Correlations in the magnitude of heartbeat increments as a measure of nonlinearity

Pedro A. Bernaola-Galván

Dpt. of Applied Physics II

University of Málaga (Spain)

Colaborators

Dpt. of Applied Physics II. University of Málaga

Pedro J. Carpena
Manuel Gómez-Extremera

Dpt. of Physical Education. University of Málaga

A. Ramón Romance-García
Javier Benítez-Porres

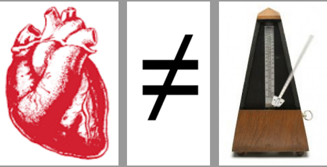
EADE University of Wales Trinity Saint David (Málaga)

Salvador Vargas

The talk

- We propose a measure nonlinearity for time series based on the analysis of the autocorrelation of the magnitude of the series
- We apply it to series of interbeat intervals (*RR*-intervals) recorded during rest and exercise

Why nonlinearity of heartbeat time series?



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- In Physics $1/f \Rightarrow$ non equilibrium, complexity, fractals, etc.

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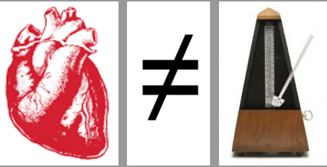

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- Ivanov et al. (1999): heartrate is **multifractal** $\Rightarrow$ **nonlinear**

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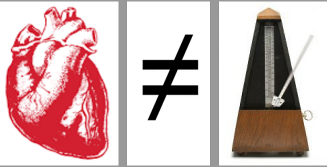

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Lack of nonlinearity $\Rightarrow$ **PROBLEMS**

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- **Nonlinear time series**
  - Generated by nonlinear dynamical equations
  - There exist correlations beyond the autocorrelation function (i.e. beyond lineal correlations)

$$C_x(\ell) = \frac{\langle x_i \cdot x_{i+\ell}\rangle - \langle x_i\rangle\langle x_{i+\ell}\rangle}{\sigma_x^2}$$

  - Multifractality
  - Non-random Fourier phases (Schreiber & Schmitz, 2000)
  - Correlations in the magnitude series

T. Schreiber & A. Schmitz: Surrogate time series. Physica D 142, 346382 (2000)

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Correlations in the magnitude

- Given a time series $\{y_i\}$, $i = 1, ..., N$ its magnitude series (also called **volatility**) is given by:

$$|x_i| = |y_{i+1} - y_i|$$

- Correlations in $|x_i|$ $\rightarrow$ **Related to nonlinearity**
- The decomposition into magnitude and sign has **Physiological meaning** (e.g. heartbeat fluctuations)
- Such correlations are quantified using DFA (Detrended Fluctuation Análysis)

Y. Ashkenazy, et al.: Magnitude and Sign Correlations in Heartbeat Fluctuations. Phys. Rev. Lett. 86, 1900-1903 (2001).

Detrended Fluctuation Analysis

- Indirect measure of correlations (actually it measures fluctuations)
- Smooths out the noise in the autocorrelation function. Is it always good?
- Only when autocorrelation function is a power law the results can be properly interpreted
- Even having power laws **there are problems with correlations in the magnitude** (Carpena et al. 2017)

We propose here a direct study of the autocorrelation function of the magnitude

P. Carpena et al.: Spurious Results of Fluctuation Analysis Techniques in Magnitude and Sign Correlations. Entropy 19(6), 261 (2017)

Model for linearity ⇒ **Linear Gaussian Noise**

- Let $\{x_i\}$ be a series of $\mathcal{N}(0,1)$ random variables with only linear correlations

Model for linearity ⇒ **Linear Gaussian Noise**

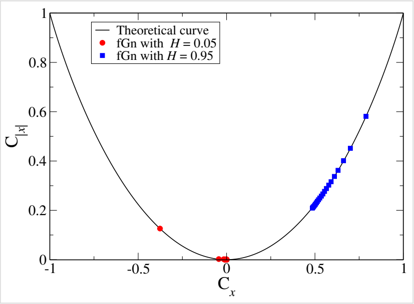
- Let $\{x_i\}$ be a series of $\mathcal{N}(0,1)$ random variables with only linear correlations
- If we denote by $C_x(\ell)$ its autocorrelation function at distance ℓ, the autocorrelation function of its magnitudes, $C_{|x|}(\ell)$, is given by:

$$C_{|x|} = \frac{2\left[C_x \arcsin C_x - 1 + \sqrt{1 - C_x^2}\right]}{\pi - 2}$$

- $C_{|x|}(\ell) \geq 0$ y $C_{|x|}(\ell) = 0 \Leftrightarrow C_x(\ell) = 0$
- For small values of C_x, we have: $C_{|x|} = \frac{1}{\pi-2} C_x^2 + \mathcal{O}(C_x^4)$

M. Gómez-Extremera et al.: Correlations in magnitude series to assess nonlinearities: Application to multifractal models and heartbeat fluctuations. Physical Review E **96**(3) 032218 (2017)

Examples of $C_{|x|}$ vs. C_x for Gaussian Noises



- The magnitude of a linear noise **can be correlated**

$$C_{|x|} \neq 0 \;\not\Longrightarrow\; \text{Nonlinearity}$$

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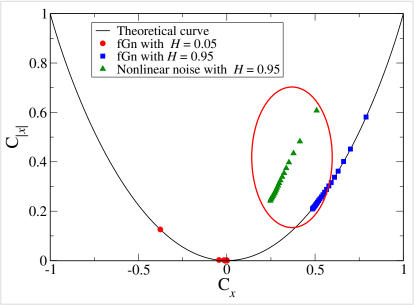

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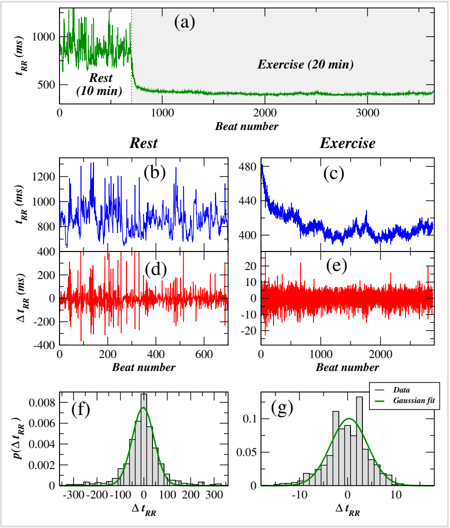
- **The deviation from the theoretical curve can be a measure of nonlinearity**

Heartrate during exercise



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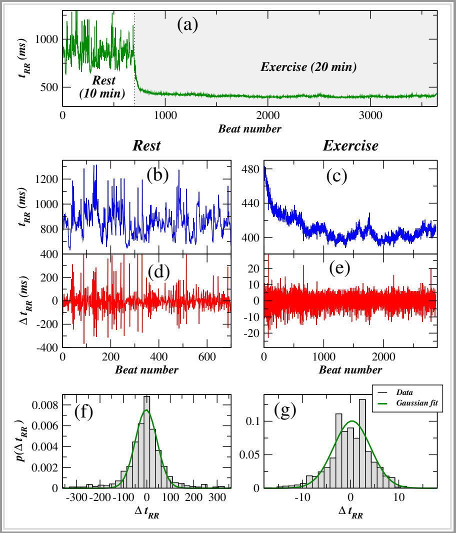

- Heartrate increases and heartrate variability is reduced
- Power spectrum is reduced, specially at low frequencies (respiration rate dominates)
- *Sample entropy* is reduced
- Short-range correlations are reduced (Not clear)
- Multifractal spectrum disappears (?)

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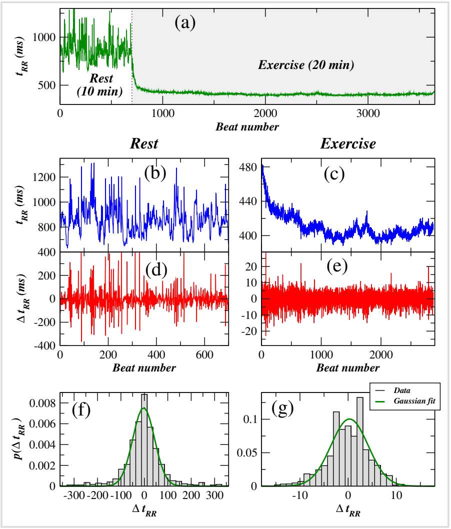

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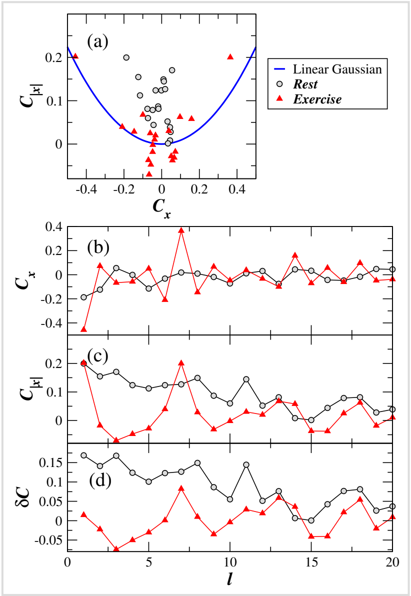
In general:
Complexity is reduced



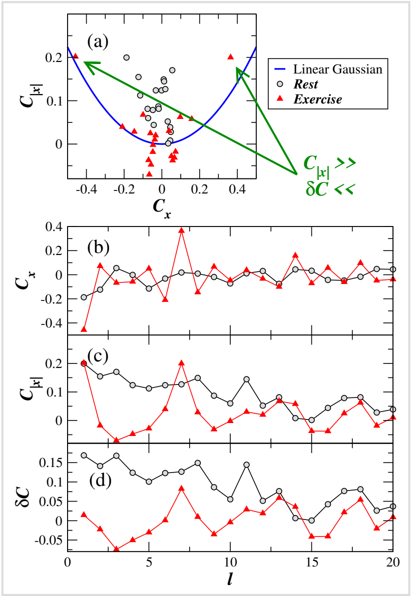
¿What about nonlinearity?

- More lineal ⇒ less complex
- Measure of nonlinearity: **deviation from linear Gaussian expectation**

$$\Delta = \sum_{\ell=1}^{\ell_{\max}} \delta C(\ell)^2$$

where:

$$\delta C(\ell) = C_{|x|}(\ell) - C_{|x|,\mathrm{linear}}\left[C_x(\ell)\right]$$



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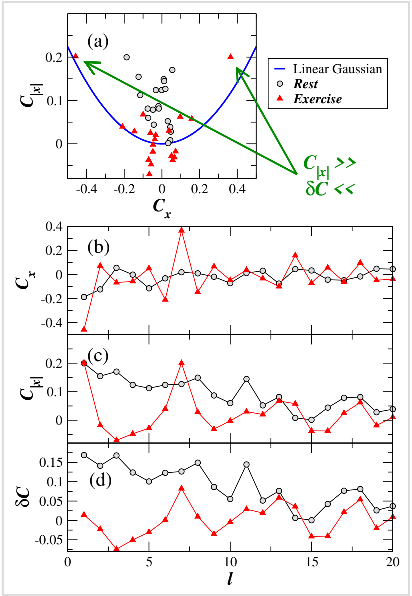

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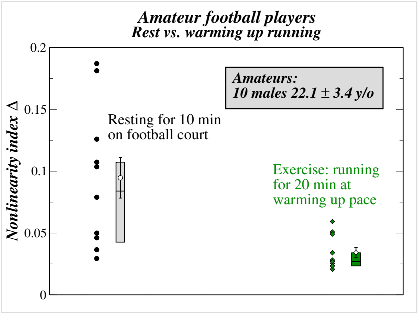
There's no assumption of scaling or fractality in the autocorrelation function

Rest vs. exercise for football players



- We choose $\ell_{\max} = 10$ beats
- Prior to the analysis data is converted into Gaussian
- For all subjects Δ is greater for rest, this is also true for group average.

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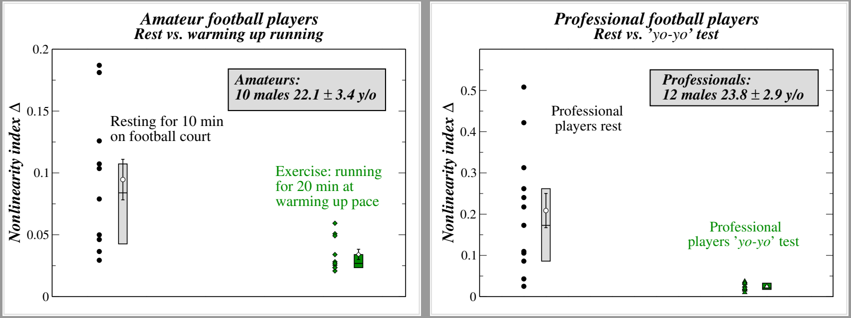

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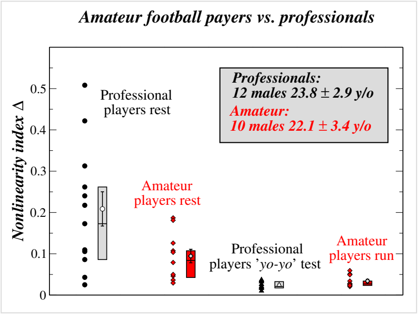

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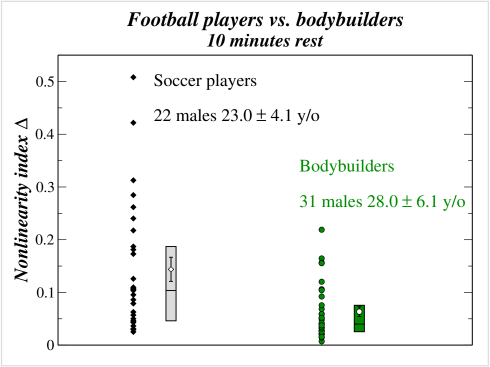
- **Higher nonlinearity during REST for professional football players** (statistically significant $p = 0.047$)

Permanent effects of exercise on HR nonlinearity (cardio vs. strength training)



- Higher nonlinearity during rest for football players (statistically significant $p = 7.6 \times 10^{-4}$)

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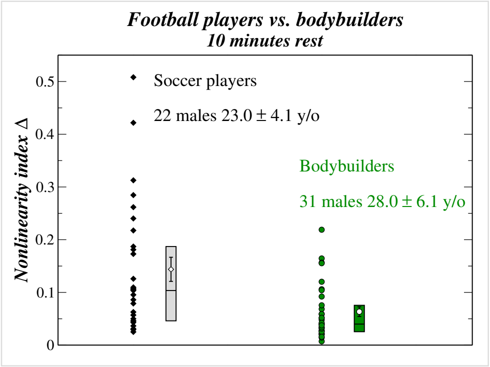

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- **Is aerobic training better for the heart?**

Thank you for your attention