GRADINGS ON LIE ALGEBRAS: MAIN RESULTS AND TOOLS

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Given G an abelian group and L a Lie algebra over a field $\mathbb{F}$, a G-grading on L is a partition into subspaces $L = \oplus_{g\in G} L_g$ such that $L_g L_h \subset L_{g+h}$ for all $g, h \in G$. Gradings on Lie algebras have been extensively studied because of their applications as well as their own interest. Among their applications we can mention symmetric spaces, related to the $\mathbb{Z}_2$-gradings, loop algebras, in connection to $\mathbb{Z}_m$-gradings, the theory of deformations or graded contractions, Lie colour algebras and so on. Gradings over $\mathbb{Z}$ turn out to appear everywhere in Algebra and Geometry. The root decomposition of a complex semisimple Lie algebra is a $\mathbb{Z}^{\text{rank } L}$-grading that has had a deep influence in structure theory of Lie algebras, in representation theory and even in particle physics (particles living in root spaces).

This well-known root decomposition is the main example of a *fine* grading. There has been an intense mathematical activity trying to determine the remaining fine gradings (that is, gradings which cannot be further refined) on simple Lie algebras. The reason for restricting only to fine gradings is that any other grading can be obtained by joining pieces of the fine ones. Most of the results concerning this topic, obtained by many authors, are compiled in [4], which is the main reference. Also, the review [2] offers a panoramic view of the main ideas used in the classification of fine gradings on the finite dimensional simple Lie algebras over an algebraically closed field $\mathbb{F}$ of characteristic zero, and tries to provide a description of the classification. But, while [2] deals with the classical case, the paper [1] revises the exceptional one, trying to describe more than to prove. The exceptional case has been difficult to attack, being the arguments completed recently (see [3, 5]), with techniques of different branches of Mathematics.

The aim of this talk cannot be to provide an explicit and complete classification of the gradings on simple finite-dimensional Lie algebras, complex and real, simply because the topic is too wide. Thus, we have selected some of the keys of the classification up to equivalence, with emphasis on the complex field case.

REFERENCES

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