

Statistical Channel Impulse Response Modeling for Optical Wireless Communication in Turbid Waters

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Abstract—Underwater optical wireless communication (UOWC) systems support high-speed, reliable and cost-effective implementations that are demanded for extending the telecommunication networks to oceans. UOWC systems suffer from attenuation and scattering processes. Particularly, the scattering process can change the direction of the emitted photons, especially in turbid waters. In this way, quantifying the signal attenuation and the time dispersion produced by absorption and scattering is a crucial work. Hence, we propose a new closed-form solution for modeling the channel impulse response for UOWC systems in turbid water that is validated through Monte Carlo simulations and can be used for system design and optimization purposes.

I. INTRODUCTION

The great challenge of extending the telecommunication networks to oceans is taking shape. The scientific community and governments are really making extra efforts to create a worldwide network of smart interconnected underwater objects and to digitally link our oceans, streams, and lakes. Particularly, this research is framed in two of the societal challenges related to the current European research trends, i.e., European Bioeconomy: the promotion of marine research and the search for green energy sources. Underwater optical wireless communication (UOWC) has been considered as a promising candidate for this purpose since it is able to offer a large variety of emerging applications such as ecological monitoring, natural resource discovery and extraction, and port security. All of them presents an inherent need for high data-rate underwater wireless links to connect submarines, autonomous underwater vehicles (AUV) or divers to enable data telemetry, control for navigation and docking. UOWC systems in the blue-green band (450-570 nm) is attractive for underwater communication links over moderate distances due to its high bandwidth in comparison with acoustic and radio-frequency (RF) waves [1].

Despite the great advantages of UOWC systems in real-time applications, these kind of systems are not without drawbacks due to the absorption and scattering processes, which can be characterized by the inherent optical properties of the water [2]. Both processes produce a strong intensity attenuation and are characterized by the radiative transfer equation (RTE). Regarding the absorption process, this one represents the loss of energy when light propagates through water. This is due to the interaction of the light beam with water molecules and other particles. The absorption effect is mitigated by using the blue-green spectrum. By other hand, the scattering process can change the direction of the emitted photons, especially in

turbid environments such as coastal and harbor waters [3], [4]. Of course, this effect produces a severe temporal dispersion that will have an undesirable impact on the performance. In addition to this, scattering can perfectly generate inter-symbol interference (ISI) by introducing errors in the decision device at the receiver and, hence, making the communication less reliable [5]. In order to quantify the intensity attenuation and the time dispersion produced by absorption and scattering, an accurate channel impulse response (CIR) modeling represents a step forward in UOWC systems design and, hence, it must be studied carefully. Based on this, the CIR analysis has been carried out in the last few years [6], [7]. In [6], the double Gamma function (DGF) is proposed to model the CIR by using four degrees of freedom, obtaining good results for a variety of UOWC scenarios. The adoption of this approach is based on the impulse response in clouds, whose channel characteristics are bit far away from the water. In [7], a new function based on a combination of exponential and arbitrary power function (CEAPF) is proposed by also using four degrees of freedom. From a physics point of view, none of these reported works was able to thoroughly explain the connection between absorption and scattering phenomena with the adopted mathematical model. Motivated by the above, we try to fill this gap by proposing an easy and interesting approach based on the albedo definition to study the CIR in turbid waters.

In this paper, we focus on the CIR modeling as the main channel characteristic for turbid harbor and estuary waters that have a very high concentration of dissolved and in-suspension matter. This is a crucial work to quantify the signal attenuation and the time dispersion produced by absorption and scattering. In this sense, an accurate and easy impulse response modeling is essential to study the performance analysis and optimize the system design in UOWC systems. We quantify the channel time dispersion for different link distances, transmitter sources such as laser diode (LD) and light emitting diode (LED), and different values of field-of-view (FOV) at the receiver-side. Unlike the prior works, we propose a new approximation for the CIR based on the mixture of two Gamma functions that fits much better with Monte Carlo simulations by using three degrees of freedom for different transmitter sources.

The remainder of this paper is arranged as follows. In Section II, the underwater channel is described. In Section III, the analysis of the CIR for turbid environments is carefully presented as well as some numerical results are illustrated in Section IV. Finally, the paper is concluded in Section V.

TABLE I: Extinction Coefficients for Turbid Waters.

Water type	$a[m^{-1}]$	$b[m^{-1}]$	$c[m^{-1}]$
Coastal	0.179	0.219	0.398
Harbor	0.366	1.824	2.190

II. CHARACTERIZATION OF THE CHANNEL

The two main phenomena that affect light propagation in water are absorption and scattering, whose spectral coefficients are $a(\lambda)$ and $b(\lambda)$, respectively, with λ being the wavelength [2]. In this way, the spectral beam attenuation coefficient is defined as

$$c(\lambda) = a(\lambda) + b(\lambda) \quad [m^{-1}]. \quad (1)$$

In line with these parameters, another important one is the angular distribution of scattering in water that is characterized by the volume scattering function $\beta(\theta)$, where θ represents the scattering angle and takes the medium into consideration. In most of the Monte Carlo simulations, the Henyey-Greenstein (HG) phase function proposed in [8] is widely used to model the scattering phase function in astrophysics, atmospheric and oceanic optics. The HG phase function is given by

$$\beta_{HG}(\mu, g) = \frac{1 - g^2}{(1 + g^2 - 2g\mu)^{3/2}}, \quad (2)$$

where $\mu = \cos \theta$, and g is the average cosine of θ , i.e., $g = \overline{\cos \theta}$. It should, however, be noted that underwater optical channel may be affected by the effect of oceanic turbulence due to variations of the water refractive index produced by the changes in temperature, pressure and salinity of the water, and misalignment errors. As proved in [9], oceanic turbulence can be considered neglected in deep waters. At the same time, the combined effect of oceanic turbulence and misalignment errors as well as potential beam blockages are beyond the scope of this work.

III. MODELING OF CHANNEL IMPULSE RESPONSE

The impulse response, $h(t)$, due to the absorption and scattering processes is more than a complex task and, hence our purpose is to find a prototypical impulse response from Monte Carlo simulations.

A. Monte Carlo Simulation

In order to analyze the channel impulse response, an approach based on Monte Carlo simulations is used to numerically evaluate the underwater channel characteristics by generating a large number of photons and simulating the interaction of each photon with the medium [10], [11]. We solve the RTE by using probabilistic methods based on Monte Carlo simulations. The simulation process is reproduced here for convenience. In this way, the parameters to be taken into consideration are the following: transmitter characteristics such as wavelength, beam width and divergence angle; distance between the transmitter and the receiver; and the receiver characteristics such as aperture size and FOV.

At the beginning, each photon is put into the underwater channel with unit weight. When a photon interacts with the medium, its direction and weight are modified due to scattering. On the one hand, the photon weight experiences

a drop according to the albedo definition such that the new weight is multiplied by b/c . On the other hand, the new photon direction is obtained from the scattering function in Eq. (2), determining a new propagation direction. This process is repeated over and over again until the photon either is considered as absorbed or reaches the receiver. The photon is absorbed when its weight is below the threshold $W_{th} = 10^{-6}$. Note that a high number of photons of 10^{11} are considered in order to get a smooth CIR due to the fact that too few photons are able to reach the receiver especially in harbor and coastal waters. This simulation program is probabilistic in nature and, hence, this has been developed to efficiently deal with a huge amount of photons thanks the RAM memory management to avoid long simulation times.

B. Previous work

For convenience, we firstly reproduce some of the reported models mentioned in Section I. As commented, the authors in [6, Eq. (10)] proposed the DGF to model the impulse response that is based on the properties of clouds. In this way, the closed-form expression is given by

$$h_1(t) = C_1 \cdot t \cdot e^{-C_2 t} + C_3 \cdot t \cdot e^{-C_4 t}, \quad (3)$$

where C_1, C_2, C_3 , and C_4 are the four parameters to be found through Monte Carlo simulations. At the same time, a new function model based on CEAPF was recently proposed in [7, Eq. (13)] as follows

$$h_2(t) = C_1 \frac{t^\alpha}{(t + C_2)^\beta} e^{-\alpha v t}, \quad (4)$$

where $C_1 > 0$, $C_2 > 0$, $\alpha > -1$, and $\beta > 0$ are the four parameters to be found through Monte Carlo simulations with v is the speed of light in water. Note that none of these impulse response models has been able to explain the connection between absorption and scattering phenomena with the adopted mathematical model.

C. Proposed CIR Analysis

From a statistical point of view, we model the CIR as a random variable that can be partitioned into two factors i.e., from the albedo definition (b/c). We define the CIR as $h = b/c$, where b and c arise, respectively, from the effect of scattering and the total effect of absorption and scattering (the extinction coefficient). Note that this approach has never been used in the literature to model the CIR in underwater optical channels. To develop a CIR model, we consider that both parameters are distributed by Gamma distributions as

$$f_b(x) = \frac{\alpha_1^{\beta_1}}{\Gamma(\beta_1)} \exp(-\alpha_1 \cdot x) \cdot x^{\beta_1 - 1}, \quad (5a)$$

$$f_c(y) = \frac{\alpha_2^{\beta_2}}{\Gamma(\beta_2)} \exp(-\alpha_2 \cdot y) \cdot y^{\beta_2 - 1}, \quad (5b)$$

where α_1 and α_2 are the shape parameters, β_1 and β_2 are the scale parameters, and $\Gamma(\cdot)$ is the Gamma function. In order to derive the CIR, we have to resolve the following conditional integral by first fixing $b = h \cdot c$ as follows

$$h(t) = \int_0^\infty f_{b|c}(t|y) f_c(y) dy, \quad (6)$$

TABLE II: UOWC System Parameters.

Parameter	Symbol	Value
Wavelength	λ	532 nm
UOWC link distance	L	{5,12,14,20,30} m
Water refractive index	n	1.3
LD divergence angle	θ_{div}	10°
LD beam waist radius	w_z	10 mm
LED divergence angle (full-angle)	θ_0	80°
Receiver aperture diameter	D	50 cm
Receiver FOV	FOV	{20°, 180°}
Number of emitted photons	N_T	10 ¹¹
Photon weight threshold	W_{th}	10 ⁻⁶

where $f_{b|c}(t|y)$ is the conditional probability given a state c . Now, by substituting Eq. (5) into Eq. (6), we obtain

$$h(t) = \frac{\alpha_1^{\beta_1} \alpha_2^{\beta_2}}{\Gamma(\beta_1)\Gamma(\beta_2)} t^{\beta_1-1} \int_0^\infty \frac{\exp[-(\alpha_1 t + \alpha_2)y]}{y^{-\beta_1-\beta_2+1}} dy. \quad (7)$$

If we particularize for $\alpha_1 = \alpha_2$, the above integral can be easily solved with the help of [12, Eq. (3.326.2)], deriving the Beta prime distribution as follows

$$h(t) = k \cdot \frac{\Gamma(\beta_1 + \beta_2)}{\Gamma(\beta_1)\Gamma(\beta_2)} \cdot \frac{t^{\beta_1-1}}{(1+t)^{\beta_1+\beta_2}}, \quad t > 0, \quad (8)$$

where $k > 0$, $\beta_1 > 0$, and $\beta_2 > 0$ are the parameters to be solved. These parameters can be found by using nonlinear least square criterion as follows

$$(k, \beta_1, \beta_2) = \arg \min \left(\int [h(t) - h_{mc}(t)]^2 dt \right), \quad (9)$$

where $h(t)$ is the impulse response model proposed in Eq. (8), and $h_{mc}(t)$ is derived from Monte Carlo simulations. The above equation is computed via curve fitting approach using specific software packages such as Mathematica (version number 11.1.1.0). It is noteworthy to mention that this CIR expression presents only three degrees of freedom in comparison with the existing literature [6], [7]. The Gamma distribution has been selected for this task in numerous occasions because it has been shown to be an excellent approximation for many propagation problems [13].

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, the proposed CIR is plotted for different water types (harbor and coastal), as shown in Table I [6], as well as for different values of UOWC link distances of $L = \{5, 12, 14, 20, 30\}$ m to see how well the new approximation fits with Monte Carlo simulations. At the transmitter-side, we use two types of transmitter source: a LD with a Gaussian beam profile with a beam waist of 10 mm and a divergence angle of 10°; and a LED with Lambertian emission with a divergence angle of 80°. At the receiver-side, we use a receiver's aperture diameter of $D = 50$ cm and different values of the receiver FOV = {20°, 180°}. For each scenario, we send at least 10¹¹ photons in order to get a smooth CIR due to the fact that too few photons are able to reach the receiver in harbor and coastal waters. The UOWC system parameters used in this paper are summarized in Table II.

In Fig. 1, it can be seen how the proposed CIR matches very well with Monte Carlo simulations regardless of water type, source type, UOWC link distance and FOV. More importantly,

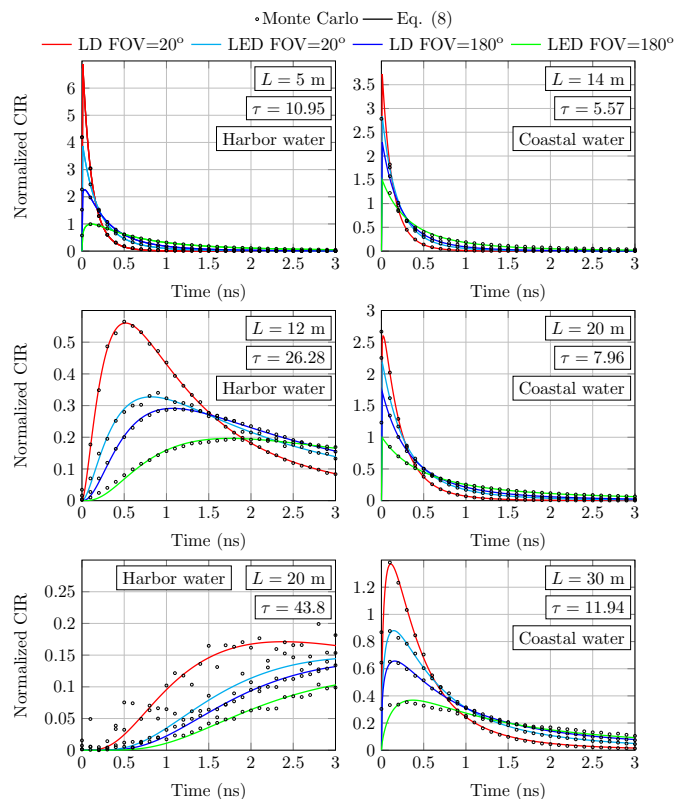


Fig. 1: Normalized channel impulse response (CIR) for coastal and harbor waters when different UOWC link distances, different transmitter sources as well as different values of FOV are considered.

this accuracy is achieved using only three degrees of freedom in comparison with [6], [7] that use four degrees of freedom each. From this figure, we can see how the channel impulse response gets spread out mainly due to the scattering process. In other words, the larger attenuation length ($\tau = L \cdot c$), the more spread out the channel impulse response will become. Hence, it is concluded that there is a strong relation between the channel impulse response and the receiver FOV. This is due to the fact that a narrow FOV results in a receiver that will not be able to detect photons since these ones are scattered multiple times as τ increases, i.e., as the water type gets more turbid. These results are in agreement with [6], [7]. In order to quantitatively measure the accuracy of the proposed approximation, we evaluate the coefficient of determination adjusted R-square (R^2) for all scenarios considered. This one is a modified version of R-squared that has been adjusted for the number of predictors in the model. In this way, it should be highlighted that the proposed CIR presents a greater robustness against the type of source, i.e. LD or LED, achieving a higher value of adjusted R^2 in comparison with the existing literature, i.e., with Eqs. (3) and (4) in most scenarios, as summarized in Table III in green color. Moreover, we must emphasize the superiority of this approach that even using a less degree of freedom is able to achieve a higher accuracy than or equal to those already reported in the literature.

In Fig. 2, the normalized received intensity is plotted as a function of differential FOV (Δ FOV) in degrees when different receiver aperture sizes of $D = \{20, 30, 40, 50, 60\}$ cm as well as different transmitter sources are assumed. In this figure, what we show is how the received photons are

TABLE III: Comparison of Different Curve Fitting Results.

R ² –Coastal water				R ² –Harbor water			
L	Eq. (3)	Eq. (4)	Eq. (8)	L	Eq. (3)	Eq. (4)	Eq. (8)
LD-FOV20°							
14	0.022	0.022	0.043	5	0.301	0.301	0.326
20	0.277	0.277	0.286	12	0.996	0.999	0.999
30	0.731	0.732	0.736	20	0.981	0.989	0.987
LD-FOV180°							
14	0.118	0.118	0.137	5	0.757	0.758	0.767
20	0.452	0.454	0.461	12	0.996	0.999	0.997
30	0.866	0.875	0.876	20	0.981	0.999	0.998
LED-FOV20°							
14	0.102	0.102	0.121	5	0.612	0.612	0.626
20	0.423	0.424	0.432	12	0.998	0.998	0.997
30	0.834	0.840	0.842	20	0.970	0.977	0.975
LED-FOV180°							
14	0.249	0.249	0.265	5	0.921	0.925	0.927
20	0.631	0.641	0.646	12	0.997	0.999	0.996
30	0.935	0.947	0.935	20	0.984	0.998	0.996

distributed from the receiver FOV point of view. In other words, as a result of scattering process, the received photons are not uniformly distributed within the receiver FOV. Through this figure, we can see how the received photons are distributed within the FOV by considering a sample space of 180° in 1° steps. This way of illustrating the received intensity proves to be particularly much more interesting than illustrating the received intensity directly as a function of the FOV, where the received intensity is saturated when a specific value of FOV is exceeded, not achieving a relevant improvement as a consequence of increasing the FOV. At the same time, the value of FOV that makes the received intensity saturated depends also on the receiver aperture size. As expected, the results corresponding to LD present a better performance than LED due to its high directivity. Finally, it is observed that there is a maximum where more photons are received that does not depend on the receiver aperture size, but it does on the water type.

V. CONCLUSION

A novel closed-form approximation for modeling the channel impulse response for UOWC systems in turbid waters has been presented and verified by Monte Carlo simulations. This new approximation represents a step forward in the characterization of UOWC channels. The simulation tool developed to emulate the behavior of photons underwater has allowed us to send a huge amount of photons on the order of 10^{11} , which represents an unprecedented milestone in the performance analysis of UOWC systems, in order to get a smooth CIR in harbor and coastal waters. By other hand, we can also conclude that the receiver FOV is a crucial parameter especially in harbor water due to the fact that the emitted photons suffer much more scattering, i.e., when the attenuation length increases. In the future, we plan to link this approximation with UOWC channel parameters for system design and optimization purposes. We will be able to mitigate the effect of ISI produced by scattering, especially when τ becomes significant, even when oceanic turbulence and pointing errors take place.

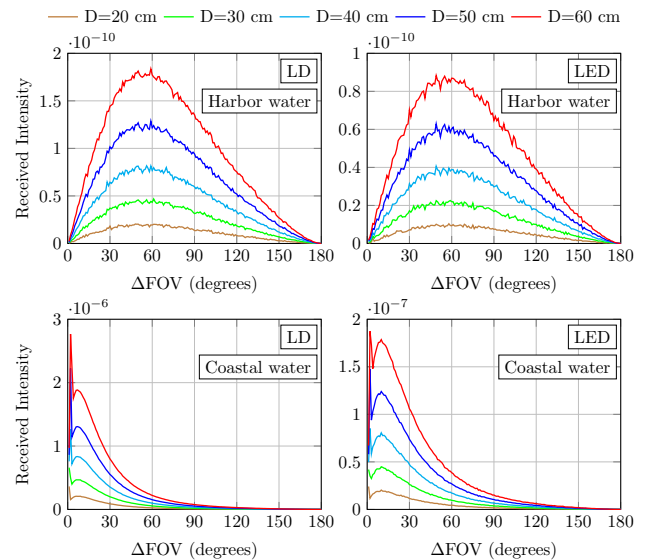


Fig. 2: Received intensity as a function of differential FOV (Δ FOV) in degrees for an UOWC link distance of $L = 20$ m when different receiver aperture sizes and different transmitter sources are considered for coastal and harbor waters.

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