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# A decision model for stochastic optimization of seasonal irrigation-water allocation

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ABSTRACT

Optimal water allocation on a seasonal basis is generally a decision taken with uncertainty regarding seasonal crop needs (unknown yield, precipitation and other environmental factors). Decision criteria, such as “irrigating for the good years of production” and "applying a little extra water just in case it is needed by the plant", are consistent with the rational behaviour of stochastic profit maximization. The motivation behind an increase in water allocation (acquiring water rights or reserving water for certain crops) is that of self-protection: it is better to maintain an extra allocation of water than to face potential yield losses due to water constraints on production in those years when potential yields exceed average levels. The stochastic optimization model presented herein is applied to maize in Spain showing that in current economic and technical conditions, the optimal stochastic water allocation under yield uncertainty is 10% higher than the irrigation dose required under certainty (historical average yield), which leads to an 8% higher expected profit than that obtained for an average-yield water application.

## 1. Introduction

Irrigation accounts for 70% of total global water withdrawals (Alexandratos and Bruinsma, 2012; Pereira and Marques, 2017) and is predicted to rise by up to 80% in many river basins and aquifers in arid regions by 2030. The reduction of water withdrawals together with an increase in efficiency in water use constitute a priority in many developed and developing regions, with metering and water pricing employed as the principal instruments to achieve the policy goal of sustainable water use (World Bank-OECD, 2018).

Water pricing has featured in the policy agenda of many institutions, such as the OECD, the World Bank, and the European Union, as an economic instrument towards inducing water saving, and increasing productivity by inducing reallocation from low- to high-value uses, as well as towards preventing wasteful water practices, while simultaneously increasing financial resources for the operation of water services. Specifically, the Water Framework Directive (European Commission, 2000) promotes the use of water pricing and the recovery of all costs. Nevertheless, the effectiveness of water pricing is subject to the implementation of volumetric metering and reliable governance systems, and the impact on water saving is related to the elasticity of demand. Scheierling et al. (2006) carried out a meta-analysis of price elasticity of agricultural water demand and reported a mean elasticity of − 0.48 with higher values for high-value crops. More recently, Zuo et al. (2015) found demand elasticity for high-security entitlements to water rights in Australia at approximately − 0.57, which lies in the lower range of elasticities reported in Australia (from −0.52 to −1.9). Therefore, estimation results show that the effectiveness of water pricing remains very limited, especially in the case of high-value crops with very inelastic water demands (Berbel and Expósito, 2020).

Low price elasticities of water irrigation demand help to explain over-allocation (holding water resources for the next campaigns) and water over-use (applying water in excess to crop needs) by farmers. This research focuses on ex-ante over-allocation understood as water acquired or reserved to be applied to the crop during the growing stage in the attempt to minimize potential yield losses. Decision criteria for the over-allocation of water, such as “irrigating for the good years of production” and "applying a little extra water just in case it is needed by the plant", are consistent with the expected behaviour of profit maximization. The motivation behind over-allocation is that of self-protection against yield uncertainty, in that it is deemed better to support an excessive water cost than to face potential yield losses due to water

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constraints on production in those years when potential yields exceed average levels.

Uncertainty constitutes a factor in almost every aspect of irrigation and agricultural activity, as driven by unpredictable climate conditions, yield variability, prices and changing economic and policy conditions. Mathematical programming applications to consider this uncertainty have seen a significant development in the last decade, including stochastic, inexact, fuzzy, and interval-based programming (Archibald and Marshall, 2018). As argued by Linker (2021), stochastic approaches constitute the most adequate instrument to explain risk-adverse behaviour in famers' decision making and are therefore more likely to be of practical interest. Nevertheless, these techniques have been mostly applied to uncertainty in climate conditions and water availability, with the aim to offer optimized irrigation schedules (Anvari et al., 2017; Pereira and Marques, 2017; Linker, 2021; Yan et al., 2021). Additionally, most stochastic water-allocation decision support models applied in the literature either come from mathematical programming with mean-variance or similar approaches, or apply stochastic dynamic programming to irrigation scheduling in order to decide water application according to varying rain, temperature, crop growing stage, etc, though not considering yield uncertainty. To the best of our knowledge there is not a similar model for water allocation as the one presented in this paper and literature has not yet explored this type of farmer behaviour regarding irrigation water management.

A greater use of inputs under production uncertainty is compatible with previous analyses of the impact of risk on the demand of agricultural inputs. Several authors, such as Sandmo (1971), Anderson et al. (1977), and Just and Pope (1979), have documented the impact of uncertainty in input and output prices. They indicate that uncertainty in prices and the exact elicitation of the production function reduces input usage following a line of reasoning like that of Magnusson (1969). Magnusson develops a model for risk-aversion behaviour when the production function is subject to uncertainty and concludes that factor demand is defined by the point when expected factor marginal productivity is equal to its "price minus a marginal risk deduction", and thus input use would be lower than in a certainty context. This result assumes that the manager is a risk averter and that the co-variance between the output and the input is positive. However, most of the theoretical models listed above refer to price uncertainty and input productivity, while the model presented in this paper assumes risk neutrality, the production function is known, and the stochastic variable is the maximum achievable yield.

In summary, this paper aims to explore the optimal strategy for allocation of seasonal water quota to an annual crop that maximizes stochastic profit under yield uncertainty. Our model has a 'normative' nature, aiming to maximize stochastic profit and offer a decision support model. The hypothesis that farmers use a similar optimizing strategy requires further research, which is out of this paper's scope. Nevertheless, we believe that our model may serve as an illustration for policy making regarding impact of yield uncertainty in the potential over-allocation of water. Water allocation is defined in this work as the decision to assign a certain volume of water in either the long term (over several years) or at seasonal level (for the current year). Specifically, this paper explores the microeconomic implications of the potential over-allocation of irrigation water based on an agro-economic model, whereby no assumptions are made regarding the stochastic distribution function of yields, and the risk attitude of farmers is taken as neutral for the simplicity of this research.

The rest of the paper is structured as follows. The following section presents the agro-economic model in a step-by-step manner. The third section shows the results of the application of the proposed model to a specific case in a water-abundant context. Finally, a brief discussion and concluding remarks are provided.

## 2. Model

Our model is based upon the well-known Food and Agriculture Organization of the United Nations (FAO) model that can be found in Steduto et al. (2012). This model has shown to illustrate in a very accurately way the crop responses to water and evapotranspiration. Additionally, this model is used in the majority of crop growth models (e.g. AQUACROP) and there is a wide consensus in the agronomic field about its use in the case of herbaceous crops, which is the focus of this paper. In the literature we can find some empirical water response functions in quadratic or other functional forms for olives (Vita Serman et al., 2021) or almonds (Moldero et al., 2021), but they are mainly used in the case of perennials (orchards).

The proposed model is presented in a step-by-step manner to facilitate readers' comprehension and to introduce factors, such as expected yield, that represent the actual situation faced by the farmer in the field with water-allocation decisions. To this end, the model considers initially constant irrigation efficiency (E = 1), though the efficiency will be later defined as a parameter (e.g., E = 0.8) as illustrated in Section 3. Additionally, the model considers the water-yield response function (demand function) under alternative stochastic distribution functions, thereby analysing the impacts of yield uncertainty on these various alternative function specifications.

### 2.1. The agronomic model with constant irrigation efficiency

The response of any crop to water supply has been well established in agronomy science. Yield response to crop evapotranspiration (*ET*) may be expressed as defined by Doorenbos and Kassam (1979):

$$\left(1-\frac{Y}{Y_m}\right)=K_y\left(1-\frac{ET}{ET_m}\right) \tag{1}$$

where $Y_m$ and $Y$ are the respective maximum and actual yields, $ET_m$ and $ET$ are the respective maximum and actual evapotranspiration, and $K_y$ is a yield response factor representing the effect of a reduction in evapotranspiration on yields. A complete review of the present knowledge regarding $K_y$ coefficients and crop response to water availability can be found in Steduto et al. (2012).

*ET* can be calculated as:

$$ET=R+E{\cdot}W \tag{2}$$

where R is the effective rainfall plus the variations in soil water storage during the crop growing cycle (mm), W is the applied irrigation (mm), and E is the irrigation efficiency (dimensionless), understood as the irrigation water that is stored in the soil ready to be evapotranspired by the crop divided by the applied irrigation.

When Eqs. (1) and (2) are combined, the result is:

$$\left(1-\frac{Y}{Y_m}\right)=K_y\left(1-\frac{E{\cdot}W+R}{W_m+R}\right) \tag{3}$$

where $W_m$ is the irrigation requirement for a maximum yield (also known as full irrigation).

This equation gives a response to water with a complex relation. Specifically, we will focus on the production function segment that is close to the maximum yield (full irrigation) and hence we may assume a constant irrigation efficiency. As subsequently discussed, the consideration of constant efficiency does not significantly affect the results. Therefore, we have following expressions:

$$\frac{Y}{Ym}=1-K_y\left(1-\frac{E{\cdot}W+R}{W_m+R}\right)=1-K_y+K_y\frac{R}{W_m+R}+K_y\frac{E{\cdot}W}{W_m+R} \tag{4}$$

This conducts to.

$$Y=Y_m\left(1-K_y+K_y\frac{R}{W_m+R}\right)+Y_mK_y\frac{E{\cdot}W}{W_m+R} \tag{5}$$

For the sake of simplicity, to derive solutions, the first and second terms are substituted with parameters $a_0$ and $a_1$:

$$Y = a_0 + a_1 W \tag{6}$$

$$a_0 = Y_m\left(1 - K_y + K_y\frac{R}{W_m + R}\right) \tag{7}$$

$$a_1 = \frac{Y_m\,K_y\,E}{W_m + R} \tag{8}$$

The interpretation of these parameters is shown in Fig. 1.

$$a_0 = rainfed\ (non - irrigated)\ yield;$$

$$a_1 = marginal\ physical\ water\ productivity$$

These parameters are constant for each crop as they depend on specific crop and location characteristics and are known 'ex-ante' by farmers. Although water is obviously not the only factor related to yield (e.g., fertilizing and other environmental conditions also affect final yield), our proposed model focuses on water as production factor, thus assuming that other factors are applied at an optimal level. This assumption can be realistic in contexts where water is the most limiting production factor since other inputs (e.g., fertilizers) are usually available in the market.

The actual yield may be equal to $Y_m$ when applied water is equal to or greater than full irrigation ($W_m$), but it can be lower according to the linear production function when applied water (W) is lower than the irrigation requirement for a maximum yield ($W_m$). Therefore, Eq. (9) represents our production function.

$$Y = \min(a_0 + a_1(W); Y_m) \tag{9}$$

In the case that the farmer moves in a world of certainty, then the value of the maximum attainable yield $Y_m$ is known and defined by Eq. (10) and represented as a continuous line in Fig. 1.

$$Y_m = a_0 + a_1(W_m) \tag{10}$$

### 2.2. Water demand model

In a certainty context under the assumption of a linear plateau production function as assumed by the standard agronomic model, the response is illustrated by Fig. 1 where the optimization problem implies that the solution to maximum yield is reached exactly when water utilization reaches $W_m$ (considering that irrigation efficiency equals 1).

Optimal water use is obtained by maximizing profit ($\pi$) as shown in Eq. (11) where 'c' is the water cost and 'p' the price of crop.

$$\pi = p \cdot Y(W) - c\,W \tag{11}$$

One consequence of the linear nature of the crop response is the solution to this equation that gives the optimal irrigation water as zero or $W_m$, as stated by Eq. (12).

$$W(\text{year'j'})\left\{\begin{array}{l} W_j = \dfrac{Y_{mj} - a_0}{a_1}\ \text{if}\ p \cdot a_1 > c \\ \quad 0,\ otherwise \end{array}\right\} \tag{12}$$

Therefore, in a certainty context with linear response (E = 1) when the marginal value of water (as given by $'p \cdot a_1'$) is greater than the cost 'c', the optimal solution implies that farmers apply the exact water dose ($W_j$) to reach the maximum yield ($Y_{mj}$). This solution implies perfect knowledge of the production function, the net available rain 'R' and yield certainty, hypothesis that are unlikely to be present in real decision context. Nevertheless, the proposed 'ex-ante' model assumes average values of parameters (i.e., $Y_m$, $K_y$, R, p, c) as the most likely values for the profit function formulation, being yield Y and W the only variables in the model. The linear nature of the production function implies that water beyond optimum will have a null marginal productivity and a negative marginal value (defined by 'c').

### 2.3. The agro-economic model under yield uncertainty

In the real world, maximum potential yield ($Y_{m,j}$) varies from yearly 'j' to year 'k' and from farmer to farmer due to uncontrollable factors linked to the weather such as temperature, humidity, and radiation, pests, and decision mistakes, among others. This is represented in Fig. 2 by the dotted lines above and below the continuous line that represents our production function where the deterministic maximum yield (named $Y_a$) is now defined by the historical average maximum yield ($\mu_m$) and converted into the stochastic maximum potential yield for year 'j' ($\widehat{Y_{mj}}$), as represented in Eq. (13).

$$\hat{Y}_{mj} = \mu_m + \varepsilon \tag{13}$$

$\widehat{Y_{mj}}$ stands for the 'ex-post' potential maximum yield (year 'j') subject to full irrigation for the year (decision variable 'W'). This potential maximum yield (as expected by the farmer for next year) can be higher or lower than historical average depending upon the growing conditions. When growing conditions are favourable, an upper yield level ($Y_h$) may be reached; in contrast, when growing conditions are adverse, a lower yield level ($Y_l$) is reached, as shown in Fig. 2.

Fig. 2 is adapted from the Babcock (1992) who modelized nitrogen application under yield uncertainty (uniform yield distribution). Fig. 2 aims to represent the yield response under varying water allocation (which is the limiting factor in our specific case). Let us imagine that farmer always expect average yield ($Y_a$) and consequently always allocates average water ($W_a$), though the real outcome for the current

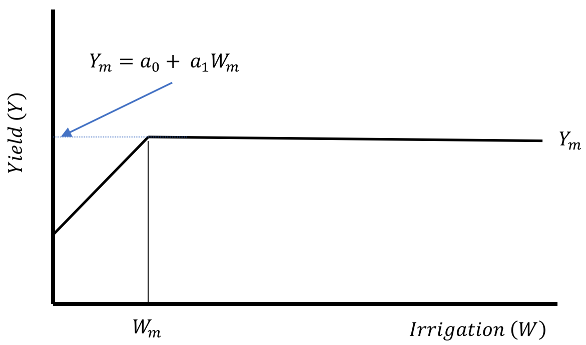


**Fig. 1.** Deterministic water response function (being $Y = a_0 + a_1 W; a_0 = Y_m\left(1 - K_y + K_y\frac{R}{W_m+R}\right); a_1 = \frac{Y_m K_y E}{W_m+R};\ K_y = crop\,coefficient; W = applied\,water; R = effective\,rail; Y_m = Maximum\,yield; W_m = Irrigation\,needs\,for\,maximum\,yield; E = 1$).

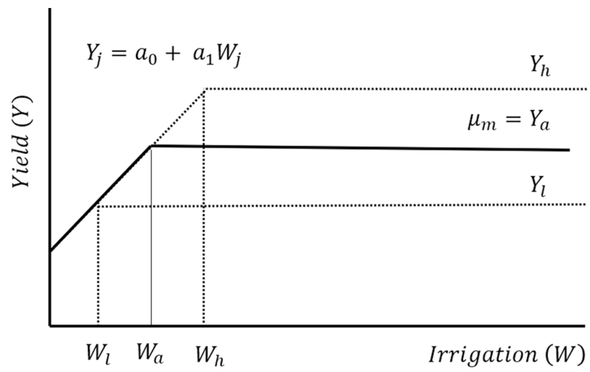


**Fig. 2.** Stochastic water response function where $Y_h$= maximum ex-ante potential yield; $Y_l$= minimum ex-ante potential yield; $\mu_m = Y_a$ = historical average yield; $W_h$; $W_a$; $W_l$, water required respectively for reaching the yield $Y_h$, $Y_a$, $Y_l$.

campaign might be either lower ($Y_l$) or higher ($Y_h$). When ex-post yield is lower ($Y_l$), the required water dose would be lower too, as given by '$W_l$'. Thus, when farmer applies the historical average dose ($W_a$) there would be an over-allocation of water ($W_a$ - $W_l$) and an economic loss (as given by the excess in applied water and its cost [c·($W_a$ - $W_l$)]. On the contrary, when the ex-post yield is higher ($Y_h$) and farmer applies the average historical dose ($W_a$) (instead of the required 'ex post' higher dose '$W_h$'), the farmer would face an "opportunity cost" (achieved yield will be $Y_a$, below the potential $Y_h$). This result implies a loss of potential profit, as represented by [p·($Y_h$-$Y_a$)-c·($W_h$-$W_a$)].

It is worth noting that, in the model, all parameters are defined and exogenous: (i) crop response to water ($K_y$), (ii) irrigation system efficiency (e.g., E = 1) and (iii) rainfed conditions (R), which are assumed constant in our model. Our interest focuses on the decision variable 'W' with the aim to maximize expected profit. Thus, with 'D' as the optimal water allocation in year 'j' (known 'ex-post' and obtained from Eq. (12)) and with 'W' (ex-ante water allocated by the farmer) as the decision variable, the equation of expected profit could be obtained. The expected profit expression is given by Gallego and Moon (1993) where the value of production is equal to the minimum of the value of the water used (W, decision variable) or the stochastic value of optimal water use that year (D, stochastic variable) determined by the stochastic yield. Thus, profit can be represented by Eq. (14):

$$\pi(W) = p\,a_1\,E(min\{W,D\}) - c_w W \tag{14}$$

This decision scenario is like the inventory management decision problem known as 'newsvendor' where there is a shortage penalty case when ex-post potential yield for year 'j' ($Y_{mj}$) requires a larger volume of water than the allocated ex-ante by the farmer and therefore there is a loss of profit due to the below-optimum supply of water. The newsvendor problem constitutes a classic problem in the literature on inventory management (Arrow et al., 1951).

Regarding Eq. (14), it is worth noting that it assumes that marginal value of production is positive [p·dY(W)/dW > c], implying that irrigation is profitable and greater than 'zero' (see Eq. (12)).

Let $f(D)$ be the probability density function (PDF) of D, whereby $F(D) = Prob(D \le W) = \int_0^x f(D)dD$ is the cumulative distribution function (CDF) of D. It is assumed that $f(D)$ *is continuous in* $[0,\infty]$ in the following proof. Therefore:

$$E[\min(D,W)] = \int_0^\infty \min(w,D)f(D)dD = \int_0^w Df(D)dD + W\int_w^\infty f(D)dw \tag{15}$$

$$\pi(W) = (pa_1 - c_w)\left(\int_0^w Df(D)dD + W \times \int_w^\infty f(D)dD\right) - c_w\int_0^w (W-D)f(D)dD \tag{16}$$

To find the maximum of $\pi(W)$, a 'W' is required that satisfies $\frac{d\pi(W)}{dW} = 0$. By using the fundamental theorem of calculus:

$$\frac{d\pi(W)}{dW} = (pa_1 - c_w)\int_w^\infty f(D)dD - c_w\int_0^w f(D)dD$$

$$= (pa_1 - c_w)(1 - F(W)) - c_w F(W) \tag{17}$$

By setting $\frac{d\pi(W)}{dW} = 0$, it is found that W* must satisfy Eq. (18) as given by:

$$F(W^*) = \frac{p\cdot a_1 - c_w}{p\cdot a_1} \tag{18}$$

The solution in Eq. (17) is known as the "critical fractile" for the well-known mathematical newsvendor problem formulated by Arrow et al. (1951). In our specific case, this ratio balances the cost of irrigation below optimum (lost income) versus the costs of excess irrigation (water cost).

## 3. Results

Using the software *Mathematica*, this section applies the model to a specific case in a water abundant context, such as that of irrigated maize in certain regions of Spain. The following parameters are based official statistics of the Spanish Ministry of Agriculture (years 2008–2018), and Ky is taken from Steduto et al. (2012).

$\mu_y = 10,000;$
$K_y = 1.25;$

$ETmax = 800\ mm;$
$R = 250\ mm;$
$w_m = 550\ mm$

Assuming $E = 0.8$, as representative value of sprinkler system efficiency (most frequent system for maize cultivation in Spain), the values of $a_0$ and $a_1$ are:

$$a_0 = 1,406\frac{kg}{ha};\ a_1 = 12.50\frac{kg}{mm\cdot ha}$$

The proposed model is tested by using various stochastic distributions of the demand function to explore the effect of yield variability in optimal irrigation water use.

$Uniform\ Yield\ (9000,\ 11000);$

$Triangular\ (9000;\ 10000;\ 11000);$

$Beta\ (\alpha = 1; \beta = 0.6; 9000; 11000);$

$N(10000;\ 1000)$

Additionally, the water demand function for a water cost in the range $(0; pa_1)$ has been estimated. Assuming a price of 0.20 EUR/kg of maize (average value in the period 2008–2018) and parameterizing water cost 'c', Fig. 3 shows the demand function for irrigation under the alternative stochastic distributions compared to the "certainty decision" based on historical average yield (vertical dotted line). Constant efficiency is assumed (E = 0.8), which can be realistic when irrigation approaches the maximum yield.

Fig. 3 illustrates the impact of various probability distributions on water demand. Beta, triangular and uniform distributions show a similar behaviour meanwhile normality assumption leads to a higher over-allocation. In this respect, there is strong evidence against assuming normality in the yield distribution (Just and Weninger, 1999).

The cost of irrigation in the regions producing maize in Spain usually lies within the range of 0.02–0.05 EUR/$m^3$ (0.20–0.50 EUR/mm·ha) (MIMAM, 2008). Fig. 3 shows that water allocation in the normal range of prices for this crop (maize), between 0 and 1,0 EUR/$m^3$, will give an

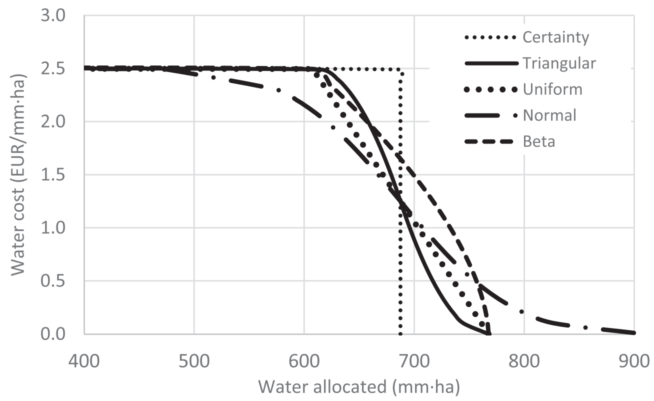


**Fig. 3.** Irrigation water demand for stochastic yield distribution (crop price 0.2 EUR/kg; Average yield=10,000 kg/ha; $K_y$ =1.25; R=250 mm; E = 0.8).

optimal stochastic allocation above the "certainty decision based on historical average". Table 1 summarizes the model results compared to certainty conditions.

Table 1 shows that assuming parameters given in Fig. 3 (crop price = 0.2 EUR/m$^3$, average yield = 10,000 kg/ha; water cost = 0.5 EUR/m$^3$), using the beta distribution function as reference and assuming a water cost of 0.05 EUR/m$^3$, farmer optimal allocation of water will be 757 mm, versus the "certainty" value based upon historical average yield (688 mm). This overallocation implies that in 80% of the years (probability defined by the critical fractile $[(p{\cdot}a_1 - c)/(p{\cdot}a_1)]$) allocated water will be above the required dose. Expected profit will be 8% over the result when applied water is the average-yield dose. When water is considered a limiting factor, results confirm that farmer's rational behaviour would be to irrigate 'for the good years' as the expected profit would significantly increase. Thus, the model predicts that rational behaviour under yield uncertainty leads to water over-allocation. The model is designed for 'ex-ante' decision support but, as the world is uncertain and observed yields are only known 'ex-post', real marginal benefit of resource allocation would be only revealed after the season is finished.

The impact of altering parameters in the model is straightforward. An increase in the variance will increase the demand at low water prices compared to the same CDF with a smaller variance. The increase in irrigation efficiency to simulate different systems what may illustrate a change of the irrigation method from furrow E = 0.60; sprinkler E = 0.80 to drip irrigation E = 0.95 will lead to an augmentation of the $a_1$ value (marginal productivity of water), thereby modifying the demand function (Fig. 4). The illustration of the impact of three different irrigation system efficiencies on the water demand function is shown in Fig. 4 for the case of a triangular stochastic distribution.

Finally, it is worthy of note that the drawing of conclusions regarding the price elasticity of demand may prove problematical since the demand function may take any of a variety of functional forms. By using alternative stochastic distribution functions, this work has aimed to offer the reader a wide analysis of the potential impacts of the selected functional form on the irrigation decision under uncertainty. In our example, representative of maize in Spain, elasticity for water prices in the range from 0.02 to 0.05 EUR/m$^3$ (generalized in Spain for maize cultivation) remains low falling in the range from − 0.04 to − 0.27 falls inside found by Scheierling et al. (2006).

The advantage of the presented model is that it determines with precision the profit maximizing optimum under yield uncertainty. Yield uncertainty may act as a 'proxy' to integrate many sources of crop-growth uncontrollable factors such as climate variables (e.g., precipitation, temperature, solar radiation) that influences yield and biological factors such as pest and diseases. All these variables may act as yield-limiting factors, including the available soil water 'R′ which is also subject to uncertainty and constitutes a critical variable in our model. Moreover, it is the main yield-limiting factor in rainfed systems. Further developments of our model will focus on the integration with other theoretical developments such as the work of Adamson and Loch (2021) that incorporates uncertainty in irrigation water supply and includes

**Table 1**
Results optimization model (being p = 0,20 EUR/kg; c = 0.05 EUR/m$^3$).

| | Water use (m$^3$/ha) | Maximum potential yield (Kg/ha) | Expected profit (EUR/ha) |
|---|---|---|---|
| [1] Historical average yield (certainty scenario) | 6,880 | 10,000 | 1,100 |
| [2] Stochastic yield Beta (α = 1;β = 0.6;9000;11000) | 7,570 | 10,863 | 1,191 |
| Increase [3] = [2] - [1] | 690 | 863 | 91 |
| Variation (%) [3]/[1] | + 10% | + 9% | + 8% |

Source: own.

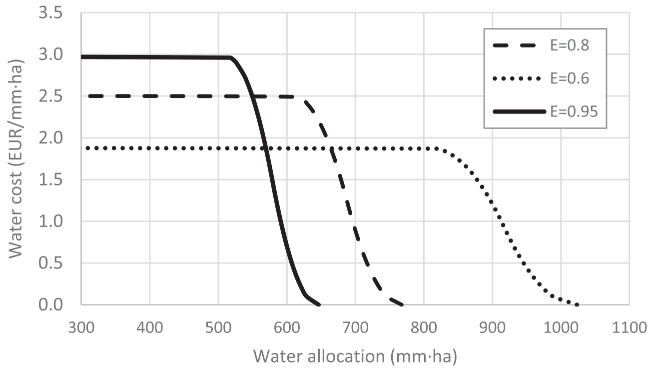


**Fig. 4.** Irrigation water demand for stochastic yield distribution and varying irrigation efficiency (E = 0.6, E = 0.8, E = 0.95). [Triangular CDF (9000;10000;11000); crop price= 0.20 €/kg; Average yield= 10,000 kg/ha; $K_y$ = 1.25; R= 250 mm].

perennials in the formulation, or the work of Xie and Zilberman (2018) that includes supply augmentation and water saving investment to optimize stochastically farm profits.

## 4. Brief discussion and concluding remarks

The farmer's decision on irrigation water allocation under yield uncertainty presented in this paper does not consider the farmer's utility and assumes risk neutrality. Our model shows that water demand, as a critical production input, increases with variance under various stochastic distribution functions, especially in the low range of water prices. Obviously, irrigation water allocation planning is a complex system, which contains multiple uncertainties that goes beyond yield uncertainty (the focus of our model) and include irrigation water supply uncertainty, soil water ('R′ in our model), temperature, wind, solar radiation that influences ET, commodity price uncertainty, and many other production factors that influence final yield. By isolating a specific factor (such as yield uncertainty as a confluence of many variables mentioned previously) we can focus on the influence of this factor in ex-ante seasonal water allocation.

Our results show that the impact of yield uncertainty on water demand will lead to an increase in water allocation compared to that required for historical average, thus following a behaviour characterized by "watering for the good years". Additionally, the example illustrated in Table 1 has shown that optimal allocation improves expected profit (by 8% over water allocation based upon average historical level). This improvement might be considered moderate and following the argument of Pannell (2006) it might be argued that "*optimizing techniques are sometimes of limited practical relevance for decision support*". Nevertheless, we believe that water is a critical input for crop production and any optimization attempt is valuable, especially in water scarce areas. In our opinion, the main policy implication is the need not only to implement modern precision technology capable of reducing yield uncertainty, but also to enhance input efficiency (Scott et al., 2014). Quantitative and qualitative impacts of over-irrigation on water resource management depend on the location of the field, since water resulting from irrigation applied in excess may return to the catchment, thereby reducing the quantitative impact of over-irrigation. Additionally, agricultural diffuse pollution should be considered as an undesirable effect that can be prevented by reducing water applied in excess. In this respect, empirical evidence shows that the implementation of irrigation technologies to enhance water efficiency (e.g., drip irrigation) may reduce the pollution load of return flows (García-Garizábal and Causapé, 2010; Lecina et al., 2010). In any case, overallocation represents a challenge for water authorities and water managers as it increases demand and quantitative pressure on the resources, that can be illustrated by the recent evolution of water trade prices analysed by Loch et al. (2021).

The limitations of the model presented in this paper include: (1) the need to investigate the link between over-allocation and potential over-irrigation; (2) the requirement for a deeper analysis into the functional form of the farmer subjective CDF of yields; and (3) an expansion of the theoretical model by removing the assumption of constant irrigation system efficiency, and also by including the possibility of applying deficit irrigation, (4) the existence of many other sources of uncertainty that affect farmer decision regarding water allocation as we have focused exclusively on yield uncertainty. The proposed model focuses on yield uncertainty and is applied to herbaceous crops. Therefore, it does not consider water supply uncertainty (either irrigation water or soil water 'R′). This represents a shortcoming of our model as illustrated by the recent increases of water price in water rights markets in Australia (MDB) are potentially caused by "*prudent (.) irrigators trying to prevent potential downside losses (.) as a response to perceptions of future supply shortages*" (Loch et al., 2021). Finally, impacts of over-allocation (as explored in this paper) and potentially related over-irrigation behaviour should be considered by policy-makers in the design of water management plans for river basins and aquifers, in order to introduce innovative management instruments, such as markets of irrigation rights, a new design of the irrigation rights system by including uncertainty and risks, and a more effective water-pricing policy, among others, to minimize undesirable impacts on increasingly scarce water resources.

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## Declaration of Competing Interest

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