability, the first derivative of output with respect to the number of firms should be positive and the second, negative. Here,

$$\frac{\partial Y}{\partial n} = \frac{1-\theta\rho}{\rho} n^{\frac{1-\theta\rho-\rho}{\rho}},$$

which is positive for

$$1 > \theta \frac{\sigma-1}{\sigma}.$$

The above is always true, as $\theta \in (0,1)$ and $\frac{\sigma-1}{\sigma} < 1$. Moreover,

$$\frac{\partial^2 Y}{\partial n^2} = \underbrace{\frac{1-\theta\rho}{\rho}}_{>0} \underbrace{\frac{1-\theta\rho-\rho}{\rho}}_{¿\text{sign}?} n^{\frac{1-\theta\rho-2\rho}{\rho}}.$$

Hence,

$$1-\theta\rho-\rho < 0$$

is needed. The above is also always true, as

$$\begin{aligned} 1-\theta\frac{\sigma}{\sigma-1} - \frac{\sigma}{\sigma-1} &< 0 \\ 1 &< \frac{\sigma}{\sigma-1}(\theta+1) \\ \frac{\sigma-1}{\sigma} - 1 &< \theta \\ -\frac{1}{\sigma} &< \theta \end{aligned}$$

and σ and θ are both positive. The stability condition ensures the non-existence of increasing returns to scale in investment, which would make the model explosive.

A.6 Algebraic details to find equation (41)

In order to derive equation (68) from the aggregate budget constraints (64) and (65),

$$P_t C_t = w_t \ell_t + (1-\lambda) n_t \pi_t(h) + \lambda e_t n_t^* \pi_t^*(f) - (1-\lambda) I_{H,t} - e_t \lambda I_{F,t}^*, \tag{64}$$

$$P_t^* C_t^* = w_t^* \ell_t^* + \frac{1}{e_t}\lambda \Pi_{H,t} + (1-\lambda)\Pi_{F,t}^* - \frac{1}{e_t}\lambda I_{H,t} - (1-\lambda) I_{F,t}^* \tag{65}$$

first, we must substitute the expressions for profits and labour income as a function of GDP (eq. 28 and 29). Therefore,

$$P_t C_t = \frac{\sigma-1}{\sigma}\theta P_{H,t}Y_{H,t} + (1-\lambda)\left(1-\frac{\sigma-1}{\sigma}\theta\right)P_{H,t}Y_{H,t} + \lambda e_t\left(1-\frac{\sigma-1}{\sigma}\theta\right)P^*_{F,t}Y_{F,t} - (1-\lambda)I_{H,t} - \lambda e_t I^*_{F,t} + (1+i_t)B_t - B_{t+1} \tag{66}$$

and

$$P^*_t C^*_t = \frac{\sigma-1}{\sigma}\theta P^*_{F,t}Y_{F,t} + \lambda\frac{1}{e_t}\left(1-\frac{\sigma-1}{\sigma}\theta\right)P_{H,t}Y_{H,t} + + (1-\lambda)\left(1-\frac{\sigma-1}{\sigma}\theta\right)P^*_{F,t}Y_{F,t} - \lambda\frac{1}{e_t}I_{H,t} - (1-\lambda)I^*_{F,t} + (1+i^*_t)B^*_t - B^*_{t+1}, \tag{67}$$

where $P_{H,t}Y_{H,t}$ is the nominal domestic output ($\mathbb{Y}$); $P^*_{F,t}Y_{F,t}$ is the foreign output ($\mathbb{Y}^*$); and I is the investment of the current period. Then, by taking cross-country differences,

$$\triangle\mathbb{C} = \triangle\mathbb{Y}\left[\frac{\sigma-1}{\sigma}\theta + (1-2\lambda)\left(1-\frac{\sigma-1}{\sigma}\theta\right)\right] - \triangle\mathbb{k}(1-2\lambda) + ((1+i_t)B_t - B_{t+1}) - e_t\left((1+i^*_t)B^*_t - B^*_{t+1}\right). \tag{68}$$

We can the latter equation in equation (40) and set the gross and net holding of bonds identically equal to zero, $B=0$. I obtain equation (41):

$$\triangle\mathbb{C} = \left(1+2\lambda\left(\frac{\sigma-1}{\sigma}\theta - 1\right)\right)(2\delta-1)\triangle\mathbb{k} - (1-2\lambda)\triangle\mathbb{k}.$$

A.7 Asymmetric countries

A.7.1 Different preferences and technologies

This section considers an extension of the model where preferences in consumption and capital intensity in Home and Foreign goods are asymmetric between countries, whereas population sizes are equal ($L = L^* = 1$). Let us assume that $\delta^* = \delta + \epsilon$ and $\gamma^* = \gamma + \tau$. Then, equation (39) in the text becomes

$$\triangle\mathbb{Y} = (2\gamma-1)\triangle\mathbb{C} + (2\delta-1)\triangle\mathbb{k} - 2e_t n^*_{t+1}P^*_{k,t}K^*_t\epsilon - 2e_t P^*_t C^*_t L^*\tau \tag{69}$$

or,

$$\triangle\mathbb{Y} = (2\gamma-1)\triangle\mathbb{C} + (2\delta-1)\triangle\mathbb{k} - 2(\delta^*-\delta)e_t n^*_{t+1}P^*_{k,t}K^*_t - 2(\gamma^*-\gamma)e_t P^*_t C^*_t L^*. \tag{70}$$

Proceeding as in the symmetric case, one has

$$\triangle\mathbb{Y} = (2\gamma - 1)\triangle\mathbb{C} + (2\delta - 1)\triangle\mathbb{k} \quad (71)$$
$$-2(\delta^* - \delta) e_t n^*_{t+1} P^*_{k,t} K^*_t - 2(\gamma^* - \gamma) e_t P^*_t C^*_t L^*,$$

$$\triangle\mathbb{Y}|_{\triangle\mathbb{C}=0} = (2\delta - 1)\triangle\mathbb{k} \quad (72)$$
$$-2(\delta^* - \delta) e_t n^*_{t+1} P^*_{k,t} K^*_t - 2(\gamma^* - \gamma) e_t P^*_t C^*_t L^*.$$

Equation (68) remains the same,

$$\triangle\mathbb{C} = \triangle\mathbb{Y}\left[\frac{\sigma-1}{\sigma}\theta + (1-2\lambda)\left(1 - \frac{\sigma-1}{\sigma}\theta\right)\right] - \triangle\mathbb{k}(1-2\lambda).$$

Combining the latter two expressions, the optimal portfolio is

$$\lambda = \frac{1 - \delta + F}{1 - \left(1 - \frac{\sigma-1}{\sigma}\theta\right)(2\delta - 1) + 2\left(1 - \frac{\sigma-1}{\sigma}\theta\right)F}, \quad \text{(asy result)}$$

where $F = \frac{\left[(\delta^*-\delta)e_t n^*_{t+1} P^*_{k,t} K^*_t + (\gamma^*-\gamma) e_t P^*_t C^*_t L^*\right]}{\triangle\mathbb{k}}$ and λ increases with F.

The optimal diversification now depends on the ratio of foreign investment ($n^*_{t+1} P^*_{k,t} K^*_t$) over country-difference investment, $\triangle\mathbb{k}$, and on the ratio of foreign consumption over country-difference investment, both weighted by the difference between the relevant preference parameters, $(\delta^* - \delta)$ and $(\gamma^* - \gamma)$ respectively. This relationships are positive when Foreign parameters, δ^* and γ^* are larger than Home counterparts and $\triangle\mathbb{k} > 0$ and negative otherwise.

This part of the portfolio shape becomes relevant when country differences in δ and γ are significant. And the portfolio is no longer constant.

A.7.2 Different elasticities of substitution

A final consideration should be made regarding another restrictive assumption. The model assumes that the elasticities of substitution between goods, σ, are identical for consumption and capital goods and regardless of being a Home or a Foreign agent. There are two reasons to keep this assumption of symmetry. The first one has already been mention: I find it useful to provide a model able to show how home bias in equities is found even for symmetric countries. Differences in σ may surely explain part of the portfolio allocation, but this would be a different explanation that I leave for future research. The second reason is a technical one. It is not possible to find an analytical solution for this extension of model or for its corresponding optimal portfolio. Therefore, using it would leave a large part of the explanation in a black box.

Notice that the introduction of different elasticities would imply: First, to have the f.o.c. for $c(f)$ in the household's problem and the f.o.c. for $k(h)$ and $k(f)$ in the minimization problem of firms modified (substituting the now general σ for the new differentiated parameters). Second and most importantly, the definition of firm's production, $y(h)$, would become much more complicated. Therefore, the profits maximization -once you substitute the demands for $c(h)$, $c(f)$, $k(h)$ and $k(f)$- would make optimal to discriminate prices when goods are sold as consumption or capital goods and for national and foreign consumers, due to different elasticities of substitution. This makes impossible to find a simple expression for aggregate profits.

A.7.3 Fully asymmetric countries

To analyse the construction of aggregate investment portfolios for the most general case, one should relax the assumption $\lambda_H = \lambda_F$. This more complex setup requires to rely on numerical solutions and addresses a different research question. Devereux and Sutherland (2011) solution method and all the recent related literature helps to answer this question.

In this section I provide two expressions, for λ_H and λ_F, that show how portfolios would be in a general asymmetric setup. For simplicity, I assume that $(1-\gamma)L = (1-\gamma^*)L^*$, $(1-\delta)L = (1-\delta^*)L^*$ and $\lambda_{i,t} = \lambda_{i,t+1}$, where $i = H, F$.

$$\lambda_F \left\{ \frac{\Theta\left(A\frac{L}{L^*}+1\right)-J}{1-\Theta\left(B\frac{L^*}{L}+1\right)} \left[\Theta B\frac{L^*}{L}+D\right] - J \right\} = A + \left[\Theta B\frac{L^*}{L} - D\right] \left[\frac{\triangle\mathbb{Y}-\triangle\mathbb{k}}{1-\Theta\left(B\frac{L^*}{L}+1\right)}\right] - J - PC,$$

$$\lambda_H \left\{1-\Theta\left(B\frac{L^*}{L}+1\right)\right\} = \triangle\mathbb{Y}-\triangle\mathbb{k} - \lambda_F\left[\Theta\left(A\frac{L}{L^*}+1\right)-J\right],$$

where $\Theta = \left(1-\frac{\sigma-1}{\sigma}\theta\right)$, $A = \frac{P_{H,t}Y_{H,t}}{L}$, $B = e_t\frac{P^*_{F,t}Y_{F,t}}{L^*}$, $J = \frac{I_{H,t}}{L}$ and $D = e_t\frac{I^*_{F,t}}{L^*}$.