A New Frequency Planning for Improved Capacity and Coverage in Sectorized OFDMA Cellular Networks

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***Abstract*—One of the main challenges in cellular systems is how to reduce Other Cells Interference (OCI) while keeping high spectral efficiency and high coverage probability. Since the beginning of cellular technologies, frequency planning with sectoring has been used to mitigate such OCI. However, in modern wireless Orthogonal Frequency Division Multiple Access (OFDMA) technologies much attention is being paid to more complex strategies like Coordintation Multi-Point transmission/reception (CoMP) techniques due to its higher performance. This paper presents a new frequency planning for sectorized cell networks. Simulation results show that this scheme considerably outperforms classical sectoring (CS) in terms of both capacity and coverage probability. In particular, for the case of hexagonal grid, the capacity improvement is around 6 b/s/Hz compared to CS while keeping a coverage probability higher than 0.9 for a 0 dB Signal to Interference plus Noise Ratio (SINR) threshold.**

I. INTRODUCTION

From operators perspective, cell planning is a trade off between throughput per unit area, coverage probability (Pc) and cost. In order to ensure a given throughput per unit area a minimum number of Base Stations (BS) are required in each region. This given throughput is not constant in the coverage area and depends on the traffic demand. The minimum number of BSs depends on the achievable throughput of each cell, and hence, the higher throughput per cell, the higher operator profits. Additionally, operators must guarantee some coverage probability to the users so Other Cells Interference (OCI) should be reduced.

Classical methods to improve cell performance have been Frequency Reuse (FR) and sectoring [1]. However, as these techniques reuse frequencies in order to reduce interference or equivalently increase coverage probability, they often decrease cell throughput. Hence, there exist a trade off between coverage probability and maximum cell throughput in the frequency planning.

In modern cellular communications standards like Long Term Evolution Advanced (LTE-A) more sophisticated techniques are proposed in order to increase cell performance, by reducing the received interference or even exploiting it as a useful signal. In the context of LTE-A these techniques are known as Coordination Multi-Point transmission/reception (CoMP). Although CoMP techniques are expected to offer higher performance than classical methods, CoMP techniques have two disadvantages [2]: (1) they require high capacity backhaul links between BSs and (2) they add complexity to BSs. For this reason simpler frequency reuse planning of cells and sectors are still appealing.

This paper presents a new frequency planning for sectorized cellular networks where the positions of the BSs follows a hexagonal grid. Classical sectoring (CS) divides the system bandwidth in N frequency bands and assigns a band to each cell following some planning algorithm. Afterwards, the frequency band assigned to each cell is divided between the M sectors of each cell so different frequency bands are assigned to different sectors [3]. In this work, it is considered the general case in which the system bandwidth is divided in L frequency bands. Q different frequency bands are assigned to each cell and allocated to the sectors, so different sectors can share the same band if $Q < M$. From now on this approach will be referred as Generalized sectoring (GS).

Our proposal includes a new frequency planning algorithm between cells and sectors, in which sectors of the same cell can reuse frequency bands. This approach makes it possible to schedule several users in the same band and cell without interfering each other, thus increasing the spectral efficiency. It is shown with simulation that the proposal considerably outperforms CS in terms of both coverage probability and capacity.

The rest of the paper is organized as follows. Section II depicts the system model for hexagonal grid and the performance metrics used in the paper. Section III defines GS and depicts the proposed frequency planning for the hexagonal case. Then, section IV illustrates performance results of several techniques. Finally conclusions are drawn in section V.

II. SYSTEM MODEL

This paper considers the downlink of a single tier cellular network where the positions of the BS follows a hexagonal grid. Cell association is based on distance i.e. serving BS is the nearest to the intended user, so Voronoi tessellation determines cell boundaries. It is considered that the bandwidth is divided in orthogonal Resource Blocks (RB) that are allocated to the users for downlink transmission. Saturated conditions are assumed, i.e. the density of users is high enough to consider that each cell and each sector has at least one user to serve. Hence it is considered that there is one user scheduled in each RB of each sector.

The system model assumes a standard power loss propagation model with path loss exponent $\alpha > 2$. BSs transmit with constant power $1/\mu$ and Additive White Gaussian Noise (AWGN) with variance σ^2 is considered at the receiver. Transmitted signals experience Rayleigh fading with mean 1, so the received signal power at a distance r from a BS is $hr^{-\alpha}$ where $h \sim \exp(\mu)$ follows a exponential distribution with mean $1/\mu$, [4]. The Signal to Interference plus Noise Ratio (SINR) at the receiver follows this expression

$$\texttt{SINR} = \frac{h_0 r_0^{-\alpha}}{\sum_{i\in\Phi/b_0} h_i r_i^{-\alpha} + \sigma^2} \tag{1}$$

where the first term of the denominator represents the interference, which is the received signal transmitted from all BSs except the serving BS (represented by the point b_0).

A. Frequency Planning

For the hexagonal grid FR divides system bandwidth in N frequency bands where N must be a rhombic number. Each cell is assigned a band following a widely known rule [3] that maximizes the co-channel distance. If CS is also applied, FR is carried out first in order to assign a band to each cell. Afterwards, the frequency band assigned to each cell is partitioned again and distributed among its M sectors. Each cell sector has a different frequency band so there is not interference between sectors of the same cell.

B. Performance metrics

As performance metrics, ergodic per cell capacity expressed in terms of (bit/sec/Hz) and coverage probability are considered. The coverage probability, is the probability that a given user reaches a minimum SINR threshold, denoted as T, hence $\text{Pc} = \mathbb{P}(\texttt{SINR} > T)$ being $\mathbb{P}(\mathcal{A})$ the probability associated with the event $\mathcal{A}$.

Regarding capacity, since saturated conditions are assumed, for FR we consider one active user in the cell of interest and transmission occupies all the bandwidth assigned to the cell. This means that capacity follows the next expression

$$C/B = \frac{1}{N}\mathbb{E}\left[\log_2(1+\texttt{SINR})\right] \text{(b/s/Hz)} \tag{2}$$

where $\mathbb{E}[\cdot]$ is the expectation operator and B is the system bandwidth, so B/N is the bandwidth assigned to the cell. If cells have M sectors, one active user in each sector occupying all the sector's bandwidth is considered. Since all sectors are identical, the cell capacity is given by $C = MC_{\text{sect}}$, where the capacity per sector is $C_{\text{sect}} = B/(MN)\mathbb{E}\left[\log_2(1+\texttt{SINR})\right]$. Hence the capacity per cell follows the same expression as (2). However, lower interference is experienced by the user in this case and the resulting per cell capacity is higher in the same conditions.

C. Generalized sectoring

Lets us consider a cellular system formed by cells with M sectors. The system bandwidth is divided in L frequency bands where Q of them are assigned to each cell following a particular planning algorithm. Therefore, $Q \leq \min(M, L)$. If $Q < M$ different sectors share the same frequency band.

As a constraint it is assumed that neighbour sectors (within the same cell) do not share the same frequency band in order to avoid intra-cell interference[1], i.e. $Q \geq 2$. Furthermore, it is not possible to fulfil such a constraint when $M\%Q = 1$ and $Q = 2$ where $\%$ represents modulus operation. Notice that CS is a particular case of GS where $L = MN$ and $Q = M$.

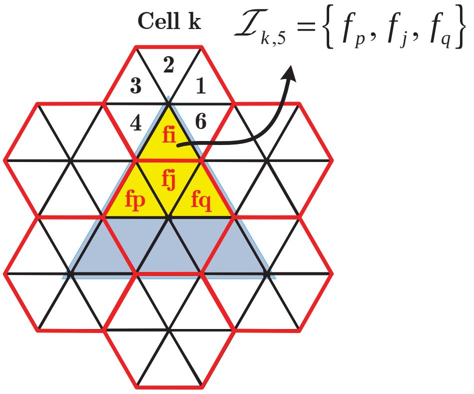

Fig. 1. Two interference free triangles for $K = 2$ (yellow) and $K = 3$ (grey) for $M = 6$ sectorized cellular network

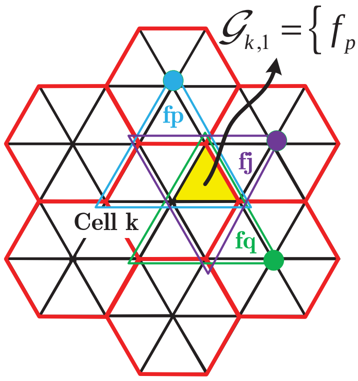

Fig. 2. Illustration of pointing sectors set $\mathcal{G}_{k,1}$ for cell k and sector 1

Assuming that all sectors are identical and there is only one active user per sector the capacity per cell is $C/B = M/L\mathbb{E}\left[\log_2(1+\texttt{SINR})\right]$ since the available bandwidth for each sector is B/L.

III. PROPOSED FREQUENCY PLANNING FOR A SECTORIZED HEXAGONAL GRID

If two planning methods obtain the same SINR Probability Density Function (PDF), the method whose ratio M/L is higher would be associated with higher capacity. Hence, it would be interesting to develop a frequency planning that reduce the interference while keeping a M/L ratio as high as possible.

In order to do that, we define a interference free region between sectors of different cells. This is a restriction and will be referred as follows. **Constraint 1**: *The frequency band allocated to each sector must be different to the frequencies allocated to other sectors inside the interference free region.*

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[1] It is assumed that the power received from secondary lobes of the antenna radiation pattern is negligible compared to the power received from the main lobe. Hence in this work interference from secondary lobes is not taken into account.

There is another constraint that aims to avoid intra-cell interference. **Constraint 2**: *Different frequency bands must be assigned to every pair of neighbor sectors that belong to the same cell*. Then, given a number of sectors per cell M, we seek the minimum number of partitions of the system bandwidth L that meets such constraints.

In particular, we focus on cellular systems with $M = 6$ sectors per cell. Thus, the interference free region is an equilateral triangle. The height of such triangle is K times the hexagon's apothem. Fig. 1 shows two interference free triangles for $K = 2$ (yellow triangle) and $K = 3$ (grey triangle) related to the 5th sector of cell k. For the $K = 2$ triangle it can be seen that $f_i \neq f_p \bigwedge f_i \neq f_j \bigwedge f_i \neq f_q$ (constraint 1). Additionally $f_j \neq f_p \bigwedge f_j \neq f_q$ as neighbor sectors of the same cell must have different frequency bands in order to avoid intra-cell interference (constraint 2). It is intuitive that as K is greater, the interference is reduced, but also the number of partitions L should be higher.

Fig. 1 illustrates the numbering of sectors inside a cell. Two sets are defined for each sector, that are related to constraint 1. One represents the frequency band assigned to other sectors inside the interference free region of sector l belonging to cell k ($\mathcal{I}_{k,l}$). This set is illustrated in Fig. 1 for the 5th sector belonging to cell k. The other set represents the frequency bands of those sectors that fall in the same interference free region and is identified as ($\mathcal{G}_{k,l}$) for sector l belonging to cell k. This latter set is shown in Fig. 2. Hence, constraint 1 imposes that the band assigned to the lth sector of cell k must not be in the set $\mathcal{I}_{k,l} \cup \mathcal{G}_{k,l}$.

In order to reduce complexity of frequency planning algorithm we have chosen $Q = 2$. Such frequency planning is an algorithm that assigns a frequency band to each sector trying to obtain a number of frequency bands L as low as possible. Hence, the algorithm starts with $L = 1$, and increments such number when it is not possible to fulfil constraints 1 and 2 for a given cell. As $Q = 2$ and $M = 6$, sectors $1, 3$ and 5 share the same frequency bands whereas $2, 4$ and 6 share another band. The band assigned to sector l of cell k is identified as $s_{k,l}$. Initially, all sectors are assigned the null frequency band $\{\emptyset\}$. The algorithm starts assigning frequency bands to each cell from 1 to C, being C the number of cells to be planned. The ordering of cells is important and affects the result. Better results are obtained if neighbour cells are ordered consecutively, i.e. cell $k-1$ and cell k are neighbours, cell k and $k+1$ are neighbours as well, etc. The algorithm description is given in Algorithm 1.

Following this algorithm for $K = 2$ yields to the frequency planning illustrated in Fig. 3 with $L = 3$. This solution has a high $M/L = 2$, which is even higher that the ratio obtained for full frequency reuse using omnidirectional antennas at the BS ($M = 1$).

The interfering cells of the sector highlighted with green color are identified in the figure with a red circle. It can be seen that the minimum distance between a interfered user and a interfering BS is $d_{\text{min}} = 2R$ being R the radius of the cell. There are two interfering BSs at this distance from the first interfered user. For CS with frequency reuse $N = 3$ and $M = 6$, the stronger interfering BSs are also two BSs at a distance $d_{\text{min}} = 2R$. However the M/L ratio is in this latter case $1/3$ whereas in the former case is 2. Intuitively this

Algorithm 1: Proposed Frequency Planning, Q=2, M=6

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Data: $\mathcal{I}_{k,l}$ frequency bands assigned to other sectors
      from interference free region of sector $l$
      belonging to cell $k$
Data: $\mathcal{G}_{k,l}$ frequency bands assigned to sectors that
      point sector $l$ belonging to cell $k$
Data: $s_{k,\{p,j,q\}}$ frequency bands assigned to sectors
      $p, j, q$ belonging to cell $k$
Result: $\mathcal{X}_{k,\{i,j,q\}} =$
      $\mathcal{F} / \{\mathcal{I}_{k,i} \cup \mathcal{I}_{k,j} \cup \mathcal{I}_{k,q} \cup \mathcal{G}_{k,i} \cup \mathcal{G}_{k,j} \cup \mathcal{G}_{k,q}\}$
begin
    $L = 1, \mathcal{F} = \{f_1\}$;
    for $k = 1 : C$ do
        //Constraint 1;
        $\mathcal{A} = \mathcal{X}_{k,\{1,3,5\}}$;
        if $\mathcal{A} = \{\emptyset\}$ then
            $L = L + 1, \mathcal{F} = \mathcal{F} \cup \{f_L\}$;
            $\mathcal{A} = \{f_L\}$;
        $\mathcal{B} = \mathcal{X}_{k,\{2,4,6\}}$;
        if $\mathcal{B} = \{\emptyset\}$ then
            $L = L + 1, \mathcal{F} = \mathcal{F} \cup \{f_L\}$;
            $\mathcal{B} = \{f_L\}$;
        //Constraint 2;
        if $\mathcal{A}/\mathcal{B} \cup \mathcal{B}/\mathcal{A} = \{\emptyset\}$ then
            $L = L + 1, \mathcal{F} = \mathcal{F} \cup \{f_L\}$;
            $\mathcal{A} = \{f_L\}$;
        else if $\mathcal{B}/\mathcal{A} = \{\emptyset\}$ then
            $\mathcal{A} = \mathcal{A}/\mathcal{B}$;
        else
            $\mathcal{B} = \mathcal{B}/\mathcal{A}$;
        $s_{k,\{1,3,5\}} = a \in \mathcal{A}$ //Assign any frequency band
        in $\mathcal{A}$;
        $s_{k,\{2,4,6\}} = b \in \mathcal{B}$ //Assign any frequency band
        in $\mathcal{B}$;
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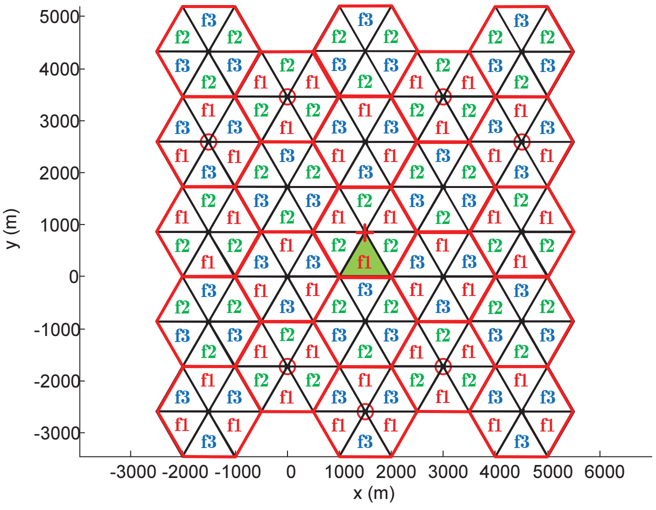

Fig. 3. Proposed frequency planning with $K = 2$ interfering free triangles for hexagonal grid with $M = 6$.

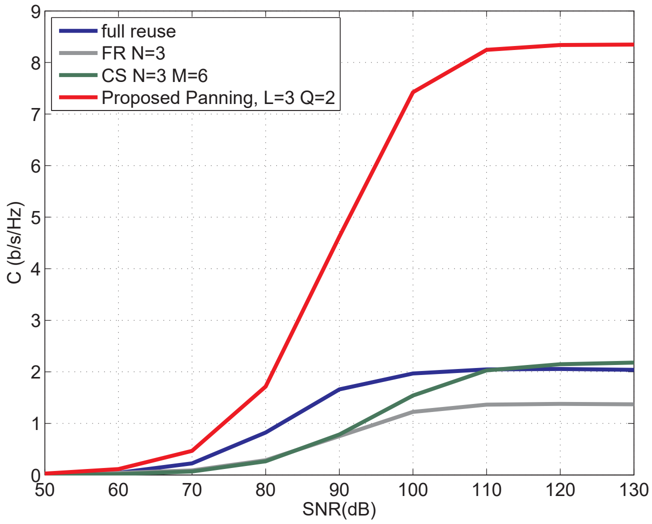

Fig. 4. Capacity of planning algorithms for hexagonal grid with $\alpha = 3$ and $R = 1$ km

reasoning suggests that the proposed frequency planning for interfering free triangle $K = 2$ has a roughly similar coverage probability while keeping higher capacity. Results will confirm this fact in next section.

IV. SIMULATION RESULTS

In this section the proposed frequency planning for the hexagonal grid is assessed and compared to conventional FR and CS.

A path loss exponent $\alpha = 3$ is assumed in all scenarios. The *transmit* Signal to Noise Ratio (SNR) is defined as the quotient between the transmit power and the noise power as $1/(\mu\sigma^2)$ following notation from section II. Hence, here the *transmit* SNR is related to transmission power instead of received power. The intensity given as expected number of BSs per unit area in the random and hexagonal grid is the same.

In case of both FR and CS, each cell is assigned a frequency band following widely known methods that maximize the distance between co-channel cells in hexagonal grid [3]. The planning for the proposed method is related to a $K = 2$ interference free triangle as shown in Fig. 3.

Fig. 4 and Fig. 5 illustrate the capacity and coverage probability associated to each planning. Compared to full reuse case, it can be observed that CS improves considerably the coverage probability. However its capacity is roughly the same as full reuse. FR offers better coverage probability than full reuse, but its capacity is lower than full reuse capacity. Thus, both FR and CS only improve coverage probability compared to the full reuse case.

The proposed method reaches a coverage probability roughly equivalent to FR, so it improves coverage probability compared to full reuse. Nevertheless, the proposed method greatly increases the capacity as well, and this is the point that makes more interesting this algorithm, since it offers a good trade off between both capacity and coverage probability.

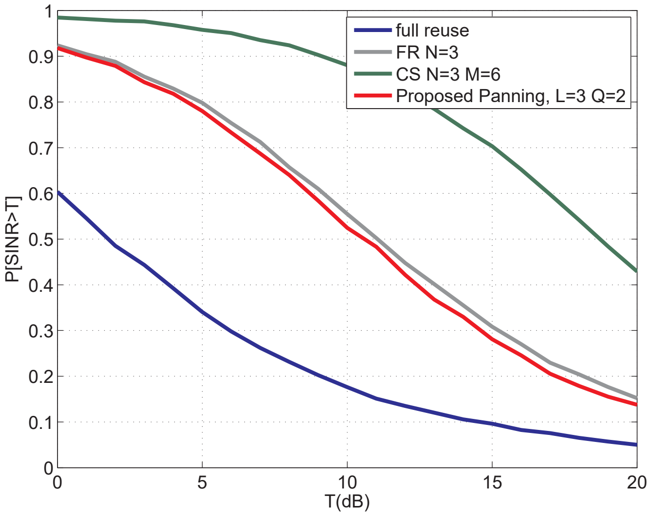

Fig. 5. Coverage probability of planning algorithms for hexagonal grid with *transmit* SNR= 130 dB $\alpha = 3$ and $R = 1$ km

V. CONCLUSIONS

In this paper a new frequency planning for a sectorized hexagonal grid is presented. On the one hand, this frequency planning reduces interference by assuring that no interference will be present in a region given by the parameter K. On the other hand, it tries to maintain the number of frequency partitions L as low as possible since this number affects the capacity. The capacity improvement of the proposed algorithm is around 6 b/s/Hz compared to CS while keeping a coverage probability higher than 0.9 for a 0 dB threshold.

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