

# Modelling asymmetric two-port reciprocal microwave structures by means of compact equivalent circuits

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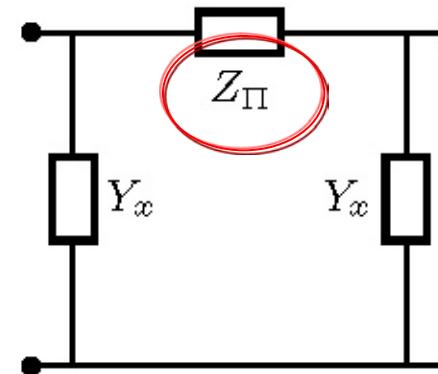
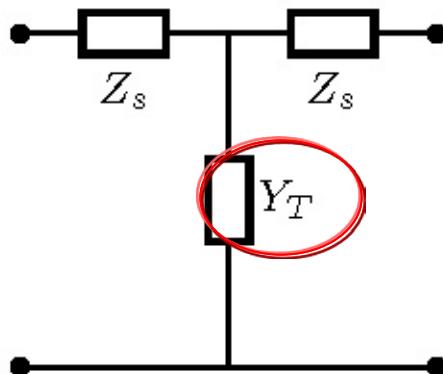
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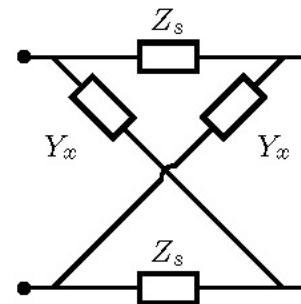
- Background:
  - Symmetric two-ports and lattice networks
  - Classic approach: Felsen-Oliner equivalent circuit
- Lattice-based non-symmetric equivalent circuits:
  - Transformer-lattice equivalent circuit (TEC)
  - Eigen-values based equivalent circuit (EEC)
  - Comparison between TEC and EEC
- Conclusions

# Symmetric two-ports (1): $\Pi$ and T networks

“In contrast, the  $\pi$  or T network equivalent of a complicated four-terminal network will sometimes require negative circuit elements in some of the arms, and hence be physically unrealizable”<sup>(1)</sup>



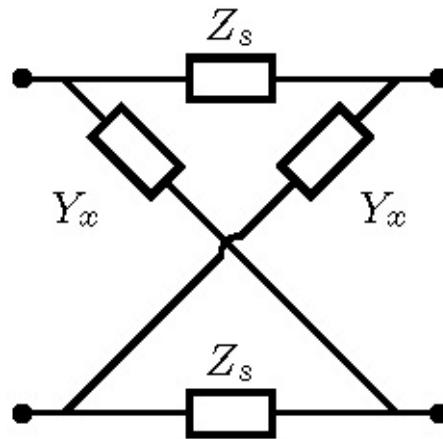
$$Y_T^{-1} = Y_x^{-1} - \frac{Z_s}{2}$$



<sup>(1)</sup> F.E. Terman, “Radio Engineers’ Handbook”, McGraw-Hill, 1943.

# Symmetric two-ports (2): the lattice network

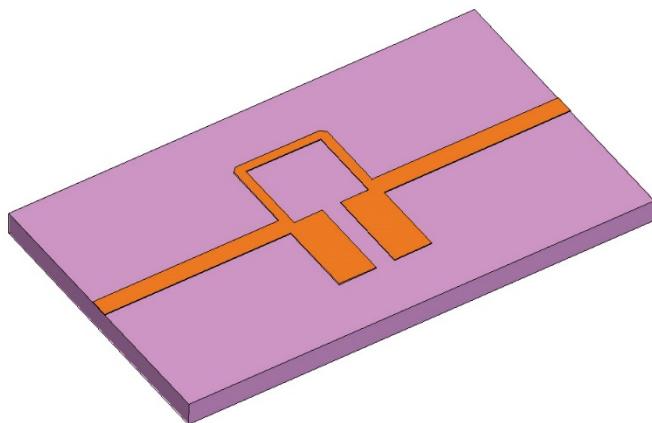
“It is possible to represent any symmetrical (reciprocal and lossy) four-terminal network by a lattice having physically realizable impedance arms”<sup>(1)</sup>



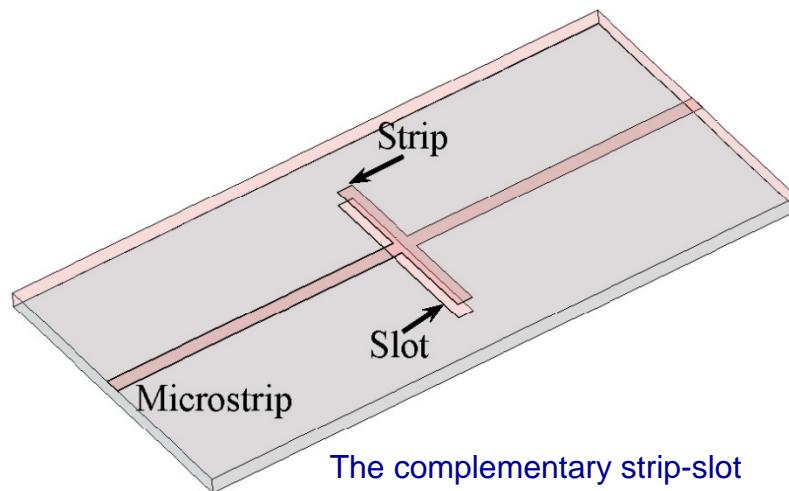
$$\boxed{Z_s = Z_{11} - Z_{12}}$$
$$Y_x = Y_{11} + Y_{12}$$

<sup>(1)</sup> F.E. Terman, “Radio Engineers’ Handbook”, McGraw-Hill, 1943.

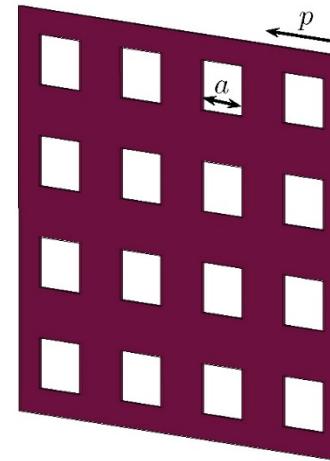
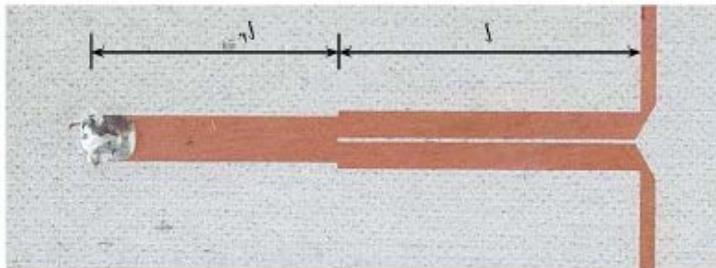
## Symmetric two-ports (3): modelling examples<sup>(2)</sup>



The stepped-impedance hair-pin resonator



The complementary strip-slot



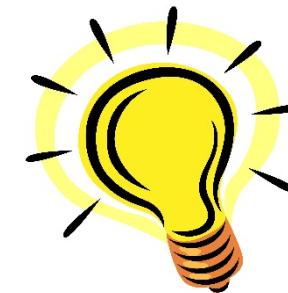
Frequency-selective surface

<sup>(2)</sup> E. Abdo-Sánchez *et al.*, "Circuit Modelling of Electromagnetic Structures by Means of the Lattice Network," Proc. XI EIEC, 93-96, 2016.

# Symmetric two-ports (4): conclusion

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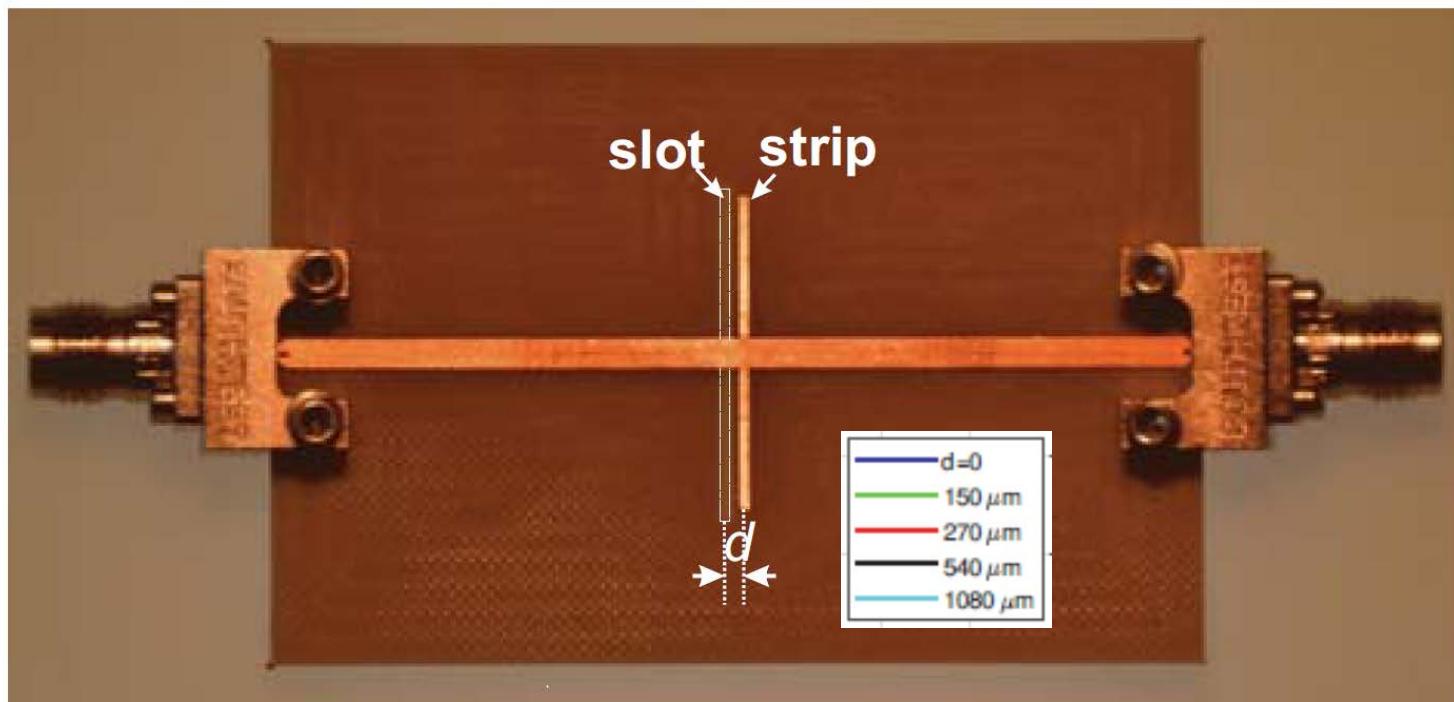
- The lattice network provides equivalent circuits with realizable elements for symmetric and reciprocal lossy electromagnetic structures:
  - ✓ Realizability
  - ✓ Orthogonal-mode decomposition
  - ✓ Deep physical insight
- Lattice network topology for non-symmetric reciprocal lossy electromagnetic structures?



# Aim of this contribution

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- To present a short description of three different approaches to the equivalent circuit modelling of non-symmetric reciprocal lossy electromagnetic structures and
- to compare their performances by modelling a misaligned complementary strip-slot element



# Classic approach: Felsen-Oliner equivalent circuit (1)

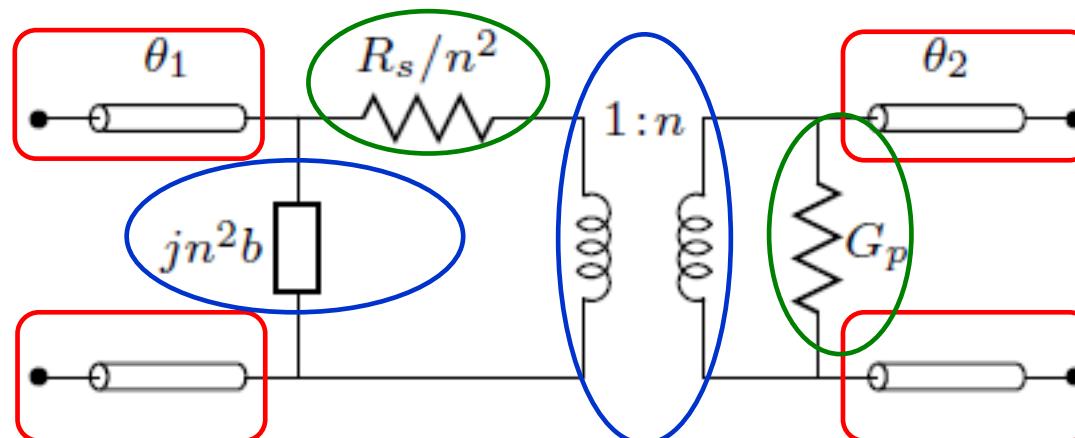
## Determination of Equivalent Circuit Parameters for Dissipative Microwave Structures\*

L. B. FELSEN†, ASSOCIATE, IRE, AND A. A. OLINER†, SENIOR MEMBER, IRE

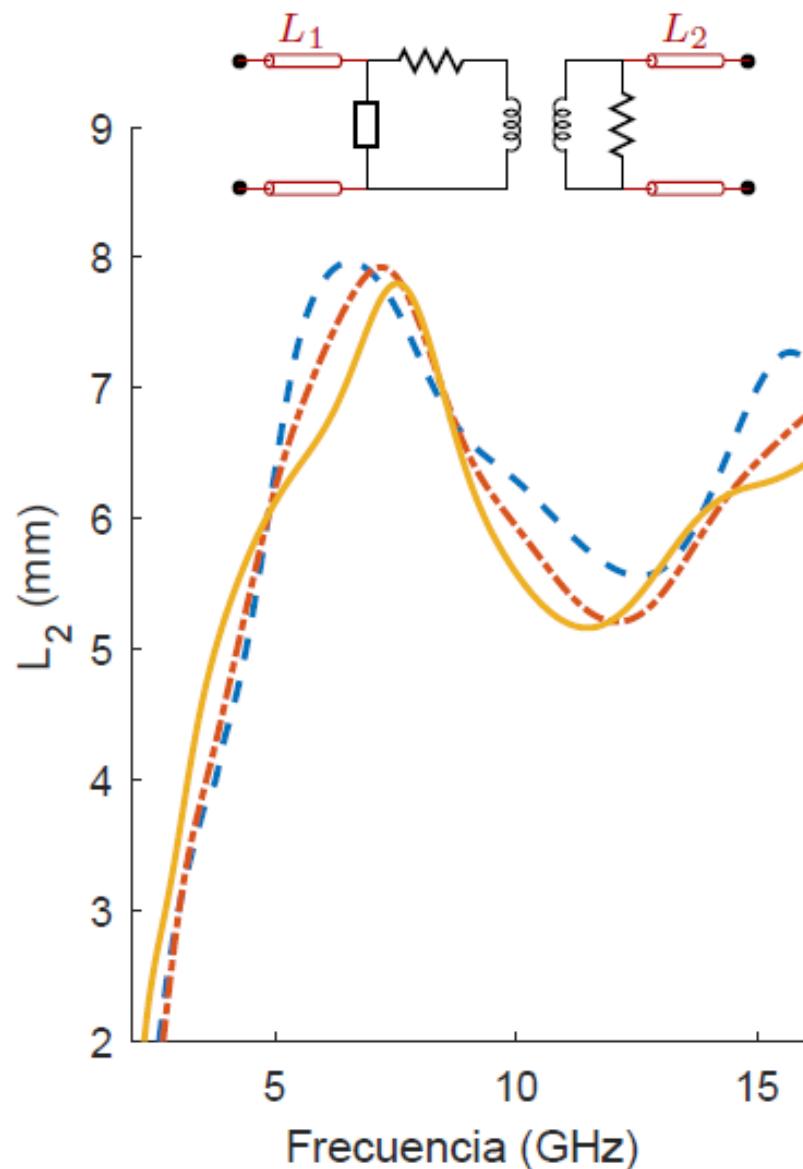
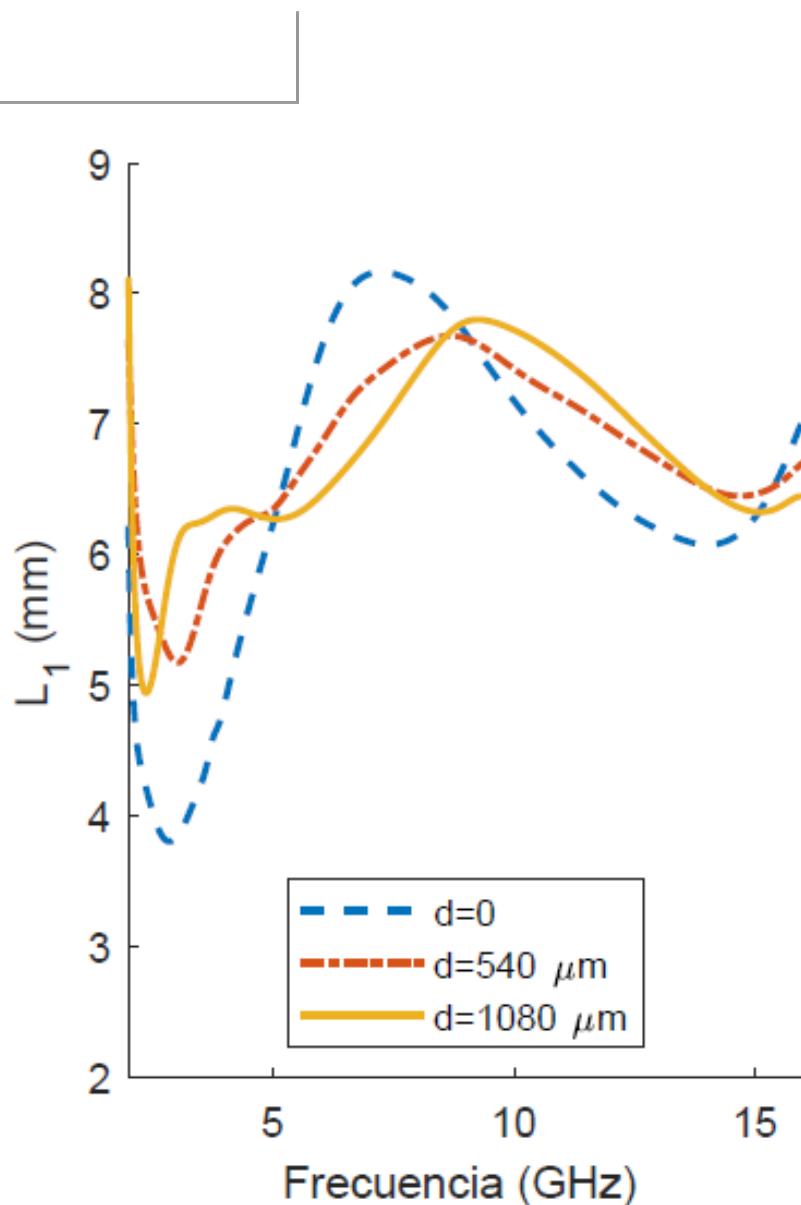
PROCEEDINGS OF THE I-R-E

February 1954

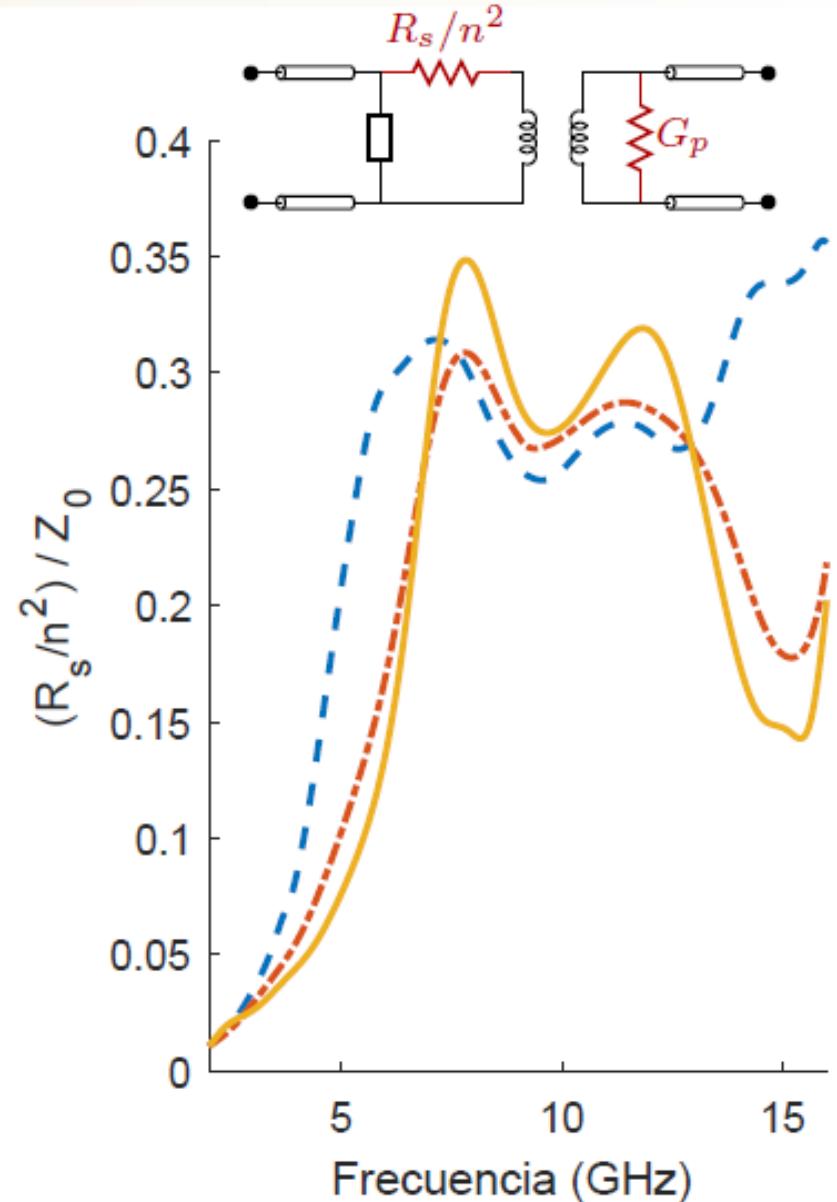
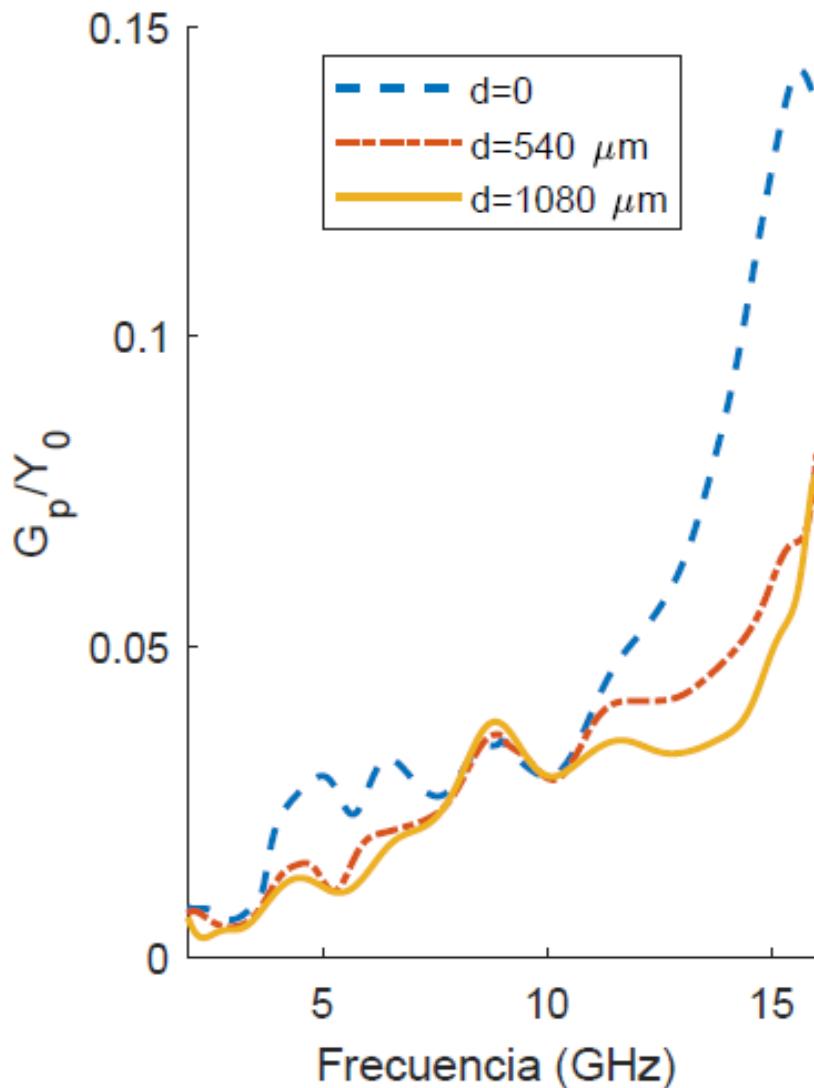
- Procedure:
  - ✓ Reference plane shift
  - ✓ Extraction of positive resistance and conductance
  - ✓ Modelling of a lossless two-port



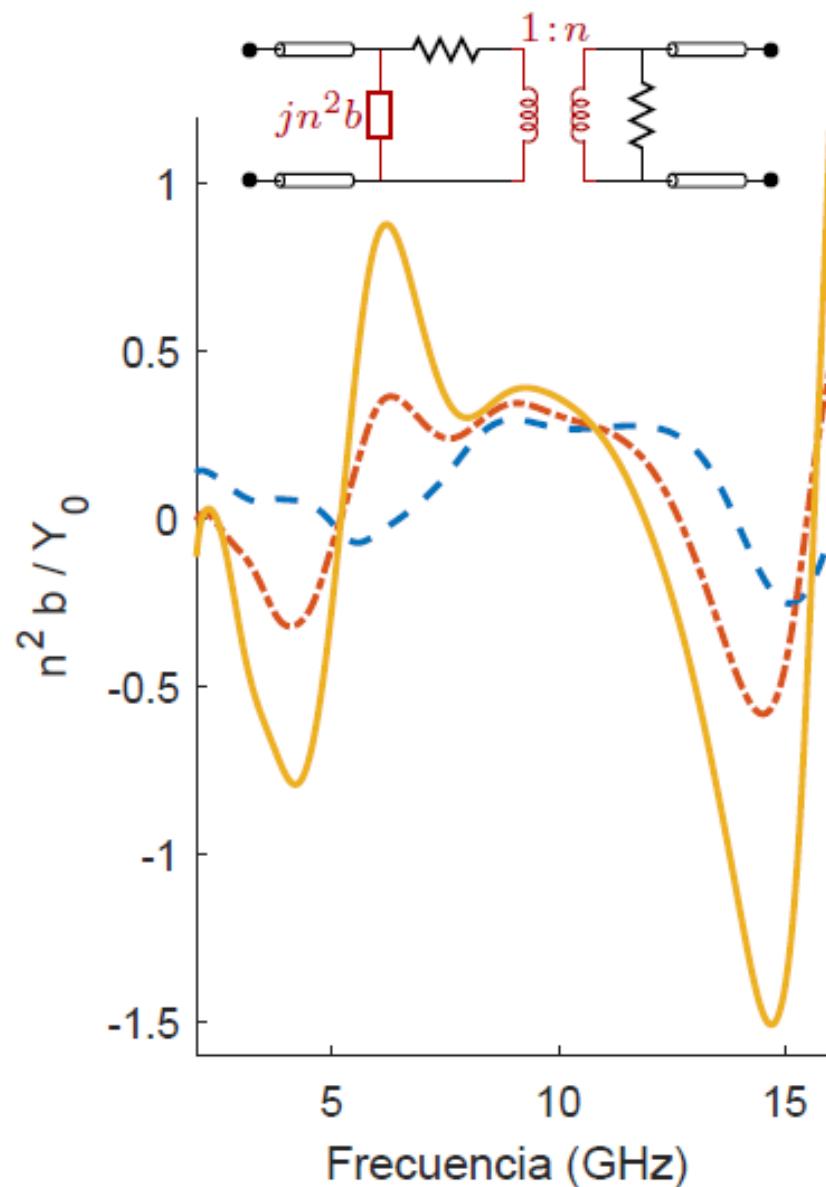
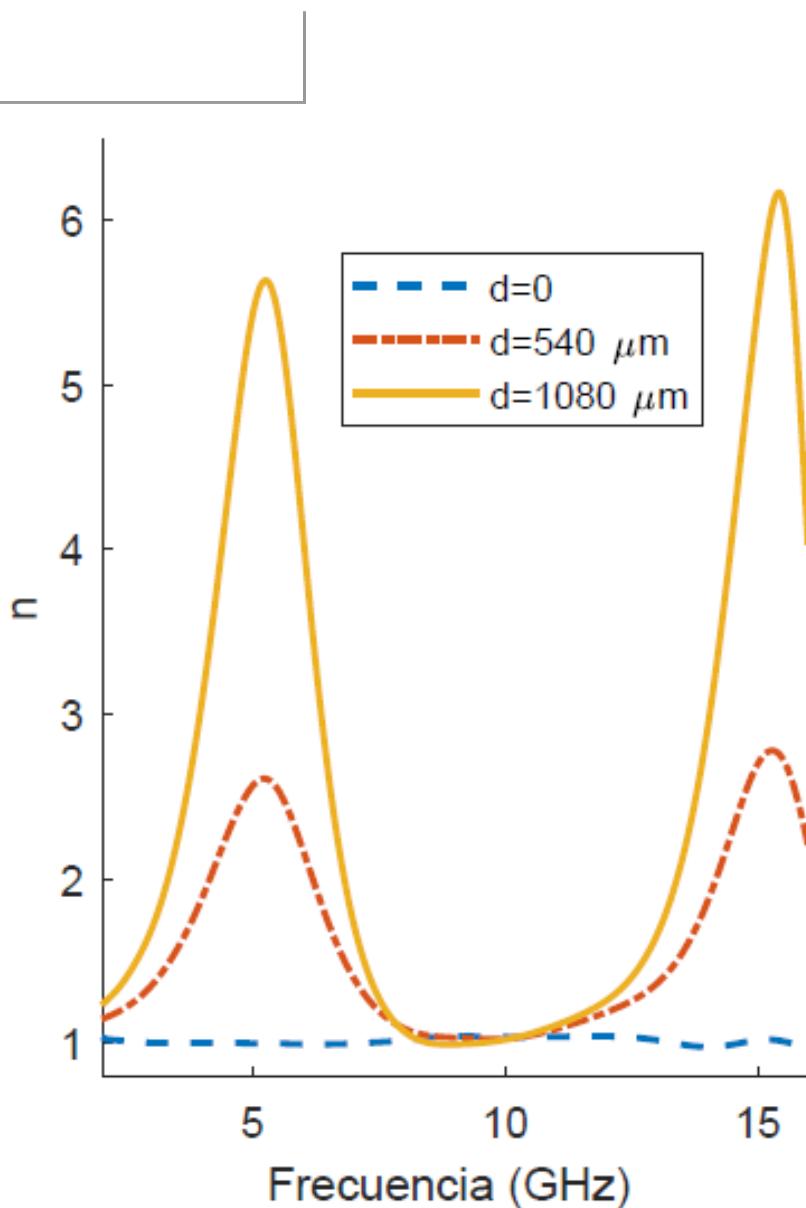
# Classic approach: Felsen-Oliner equivalent circuit (2)



## Classic approach: Felsen-Oliner equivalent circuit (3)



# Classic approach: Felsen-Oliner equivalent circuit (4)



# Felsen-Oliner equivalent circuit (5): conclusions

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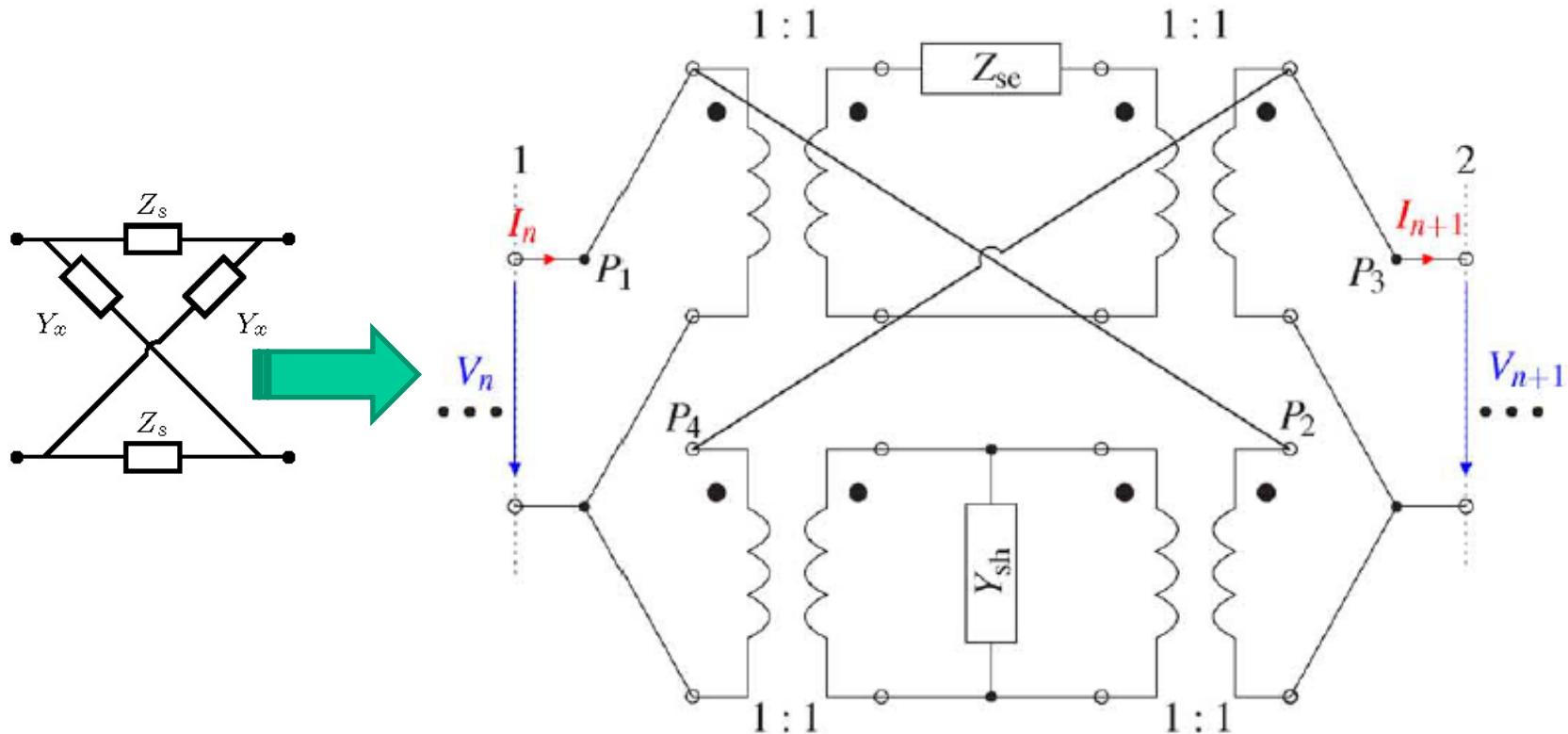
- Positive resistances and conductances with some physical meaning 
- Strong variation of the transformer ratio with the degree of asymmetry 
- Non-realizable susceptance 
- It does not converge to the lattice network in the symmetric case: realizability not guaranteed 

# Transformer-lattice equivalent circuit. TEC (1)

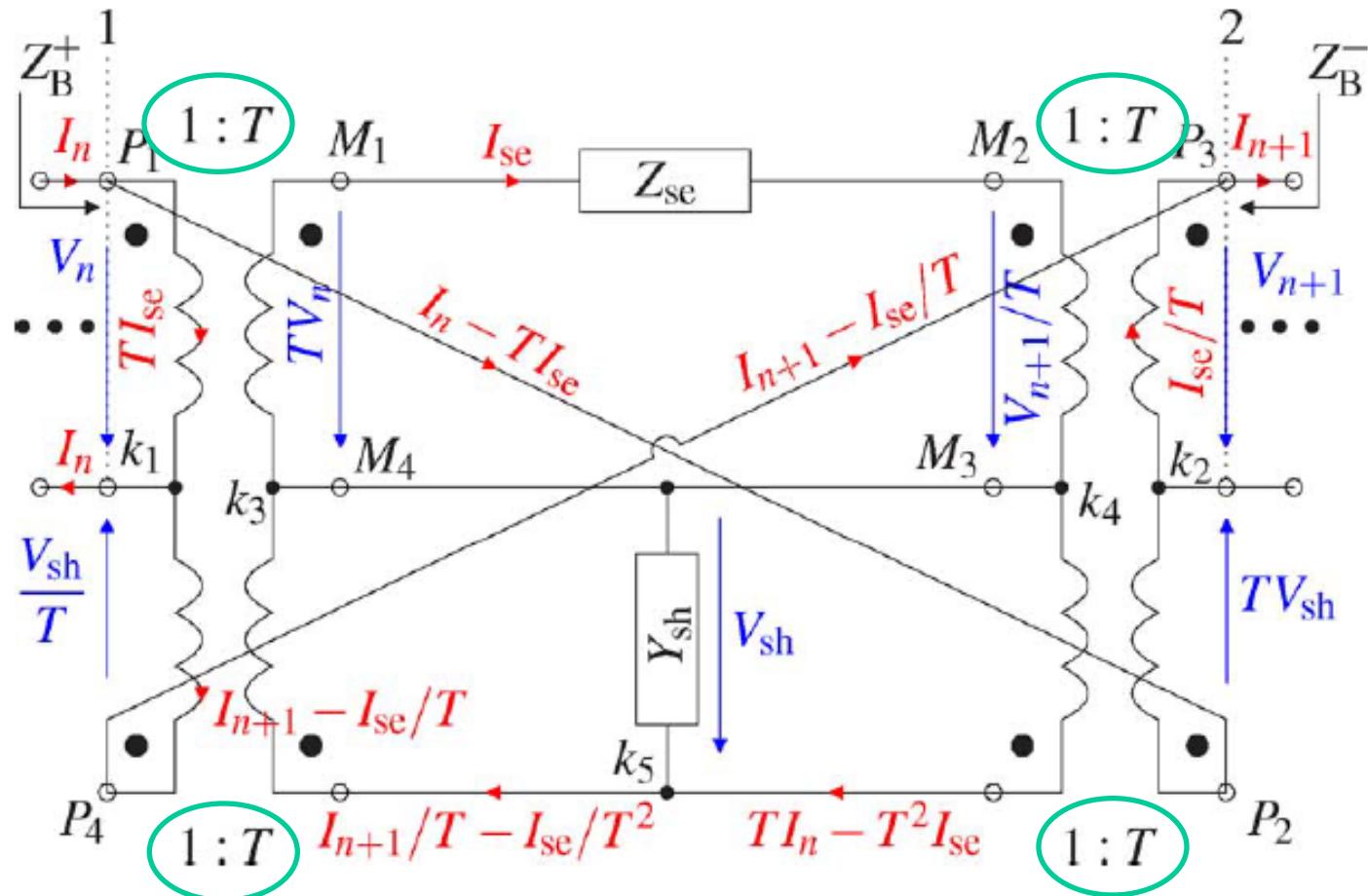
## Transversal Asymmetry in Periodic Leaky-Wave Antennas for Bloch Impedance and Radiation Efficiency Equalization Through Broadside

Simon Otto, *Member, IEEE*, Amar Al-Bassam, Andreas Rennings, *Member, IEEE*, Klaus Solbach, and Christophe Caloz, *Fellow, IEEE*

IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, VOL. 62, NO. 10, OCTOBER 2014



# Transformer-lattice equivalent circuit. TEC (2)



$$T = \sqrt{\frac{y_{21} - y_{11}}{y_{21} - y_{22}}} = \sqrt{\frac{z_{21} + z_{11}}{z_{21} + z_{22}}}$$

$$Z_{se} = \frac{y_{11} + y_{22} - 2y_{21}}{(y_{21} - y_{11})(y_{21} - y_{22})}$$

$$Y_{sh} = \frac{z_{11} + z_{22} + 2z_{21}}{(z_{21} + z_{11})(z_{21} + z_{22})}$$

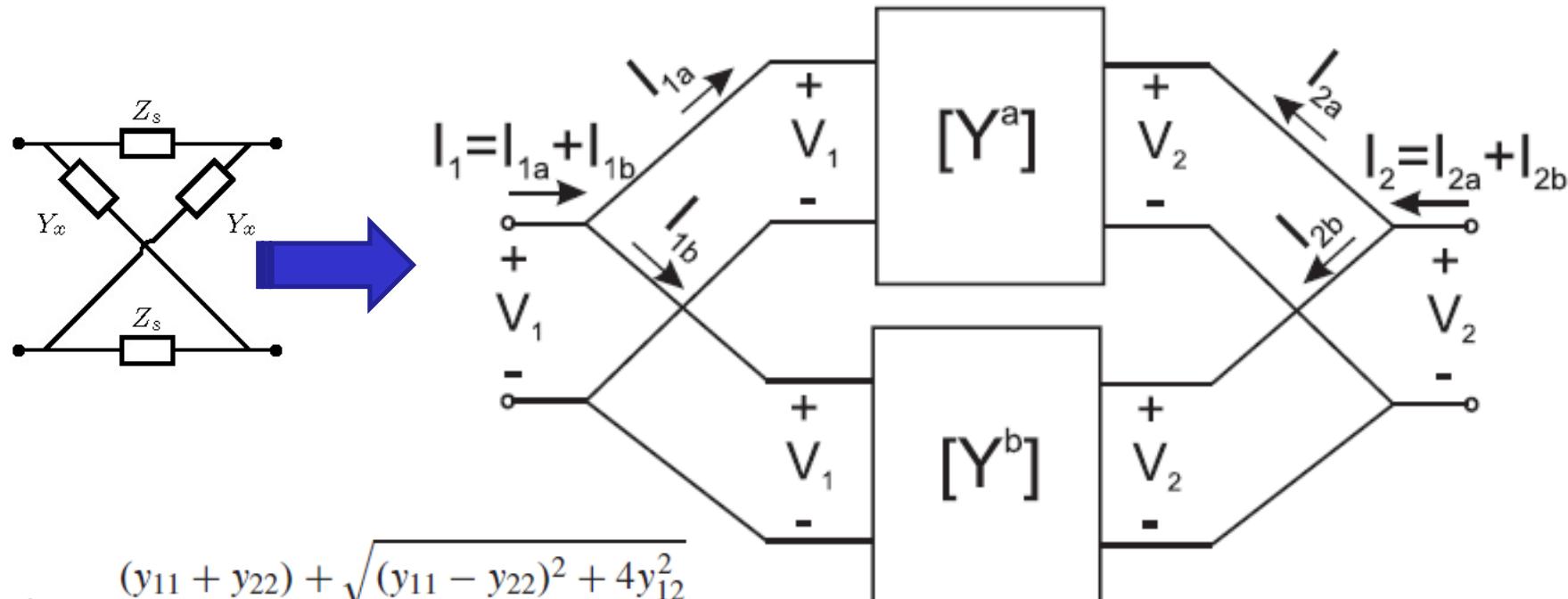
# Eigenstate lattice equivalent circuit EEC (1)

## Equivalent Circuits for Nonsymmetric Reciprocal Two Ports Based on Eigenstate Formulation

Elena Abdo-Sánchez, *Member, IEEE*, Carlos Camacho-Peña, *Senior Member, IEEE*,

Teresa M. Martín-Guerrero, and Jaime Esteban

IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, VOL. 65, NO. 12, DECEMBER 2017



$$\lambda_1 = \frac{(y_{11} + y_{22}) + \sqrt{(y_{11} - y_{22})^2 + 4y_{12}^2}}{2}$$

$$\lambda_2 = \frac{(y_{11} + y_{22}) - \sqrt{(y_{11} - y_{22})^2 + 4y_{12}^2}}{2}$$

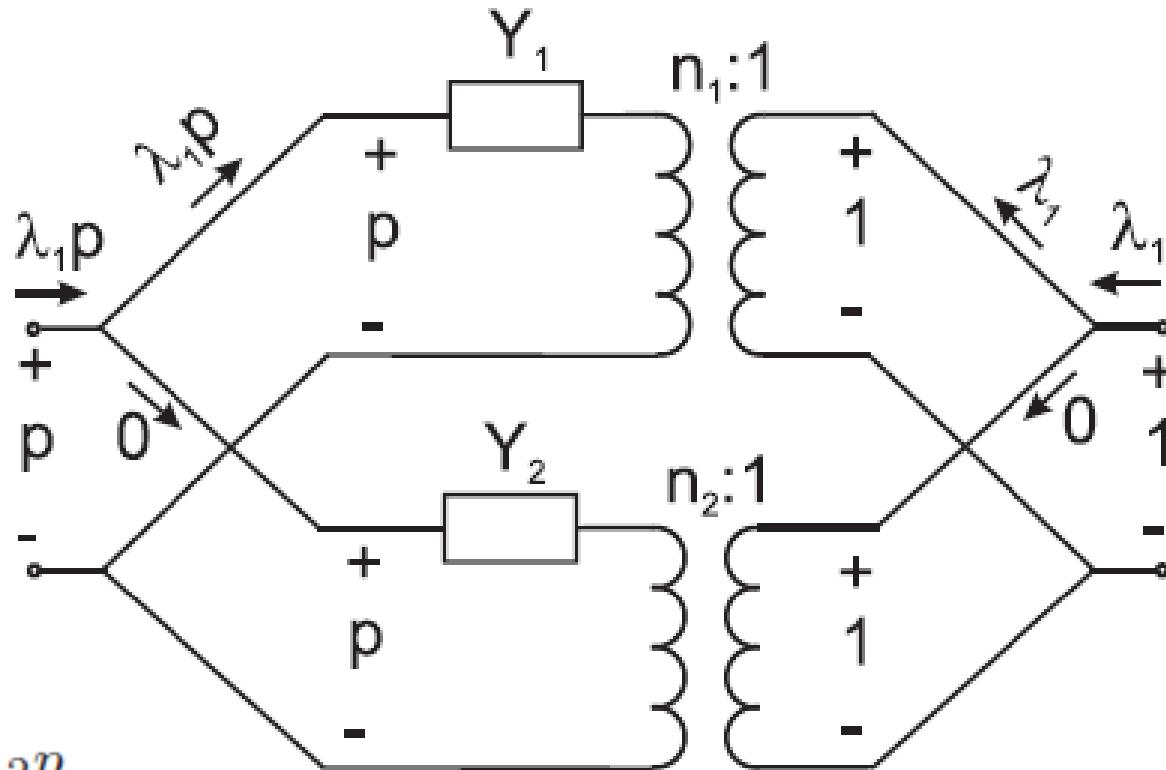
## Eigenstate lattice equivalent circuit. EEC (2)

$$n_1 = \frac{-1}{p}$$

$$n_2 = p$$

$$Y_1 = p^2 \frac{y_{22} + y_{12}p}{1 + p^2}$$

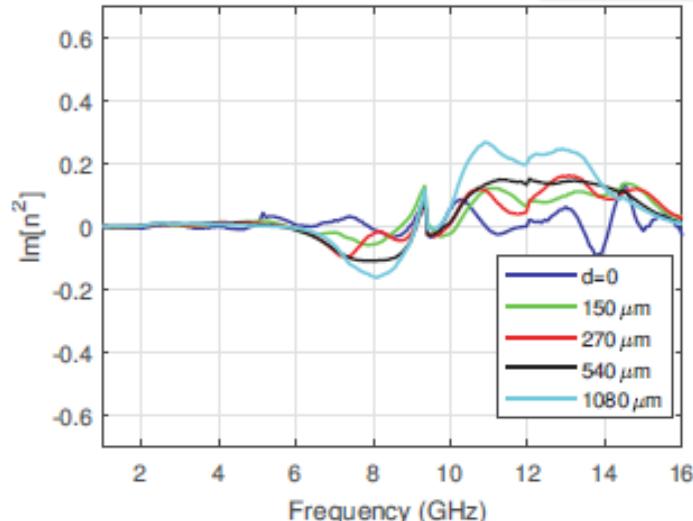
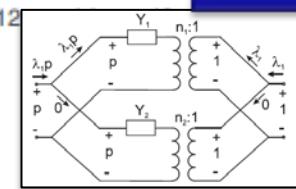
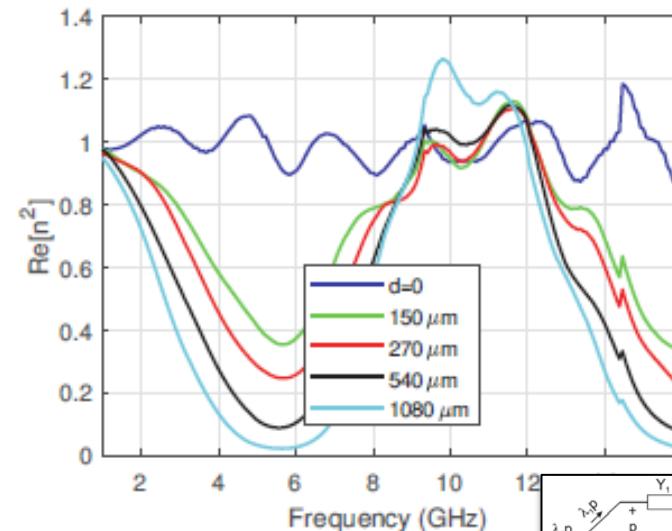
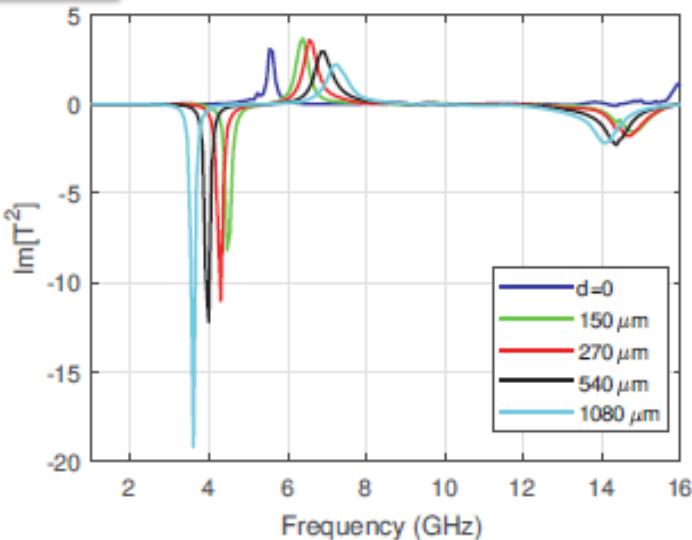
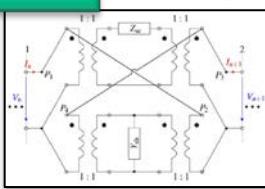
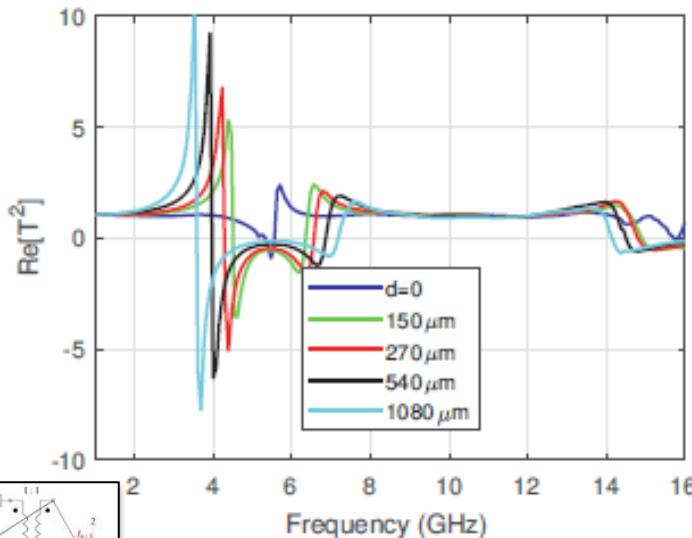
$$Y_2 = \frac{py_{22} - y_{12}}{p(1 + p^2)}$$



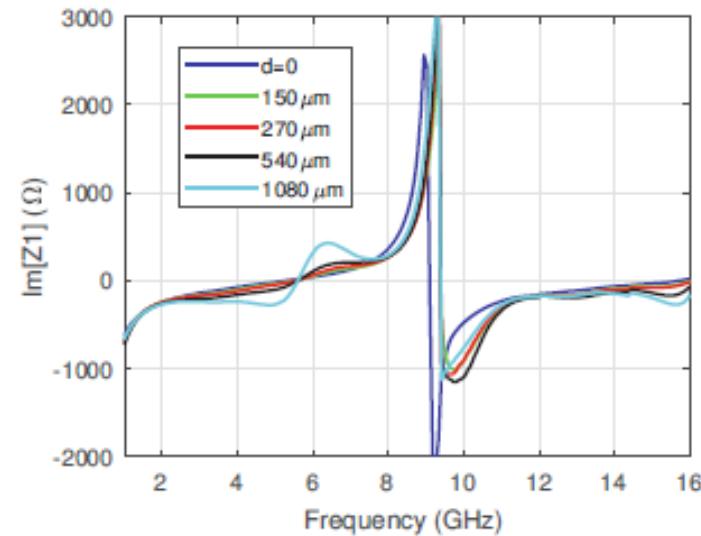
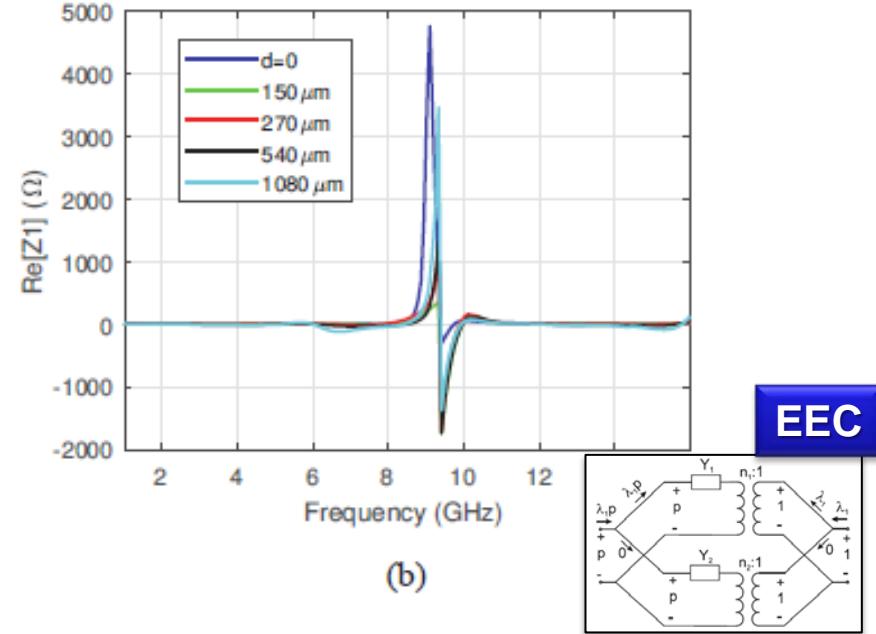
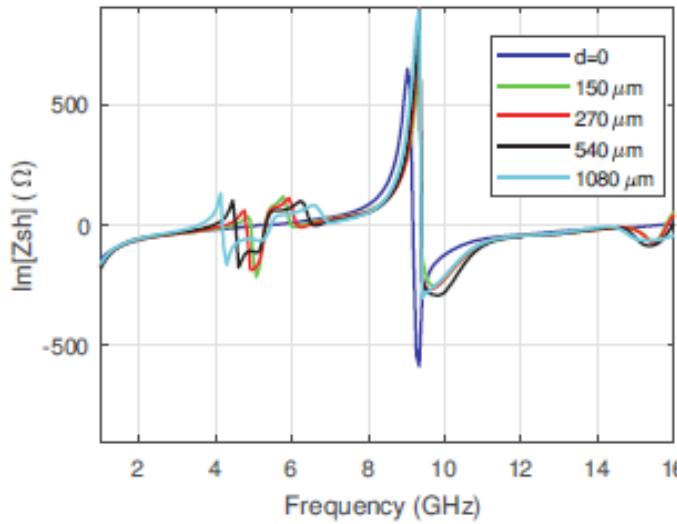
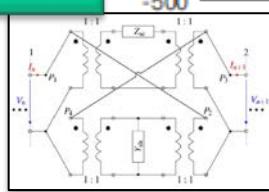
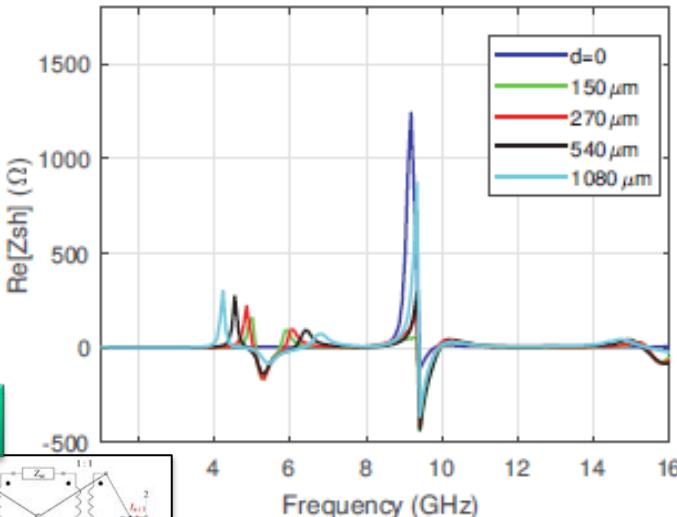
$$p = \frac{(y_{11} - y_{22}) + \sqrt{(y_{11} - y_{22})^2 + 4y_{12}^2}}{2y_{12}}$$

Reference plane shift → real transformer ratios and power orthogonality

# Lattice-inspired equivalent circuits: results (1)

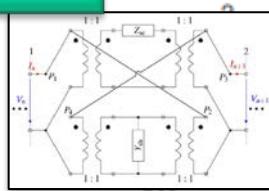


# Lattice-inspired equivalent circuits: results (2)

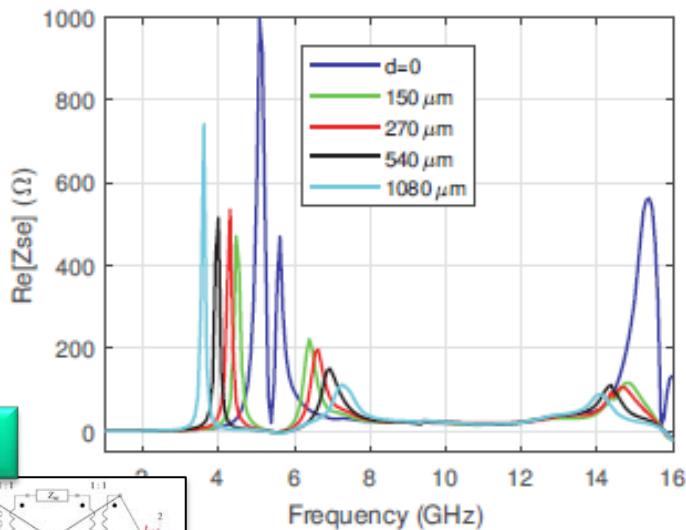


# Lattice-inspired equivalent circuits: results (3)

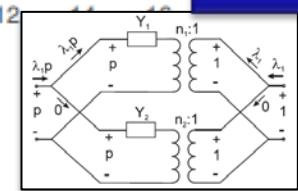
TEC



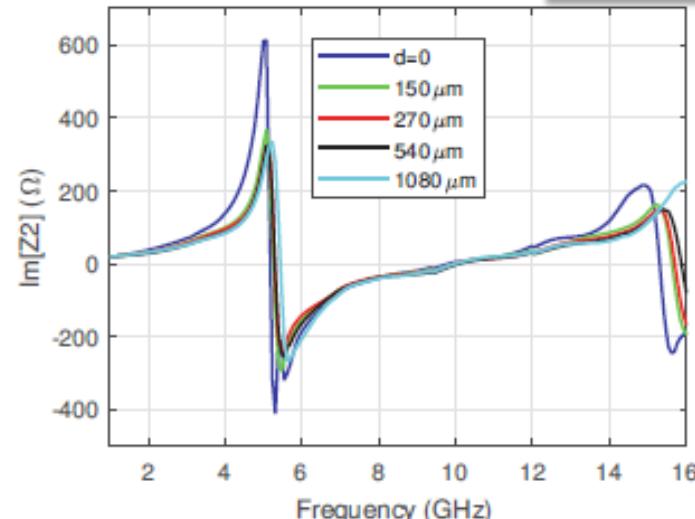
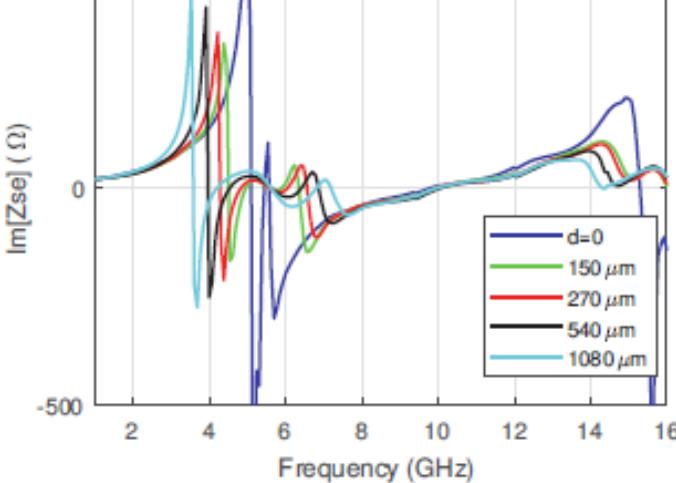
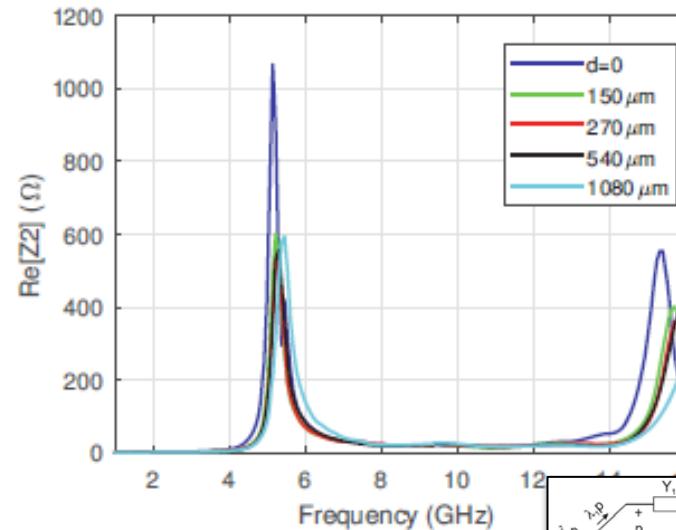
(c)



EEC



(c)



# Lattice-inspired equivalent circuits: conclusions

- Both equivalent circuits converge to the lattice network in the symmetric case
- Better performance of the eigenstate-based network:
  - Asymmetry mainly absorbed by the transformer
  - Impedances practically the same as for the symmetric case
  - Better modelling of the underlying physics of the structure
- Further comparison using different structures



Does a network topology exist that guarantees the realizability of the elements in the case of non-symmetric reciprocal lossy electromagnetic structures?

# Acknowledgment

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- European Union's Horizon 2020 Research and Innovation Programme under the Marie Skłodowska-Curie Grant Agreement No. 706334
  
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**THANK YOU FOR YOUR ATTENTION!**