

Teaching the residue theorem and its applications with a CAS

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The residue theorem is one of the most interesting result in Complex Analysis which allows not only computations in \mathbb{C} , the Field of Complex Numbers, but also provides many applications in the Field of Real Numbers \mathbb{R} .

In [1] we introduced the file `RESIDUE.MTH`, developed in the CAS `DERIVE` which main objective was to provide tools for solving integration problems in Complex Analysis using the residue theorem.

In this talk we present the library `ResidueApplications`, that was initially developed in `DERIVE` since Engineering students in the University of Málaga are still using this software in computer lectures. However, we are migrating this library to `PYTHON` using the symbolic mathematics library `SYMPY`. This way it will be also possible to use this package in other CAS as `SAGEMATH`.

The main goals of the `ResidueApplications` library are not only to provide some important applications of the Residue theorem but also to use it as a pedagogical tool for Engineering students.

`ResidueApplications` can be used as a tutorial in the teaching and learning process of this topic since it provides the results step by step allowing the students to check their computations when they solve an exercise. When developing this package, we were not interesting only in the computations of residues and their applications (which can be easily done using standards functions in different CAS) but mainly on its pedagogical use. In addition of the step by step facility, using this library, the students also can develop their own programs to deal with different applications. This way, the student are the protagonist of their self-learning process. For example, If the students develop a program to compute the residues of a function, they will be better prepared to understand this topic.

The programs developed in this tutorial can be grouped in the following blocks:

1. Compute of residues.

2. Compute of complex integrals using the residue theorem.
3. Applications of the residue theorem to compute integrals in \mathbb{R} :
 - (a) Trigonometric integrals.
 - (b) Improper integrals.

In previous ACA conferences we dealt with the application of the residue theorem to compute improper integrals (see [1] and [2]). In this talk, although we will present an overview of the whole tutorial, we will focus mainly in the computation of trigonometric integrals.

Keywords

Residue theorem, Trigonometric integrals, Improper integrals, Stepwise tutorial, CAS, DERIVE, PYTHON, SYMPY

References

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